Axion Induced Oscillating Electric Dipole Moments

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A space-time filling coherent oscillating axion field will cause any magnetic source or field to become an electric source or field through the anomalous coupling of the axion to electromagnetic fields.

The electron will develop an oscillating electric dipole moment of strength $2g_{a\gamma\gamma}\theta_0\mu_{Bohr}\overrightarrow{s}$ where $a/f = \theta_0 \cos(m_a t)$.

Axions:

The axion is a pNGB associated with the spontaneous breaking of Peccei-Quinn symmetry.

Typically the PQ symmetry breaks at a high scale f_a

At the QCD scale, instantons activate the U(1) axial current anomaly.

The axion acquires a potential and forms a VEV which cancels the QCD CP-violating phase $\theta_{\rm QCD}$

.

Small oscillations about this minimum are associated with the axion mass and can constitute dark matter.

Axions

The axion kinetic terms and potential take the form:

$$S_a = \int d^4x \;\; \tfrac{1}{2}(\partial_t a)^2 - \tfrac{1}{2}(\nabla a)^2 - \tfrac{1}{2} m_a^2 a^2$$

The axion mass is controlled by instanton effects that lead to mixing with the pseudoscalar nonet of mesons. The axion mass is then given by:

$$m_a^2 f_a^2 = \; c m_\pi^2 f_\pi^2$$

The prefactor, c, is $c = \frac{\sqrt{z}}{1+z}$ where $z = \frac{m_u}{m_d} \approx 0.5$ and vanishes as m_u or $m_d \to 0$

QCD: $m_a = m_\pi f_\pi / f_a \simeq 0.6 \times 10^{-14} \times (10^{12}/f_a)$ GeV.

$$\left(0.6 \times 10^{-14} \times (10^{12}/f_a)\right)^{-1} \times (0.2 \times 10^{-13}) = 3.3 \; (f_a/10^{12}) \text{ cm}$$

Axions

The axion is actually an "angular variable" in the effective action on scales much less than f_a

It is useful to write axion expressions in terms of the angle variable $\theta(x,t) = a(x,t)/f_a$.

The axion kinetic term can be written:

$$S_a = \int d^4x \;\; \frac{f_a^2}{2}(\partial_t\theta(x,t))^2 - \frac{f_a^2}{2}(\nabla\theta(x,t))^2 - \frac{1}{2}f_a^2 m_a^2 \theta^2(x,t)$$

Axions

Assume a cosmic axion field: $\theta(x,t) \approx \theta_0 \cos(m_a t)$

The axion energy density is:

$$E_a = \frac{f_a^2}{2}(\partial_t \theta(x,t))^2 + \frac{1}{2} f_a^2 m_a^2 \theta^2(x,t) = \frac{1}{2} f_a^2 m_a^2 \theta_0^2$$

Equate this to the galactic halo dark matter density:

$$\frac{1}{2} \frac{z}{(1+z)^2} (m_\pi f_\pi)^2 \theta_0^2 = 0.3 \text{ GeV/cm}^3$$

Hence: $\theta_0 \simeq 3.6 \times 10^{-19}$ independent of f_a (!)

Axionic Electrodynamics

The axion couples to the electromagnetic field via the U(1) axial current anomaly:

$$\frac{1}{4} g_{a\gamma\gamma} \left(\frac{a}{f}\right) F_{\gamma\mu\nu} \widetilde{F}_{\gamma}^{\mu\nu} = -g_{a\gamma\gamma} \left(\frac{a}{f}\right) \vec{E} \cdot \vec{B}$$

Where: $\widetilde{F}_{\gamma}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\gamma\rho\sigma}$ and $g_{a\gamma\gamma}$ = anomaly coefficient.

$g_{a\gamma\gamma} \approx 8.3 \times 10^{-4}$ DFSZ

M. Dine, W. Fischler and M. Srednicki, Phys. Lett. B **104**, 199 (1981); A. R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 260 (1980) [Yad. Fiz. **31**, 497 (1980)].

$g_{a\gamma\gamma} \approx -2.3 \times 10^{-4}$ KSVZ

J. Kim, Phys. Rev. Lett. 43 (1979) 103; M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B166 (1980)493

See the PDG article.

Axionic Electrodynamics

The action for axion electrodynamics

$$S = \int d^4x \ \tfrac{1}{2}\left(\vec{E}^2 - \vec{B}^2\right) - g_{a\gamma\gamma}\theta(t)\vec{E}\cdot\vec{B}$$

Note that $g_{a\gamma\gamma}\theta(t)\vec{E}\cdot\vec{B}$ is a total divergence in the limit

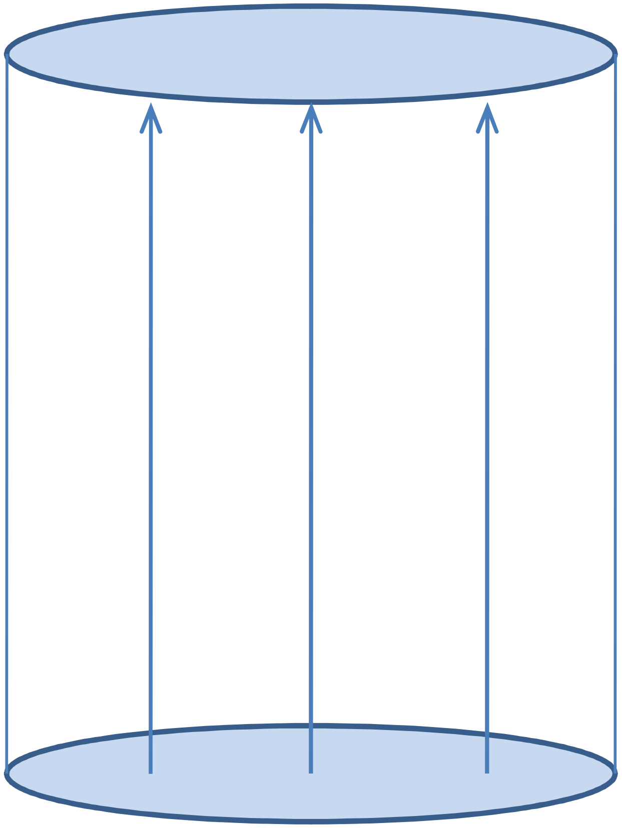
that $\theta(t) \to \text{constant}$.

The axion anomaly can we written in two ways:

$\frac{1}{8} g_{a\gamma\gamma}\left(\frac{a}{f}\right)\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$ $\qquad$ $\frac{1}{2} g_{a\gamma\gamma}\epsilon^{\mu\nu\rho\sigma}\partial_\mu\left(\frac{a}{f}\right)A_\nu\partial_\rho A_\sigma$

Gauge inv. but not chiral. $\qquad$ Chiral inv. but not gauge inv.

Axionic Electrodynamics
Example: RF Cavity Detector



Conducting cylindrical walls

Longitudinal constant applied magnetic field, B_0

Axionic Electrodynamics
Example: RF Cavity Detector

To detect an electromagnetic signature of the cosmic axion $\theta(t) = \theta_0 \cos(m_a t)$
we need a large applied background magnetic field, $\vec{B}_0$

Maxwell's equations for "response fields" $\vec{E}_r$ and $\vec{B}_r$

Maxwell (1) $\quad \vec{\nabla} \times \vec{B}_r - \partial_t \vec{E}_r = -g_{a\gamma\gamma} \vec{B}_0 (\partial_t \theta)$ <- anomaly

Maxwell (2) $\quad \vec{\nabla} \times \vec{E}_r + \partial_t \vec{B}_r = 0$

and $\vec{\nabla} \cdot \vec{B}_r = \vec{\nabla} \cdot \vec{E}_r = 0.$

The vector potential in Coulomb gauge likewise satisfies: $\partial_t^2 \vec{A}_r - \vec{\nabla}^2 \vec{A}_r = -g_{a\gamma\gamma} \vec{B}_0 (\partial_t \theta).$ where $\vec{E}_r = -\partial_t \vec{A}_r.$

Axionic Electrodynamics
Example: RF Cavity Detector

The "particular solution": $\vec{E} = g_{a\gamma\gamma}\tilde{\theta}(t)\vec{B}_0$

The homogenous solution (lowest electric mode):

$$\vec{E} = E_0 J_0(\rho\omega)\hat{z}e^{i\omega t} \qquad \vec{B} = E_0 J_1(\rho\omega) i e^{i\omega t}\,\hat{\phi}$$

Axionic Electrodynamics

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The resulting net solution:

$$\vec{B} = kJ_1(\rho m_a)B_0\,\frac{\partial_t\widetilde{\theta}}{m_a}\,\hat{\phi} \qquad \vec{E} = (kJ_0(\rho m_a) + g_{a\gamma\gamma})B_0\widetilde{\theta}\hat{z}$$

Boundary conditions: $\vec{E}(\rho = R) = 0$

Axionic Electrodynamics
Example: RF Cavity Detector

The resonant cavity solution:

$$\vec{B} = -g_{a\gamma\gamma}\frac{J_1(\rho m_a)}{J_0(Rm_a)}B_0\hat{\phi}\left(\frac{\partial_t\tilde{\theta}}{m_a}\right) \qquad \vec{E} = -g_{a\gamma\gamma}\left(\frac{J_0(\rho m_a)}{J_0(Rm_a)} - 1\right)B_0\hat{z}\tilde{\theta}$$

Note resonance at: $J_0(Rm_a) = 0$

Damping will arise from cavity wall resistance, etc., modifies the peak solution to:

$$\vec{B} = -g_{a\gamma\gamma}\frac{J_1(\rho m_a)}{F}B_0\hat{\phi}\left(\frac{\partial_t\tilde{\theta}}{m_a}\right) \qquad \vec{E} = -g_{a\gamma\gamma}\left(\frac{J_0(\rho m_a)}{F} - 1\right)B_0\hat{z}\tilde{\theta}$$

$$F = \sqrt{(J_0(Rm_a))^2 + c/Q^2}$$

Axionic Electrodynamics
Example: RF Cavity Detector

A signal can be extracted from the cavity by inserting a small loop of wire that is threaded by the flux of the response magentic field, or equivalently, encircled by the electric field. The voltage induced in a typical loop of length L:

$$V = \oint \vec{E} \cdot d\vec{l} = -\int \partial_t \vec{B} \cdot d(\overrightarrow{\text{area}}) \sim -\frac{g_{a\gamma\gamma} B_0 L}{F}\theta$$

Quantitatively, for $B_0 = 1$ Tesla, $g_{a\gamma\gamma} = 10^{-3}$, and $L = 10$ cm, $\widehat{\theta}_0 = 3.6 \times 10^{-19}$ is cosmological and independent of f_a we find that the scale of the magnetic response field is $|\vec{B}| = |g_{a\gamma\gamma} \frac{J_1(\rho m_a)}{F} B_0 \left(\frac{\partial_t \widehat{\theta}}{m_a} \right)| \sim Q g_{a\gamma\gamma} B_0 \widehat{\theta}_0$ $= 1.3 \times 10^{-17} Q$ gauss. Likewise, the induced voltage in the loop is $V \sim Q g_{a\gamma\gamma} B_0 L \theta_0$

$\sim 4.0 \times 10^{-14}\, Q$ volts. A good RF cavity with $Q \sim 10^5$ can bring these signals into a detectable range. The main issue is RF noise.

Axionic Electrodynamics
Example: RF Cavity Detector

An aside:

Q: Is there enough energy in the axion vacuum to fill an RF cavity to its maximum energy?

We can overestimate the cavity energy by assuming that it is a volume of free space with the induced oscillating electric field $\vec{E}_r = g_{a\gamma\gamma}\vec{B}_0\theta(\tau)Q$, where we assume amplification by a factor of Q. This is an overestmate, because the conducting walls excite the homogeneous solution (to match conducting boundary conditions at the wall) that tends to short out the axion signal $\vec{E}_r$ (see below).

The galactic halo energy density is $\rho_{gal} = 0.3\ \mathrm{GeV/cm}^3 =$
$(0.3)(0.2 \times 10^{-13})^3 \left(\mathrm{GeV/cm}^3\right)(\mathrm{GeV} \times \mathrm{cm})^3 = 2.4 \times 10^{-42}(\mathrm{GeV})^4$

The electromagnetic energy density in the cavity is: $\rho_{cavity} \sim \vec{E}_r^2 \sim \left(g_{a\gamma\gamma}\vec{B}_0\theta_0 Q\right)^2$

Here: $\theta_0 \simeq 3.6 \times 10^{-19}$, and $\vec{B}_0 \sim 10$ Tesla $\sim 10^5$gauss;

1 gauss =1 esu/cm^2 = $7.1 \times 10^{-20}\ \mathrm{GeV}^2$, so $\vec{B}_0 \sim 7.1 \times 10^{-15}\ \mathrm{GeV}^2$;
the axion anomaly coupling is $g_{a\gamma\gamma} \sim 10^{-3}$; We assume $Q = 10^6$ (!).

Thus, $\vec{E}_r^2 \sim \left(g_{a\gamma\gamma}\vec{B}_0\theta_0 Q\right)^2 \sim (10^{-3} \times 7.1 \times 10^{-15} \times 3.6 \times 10^{-19} \times 10^6)^2$
$\sim 6.6 \times 10^{-60}(\mathrm{GeV})^4$. Therefore ρ_{cavity}/ρ_{gal} 2.7×10^{-18} hence there is an enormouse free energy of axions in the halo that is being converted to our cavity signal energy.

Yes !!!

A Puzzle

Our solution involves $\theta(x,t) \approx \theta_0 \cos(m_a t)$

Suppose we take the limit $m_a \to 0$

Then $\theta(t) \to \text{constant}$

Does this imply $\vec{E}_r = g_{a\gamma\gamma}\vec{B}_0\theta(t) \to (\text{constant})$???

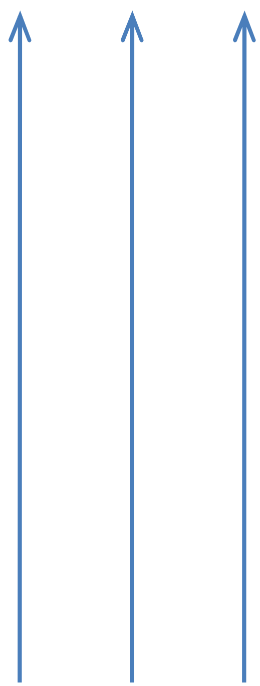
This is inconsistent with the anomaly written
In the non g.i. form: $\frac{1}{2} g_{a\gamma\gamma}\epsilon^{\mu\nu\rho\sigma}\partial_\mu\left(\frac{a}{f}\right)A_\nu\partial_\rho A_\sigma$

???

Resolution

$\vec{B}_0$

Consider and infinite Universe with background axion and magnetic field (an infinite cavity):



$$\text{Maxwell (1)} \qquad \vec{\nabla} \times \vec{B}_r - \partial_t \vec{E}_r = -g_{a\gamma\gamma} \vec{B}_0 (\partial_t \theta)$$

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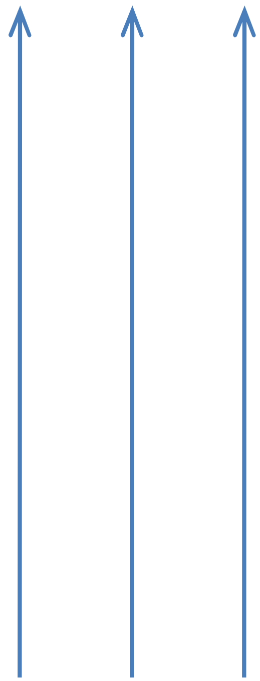

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Particular solution:

$$\vec{E}_r = g_{a\gamma\gamma} \vec{B}_0 \int_0^t d\tau \, \partial_\tau \theta(\tau) \qquad\qquad \vec{B}_r = 0$$

This "particular solution" $\vec{E}_r = g_{a\gamma\gamma}\vec{B}_0 \int_0^t d\tau\, \partial_\tau \theta(\tau)$

has the property that it vanishes as: $\partial_\tau \theta(\tau) \to 0$

(The vector potential satisfies $\vec{A}_r = g_{a\gamma\gamma}\vec{B}_0 \int_0^t d\tau' \int_0^{\tau} d\tau\, \partial_\tau \theta(\tau)$)

Therefore, whenever we write the sloppy form "$\vec{E}_r = g_{a\gamma\gamma}\vec{B}_0 \theta(t)$" we

really intend $\vec{E}_r = g_{a\gamma\gamma}\vec{B}_0 \int_0^t d\tau\, \partial_\tau \theta(\tau)$ which manifestly goes to zero as $m_a \to 0$.

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Nonetheless, our infinite universe has acquired an oscillating electric field that is parallel to the fixed magnetic field and oscillated with the axion field.

To see this more explicitly, let us consider a source term for the magnetic field added to the free Maxwell action

$$S = \int d^4x \;\; \tfrac{1}{2}\left(\vec{E}^2 - \vec{B}^2\right) + \vec{m} \cdot \vec{B}$$

The potential energy density is $V = \frac{1}{2}\vec{B}^2 - \vec{m} \cdot \vec{B}$
leading to the energy minimum $\vec{B}_0 = \vec{m}$ and therefore
the action becomes: $\rightarrow \int d^4x \;\; \frac{1}{2}\left(\vec{E}^2 - \vec{B}_0^2\right)$.

Consider the full action with the anomaly:

$$S = \int d^4x \ \tfrac{1}{2}\left(\vec{E}^2 - \vec{B}^2\right) + \vec{m} \cdot \vec{B} - g_{a\gamma\gamma}\theta(t)\vec{E} \cdot \vec{B}$$

$$\rightarrow \int d^4x \ \tfrac{1}{2}\left(\vec{E}^2 - \vec{B}_0^2\right) - g_{a\gamma\gamma}\theta(t)\vec{E} \cdot \vec{B}_0 + O(g_{a\gamma\gamma}^2\theta(t)^2)$$

We can write the anomaly in the presence of our magnetic field as an electric dipole term:

$$\rightarrow \int d^4x \ \tfrac{1}{2}\left(\vec{E}^2 - \vec{B}_0^2\right) - \vec{E} \cdot \vec{p}(t)$$

Varying the action wrt $\vec{A}$ (where $\vec{E} = -\partial_t \vec{A}$) implies

$$\partial_t \vec{E} = \partial_t \vec{p}(t) = g_{a\gamma\gamma} \partial_t \theta(t) \vec{B}.$$

Hence: $\vec{p}(t) = g_{a\gamma\gamma} \tilde{\theta}(t)\, \vec{B}_0$ (so $\vec{p}(t) \to 0$ as $m_a \to 0$).

Turning on the axion in this universe produces an electric field:

$$\vec{E}_r = g_{a\gamma\gamma} \vec{B}_0 \tilde{\theta}(t) = \vec{p}(t)$$

The effect of the axion is to produce a time dependent electric dipole.

The effect of the axion is to produce
a time dependent electric dipole.

Theorem:

The cosmic oscillating axion field will cause any magnetic dipole moment to become an oscillating electric dipole moment

Related: Magnetic monopoles acquire electric charge in presence of a nonzero θ angle, "Witten Effect."

DYONS OF CHARGE $e\theta/2\pi$

E. WITTEN [1]
CERN, Geneva, Switzerland

Received 11 August 1979

It is shown that in *CP* non-conserving theories, the electric charge of an 't Hooft–Polyakov magnetic monopole will not ordinarily be integral, or even rational in units of the fundamental charge e. If a non-zero vacuum angle θ is the only mechanism for *CP* violation, the electric charge of the monopole is exactly calculable and is $-e\theta/2\pi$, plus an integer. If there are additional *CP* violating interactions, the monopole charge must be computed as a power series in the coupling constant. These results apply in realistic theories such as SU(5).

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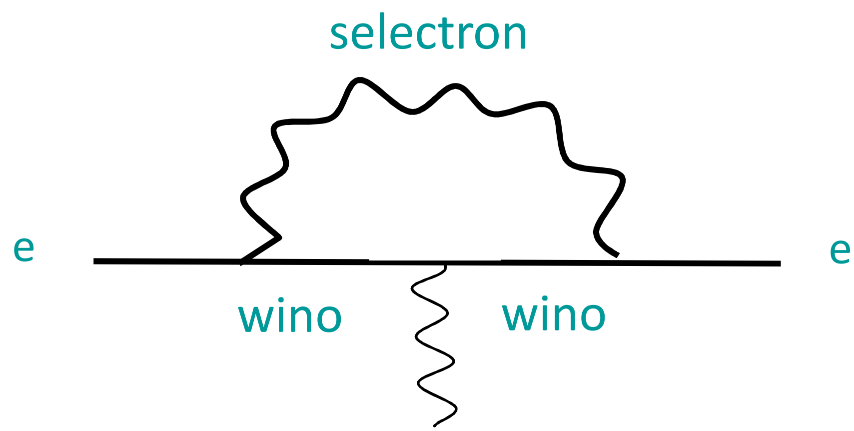
A magnetic monopole-antimonopole pair will have a magnetic dipole; this becomes an electric dipole moment For nonzero θ angle. Oscillating axion field implies a nonzero oscillating θ angle.

e.g., EDM's are a powerful probe:



$$d_e = \frac{e\, m_e}{\Lambda^2}$$

$$\frac{e\, m_e}{\Lambda^2}\, \bar{\Psi}\, \gamma_5\, \sigma_{\mu\nu} F^{\mu\nu}\, \Psi$$

$$1/(\Lambda)^2 = (\alpha \sin(\gamma) / 4\pi \sin^2 \theta)\, (1/M_{selectron})^2$$

Current limit: $d_e < 10^{-28}$ e-cm

$M_{selectron} > 10$ TeV $(\sin(\gamma))^{1/2}$

Future limit: $d_e < 10^{-29}$ e-cm -- 10^{-32} e-cm ?

Similar constraints from flavor physics

A simple Feynman Diagram demonstration of induced electric dipole moment for the elctron:

We begin by writing the axion anomaly in terms of vector potentials and integrating by parts:

$$\frac{1}{2} g_{A\gamma\gamma} \int d^4x \; \epsilon_{\mu\nu\rho\sigma} \partial^\mu \theta(x) A^\nu(x) \partial^\rho A^\sigma(x)$$

Dirac operator of the magnetic moment of the electron:

$$\frac{ie}{2m_e} \int d^4x \; \overline{\psi}(x) \sigma_{\alpha\beta} \psi(x) \; \partial^\alpha A^\beta(x)$$

Feynman Diagram

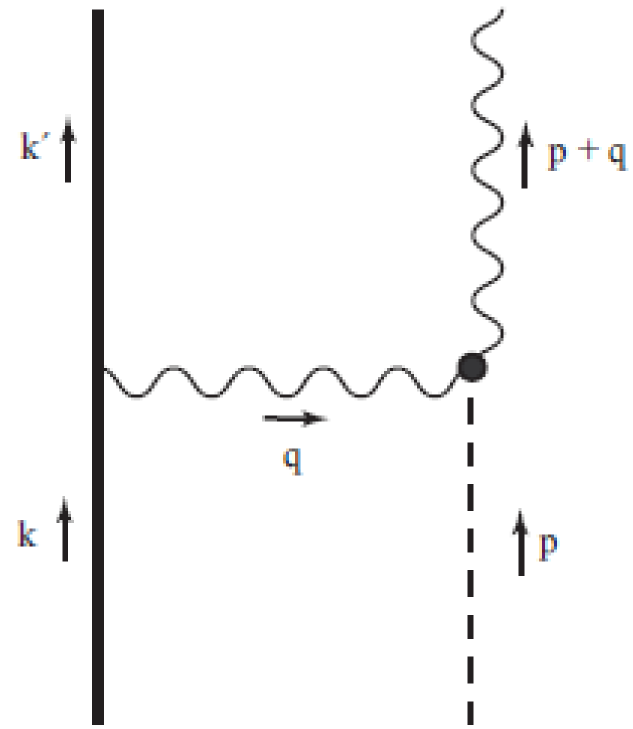


FIG. 1: Feynman diagram for axion induced electric dipole moment. Photon q emitted from electron magnetic moment. Solid dot is the axion anomaly interaction, $\theta \overrightarrow{E} \cdot \overrightarrow{B}$. Dashed line: incoming axion θ, p. Outgoing photon: $p + q$, ϵ_μ. Solid line: incoming electron k; recoil electron, k'.

Feynman Diagram

In momentum space we have:

$$\frac{ie}{2m_e} g_{A\gamma\gamma}\theta_0 \, p^\mu \epsilon^\nu (p+q)^\rho \epsilon_{\mu\nu\rho\sigma} \times$$
$$\frac{1}{q^2}\left(g^{\sigma\tau} - \lambda\frac{q^\sigma q^\tau}{q^2}\right) q^\omega \overline{u}(k')\sigma_{\tau\omega}u(k)$$

The dipole moments are defined by going to the electron rest-frame $k = (m_e, \overrightarrow{0})$. The electron is very heavy compared to the axion, and is therefore essentially stationary, and absorbs 3-momentum but not energy (zero recoil). We assume in the electron rest frame that the axion field momentum is approximately pure timelike, $p_\mu = (m_a, \overrightarrow{0})$ and $|\overrightarrow{q}| \approx m_a << m_e$, $k_0 \approx k_0'$, and

Feynman Diagram

Since p^μ is timelike, we have $p^\mu \epsilon_{\mu\nu\rho\sigma} = m_a \epsilon_{0\nu\rho\sigma}$ and we'll pass to $D = 3$ latin spatial indices, $\epsilon_{0\nu\rho\sigma} \rightarrow \epsilon_{ijk}$. Also, $\overline{u}(k')\sigma_{\tau\omega}u(k) \rightarrow \epsilon_{ijk}\chi^\dagger\sigma_k\chi$ ($(\tau\omega) \leftrightarrow (ij)$ are likewise spatial in the nonrelativistic limit). The Dirac four-component spinor ψ has been replaced by the two-component Pauli spinor, χ, with Pauli matrices σ^k. The amplitude becomes:

$$-\frac{ie}{2m_e} g_{A\gamma\gamma}\theta_0 \, m_a \, \frac{1}{\vec{q}^{\,2}} \epsilon^i q^j q^l \epsilon_{ijk} \epsilon^{klm} \chi^\dagger \sigma_m \chi$$

$$= g_{A\gamma\gamma}\theta_0 \, \mu_{Bohr} \chi^\dagger \sigma_i \chi \cdot m_a \epsilon^i$$

In Coulomb gauge with vector potential $\vec{A}$, the electric field is given by $\vec{E} = -\partial_t \vec{A} = m_a \vec{\epsilon}$. Our final result can be written as an effective interaction for the nonrelativistic electron as:

$$\int d^4x \; 2g_{A\gamma\gamma}\theta(t) \, \mu_{Bohr} \, \chi^\dagger \frac{\vec{\sigma}}{2} \chi(x) \cdot \vec{E}(x,t)$$

Feynman Diagram

The result is:

$$\approx 1.4 \times 10^{-32}(g_{A\gamma\gamma}/10^{-3})\cos(m_a t) \text{ e-cm.}$$

result is two orders of magnitude greater than the typical result expected for the nucleon, $d_N \sim 3.67 \times 10^{-35}\cos(m_a t)$ e-cm [11], and within four orders of magnitude of the DC limit on the EDM of the electron, $d_e \leq 8.7 \times 10^{-29}$ e-cm, [13].

Axionic Electrodynamic Radiation

Consider a compact local magnetic field configuration $\vec{B}_0$
for a magnetic dipole in the presence of the cosmic axion

Maxwell (1) $\quad \vec{\nabla} \times \vec{B}_r - \partial_t \vec{E}_r = -g_{a\gamma\gamma} \vec{B}_0 \left(\partial_t \widehat{\theta} \right)$

Maxwell (2) $\quad \vec{\nabla} \times \vec{E}_r + \partial_t \vec{B}_r = 0$

and $\quad \vec{\nabla} \cdot \vec{B}_r = \vec{\nabla} \cdot \vec{E}_r = 0.$

Equivalently: $\quad \partial_t^2 \vec{A}_r - \vec{\nabla}^2 \vec{A}_r = -g_{a\gamma\gamma} \vec{B}_0 \left(\partial_t \widehat{\theta} \right)$

SOLVE WITH RETARDED GREEN'S FUNCTIONS

$$G(r, t; r' t') = \frac{1}{|\vec{x} - \vec{x}'|} \delta(t - t' - |\vec{x} - \vec{x}'|)$$

Axionic Electrodynamics

Radiation

Messy:

Hence,

$$\vec{B}_r(x,t) = i m_a g_{a\gamma\gamma}\theta_0 \exp(i m_a t)\int d^3\vec{x}'\,\frac{1}{|\vec{x}-\vec{x}'|}\exp(-i m_a|\vec{x}-\vec{x}'|)\,\vec{\nabla}_{x'}\times\vec{m}_0\left(\vec{x}'\right)$$

Integrate by parts:

$$= i m_a g_{a\gamma\gamma}\theta_0 \exp(i m_a t)\,\vec{\nabla}_x\times\int d^3\vec{x}'\left(\frac{1}{|\vec{x}-\vec{x}'|}\exp(-i m_a|\vec{x}-\vec{x}'|)\right)\vec{m}_0\left(\vec{x}'\right)$$

Looks just like the curl of Hertzian dipole E field.

Integrate, to obtain:

$$\vec{B}_r(x,t) = i m_a g_{a\gamma\gamma}\theta_0 \exp(i m_a t)\,\vec{\nabla}\left(\frac{1}{|\vec{r}|}\exp(-i m_a|\vec{r}|)\right)\times\vec{m}$$

$$= -i m_a g_{a\gamma\gamma}\theta_0 \exp(i m_a t)\left(\frac{\vec{r}}{r^3}+\frac{i m_a\vec{r}}{r^2}\right)\times\vec{m}\,\exp(-i m_a|\vec{x}|)$$

[using $\vec{\nabla}r = \frac{\vec{r}}{r}$ $\quad\vec{\nabla}\frac{1}{r} = -\frac{\vec{r}}{r^3}$]

Jackson defines: $\vec{E}_r(x,t) = \frac{1}{k}\vec{\nabla}\times\vec{B}_r(x,t) = \frac{1}{i m_a}\vec{\nabla}\times\vec{B}_r(x,t)$ hence:

$$\vec{E}_r(x,t) = -\frac{1}{i m_a} i m_a g_{a\gamma\gamma}\theta_0 \exp(i m_a t)\,\vec{\nabla}\times\left[\left(\frac{\vec{x}}{x^3}+\frac{i m_a\vec{x}}{x^2}\right)\times\vec{m}\,\exp(-i m_a|\vec{x}|)\right]$$

Compute:

$[-g_{a\gamma\gamma}\theta_0 \exp(i m_a t)]\cdot$

$$\vec{\nabla}\times\left[\left(\frac{\vec{x}}{x^3}+\frac{i m_a\vec{x}}{x^2}\right)\times\vec{m}\,\exp(-i m_a|\vec{x}|)\right]$$

[[messy $= -\vec{m}\exp(-i m_a|\vec{x}|)\left(\vec{\nabla}\cdot\left(\frac{\vec{x}}{x^3}+\frac{i m_a\vec{x}}{x^2}\right)\right)+\exp(-i m_a|\vec{x}|)\left(\vec{m}\cdot\vec{\nabla}\right)\left(\frac{\vec{x}}{x^3}+\frac{i m_a\vec{x}}{x^2}\right)$
$+\left(\vec{\nabla}\exp(-i m_a|\vec{x}|)\right)\times\left(\frac{\vec{x}}{x^3}+\frac{i m_a\vec{x}}{x^2}\right)\times\vec{m}$

$$\vec{\nabla}\cdot\vec{m} = 0 \qquad \vec{\nabla}r = \frac{\vec{r}}{r} \qquad \vec{\nabla}\cdot\frac{\vec{r}}{r^3} = 4\pi\delta^3(\vec{r}) \qquad \vec{\nabla}\cdot\frac{\vec{r}}{r^2} = \frac{3}{r^2}-\frac{2\vec{r}}{r^4}\cdot\vec{r} = \frac{1}{r^2}$$

$$\vec{\nabla}\frac{1}{r^3} = -\frac{3\vec{r}}{r^5} \qquad \left(\vec{m}\cdot\vec{\nabla}\right)\frac{\vec{r}}{r^3} = \vec{m}\frac{1}{r^3}-3\frac{\vec{r}}{r^5}\vec{m}\cdot\vec{r} \qquad \left(\vec{m}\cdot\vec{\nabla}\right)\frac{\vec{r}}{r^2} = \vec{m}\frac{1}{r^2}-2\vec{m}\cdot\vec{r}\frac{\vec{r}}{r^4}$$

$$= \left(-\vec{m}\left(4\pi\delta^3(\vec{r})+\frac{i m_a}{r^2}\right)+\left(\frac{\vec{m}}{r^3}-3\frac{\vec{r}\,(\vec{m}\cdot\vec{r})}{r^5}+i m_a\left(\frac{\vec{m}}{r^2}-2\frac{\vec{r}\,\vec{m}\cdot\vec{r}}{r^4}\right)\right)\right)-i m_a\frac{\vec{r}}{r}\times\left(\frac{\vec{r}}{r^3}+\frac{i m_a\vec{r}}{r^2}\right)\times\vec{m}$$

Axionic Electrodynamic Radiation

Resulting radiation fields: $\theta(t) = \exp(im_a t)$

$$\vec{E}_r(x,t) = -g_{a\gamma\gamma}\theta(t)\exp(-im_a|\vec{x}|) \cdot \left(-\vec{m}\left(4\pi\delta^3(\vec{r})\right) + (1 + im_a r)\left(\frac{\vec{m}}{r^3} - 3\frac{\vec{r}\,(\vec{m}\cdot\vec{r})}{r^5}\right) - m_a^2\left(\frac{\vec{m}}{r} - \frac{\vec{r}}{r^2}\frac{\vec{m}\cdot\vec{r}}{r}\right)\right)$$

$$\vec{B}_r(x,t) = g_{a\gamma\gamma}\partial_t\theta(t)\exp(-im_a|\vec{x}|)\,\vec{m} \times \left(\frac{\vec{r}}{r^3} + \frac{im_a\vec{r}}{r^2}\right)$$

Axionic Electrodynamic Radiation

Resulting radiation fields:

$$\theta(t) = \exp(i m_a t)$$

$$\vec{E}_r(x,t) = -g_{a\gamma\gamma}\theta(t)\exp(-i m_a|\vec{x}|) \cdot \left(-\vec{m}\,\left(4\pi\delta^3(\vec{r})\right) + (1 + i m_a r)\left(\frac{\vec{m}}{r^3} - 3\frac{\vec{r}\,(\vec{m}\cdot\vec{r})}{r^5}\right) - m_a^2\left(\frac{\vec{m}}{r} - \frac{\vec{r}}{r^2}\,\frac{\vec{m}\cdot\vec{r}}{r}\right)\right)$$

$$\vec{B}_r(x,t) = g_{a\gamma\gamma}\partial_t\theta(t)\exp(-i m_a|\vec{x}|)\,\vec{m} \times \left(\frac{\vec{r}}{r^3} + \frac{i m_a\vec{r}}{r^2}\right)$$

Near Zone:

"Hertzian" Electric Dipole:

$$\vec{E}_r(x,t) \to -g_{a\gamma\gamma}\theta(t)\left(-\vec{m}\,4\pi\delta^3(\vec{r}) + \left(\frac{\vec{m}}{r^3} - 3\frac{\vec{r}\,(\vec{m}\cdot\vec{r})}{r^5}\right)\right)$$

$$\vec{B}_r(x,t) = g_{a\gamma\gamma}\partial_t\theta(t)\left(\vec{m} \times \frac{\vec{r}}{r^3}\right) \to 0$$ (less singular than $\vec{E}_r(x,t)$)

Axionic Electrodynamic Radiation

Resulting radiation fields:

$$\theta(t) = \exp(i m_a t)$$

$$\vec{E}_r(x,t) = -g_{a\gamma\gamma}\theta(t)\exp(-i m_a|\vec{x}|) \cdot \left(-\vec{m}\left(4\pi\delta^3(\vec{r})\right) + (1 + i m_a r)\left(\frac{\vec{m}}{r^3} - 3\frac{\vec{r}\,(\vec{m}\cdot\vec{r})}{r^5}\right) - m_a^2\left(\frac{\vec{m}}{r} - \frac{\vec{r}}{r^2}\frac{\vec{m}\cdot\vec{r}}{r}\right)\right)$$

$$\vec{B}_r(x,t) = g_{a\gamma\gamma}\partial_t\theta(t)\exp(-i m_a|\vec{x}|)\,\vec{m} \times \left(\frac{\vec{r}}{r^3} + \frac{i m_a \vec{r}}{r^2}\right)$$

Far zone:

$$\vec{E}_r(x,t) = g_{a\gamma\gamma} m_a^2 \theta(t)\exp(-i m_a|\vec{x}|)\left(\frac{\vec{m}}{r} - \frac{\vec{r}}{r^2}\frac{\vec{m}\cdot\vec{r}}{r}\right)$$

$$\vec{B}_r(x,t) = -m_a^2 g_{a\gamma\gamma}\theta(t)\exp(-i m_a|\vec{x}|)\left(\vec{m} \times \left(\frac{\vec{r}}{r^2}\right)\right)$$

Axionic Electrodynamic Radiation

Resulting radiation fields: $\theta(t) = \exp(im_a t)$

$$\vec{E}_r(x,t) = -g_{a\gamma\gamma}\theta(t)\exp(-im_a|\vec{x}|)$$
$$\cdot\left(-\vec{m}\,\left(4\pi\delta^3(\vec{r})\right) + (1+im_a r)\left(\frac{\vec{m}}{r^3} - 3\frac{\vec{r}\,(\vec{m}\cdot\vec{r})}{r^5}\right) - m_a^2\left(\frac{\vec{m}}{r} - \frac{\vec{r}}{r^2}\,\frac{\vec{m}\cdot\vec{r}}{r}\right)\right)$$

$$\vec{B}_r(x,t) = g_{a\gamma\gamma}\partial_t\theta(t)\exp(-im_a|\vec{x}|)\,\vec{m}\times\left(\frac{\vec{r}}{r^3} + \frac{im_a\vec{r}}{r^2}\right)$$

Poynting:

$$= \frac{1}{2}g_{a\gamma\gamma}^2 m_a^4\theta_0^2\left[-\left(\frac{\vec{r}}{r^2}\right)\left(\frac{\vec{m}^2}{r} - \frac{(\vec{m}\cdot\vec{r})^2}{r^3}\right)\right]$$

Axionic Electrodynamic Radiation

Power ~ $\frac{1}{2} g_{a\gamma\gamma}^2 m_a^4 \theta_0^2 \frac{\vec{m}^2}{r^2}$ $\quad \vec{m} \sim B_0 \times$ (Volume)

Note that $\vec{m}$ has dimensions of $\mu_{Bohr} = 1/m$ (Bohr magneton)

For ADMX: $\vec{m} \sim B_0 \times$ (Volume)

at 1 Tesla, 10 cm, get = 1.8×10^{-26} watts

Pauli-Schroedinger Calculation

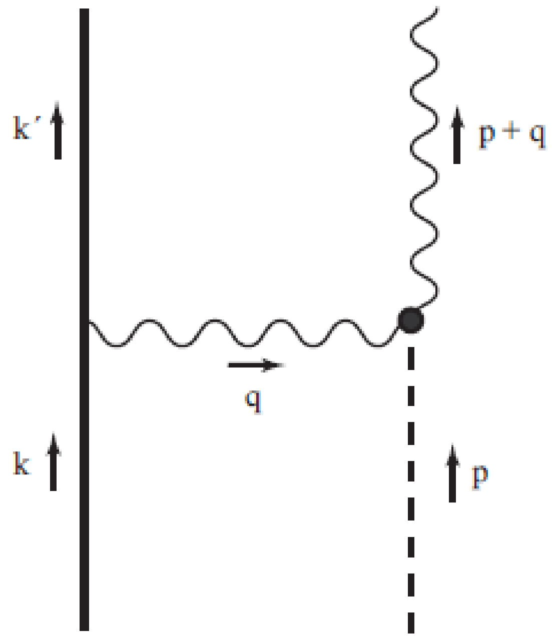


FIG. 1: Feynman diagram for axion induced electric dipole moment; solid dot is axion anomaly interaction, $\theta \vec{E} \cdot \vec{B}$; dashed line, incoming axion θ, p; outgoing photon, $p+q$, ϵ_μ; solid line, incoming electron k, solid line, recoil electron, k'.

Pauli-Schroedinger Calculation

Pauli-Schroedinger Action

$$\int d^4x \;\; \frac{1}{2m}\psi^\dagger\vec{\sigma}\cdot\left(i\vec{\partial}-e\vec{A}\right)\vec{\sigma}\cdot\left(i\vec{\partial}-e\vec{A}\right)\psi - i\psi^\dagger\partial_t\psi$$

-> magnetic moment

$$-2i\mu_B\psi^\dagger B_k\frac{\sigma_k}{2}\psi \rightarrow \;\; g = 2$$

Compute Feynman diagram:

$$= T\int d^4x\frac{ie}{2m}\psi^\dagger\epsilon_{ijk}\sigma_k\psi\;\partial_i A_j\;\int d^4y\;\frac{1}{2}\partial_0\theta\epsilon^{lmn}A_l\partial_m A_n$$

$$= \frac{ie}{2m}\int d^4y\int d^4x\;\psi^\dagger\sigma_k\psi\;\;\partial_i\,D(x-y)\;\partial_0\theta\;\partial_m A_n\;\epsilon_{ijk}\epsilon_{jmn}$$

Pauli-Schroedinger Calculation

Pauli-Schroedinger:

Use: $\epsilon_{ijk}\epsilon_{jmn} = -\epsilon_{jik}\epsilon_{jmn} = -\delta_{im}\delta_{kn} + \delta_{in}\delta_{km}$

$$= -\frac{ie}{2m}\int d^4y \int d^4x\ \psi^\dagger\sigma_k\psi\ \partial_i\ D(x-y)\ \partial_0\theta\ \partial_m A_n\ (\delta_{im}\delta_{kn} - \delta_{in}\delta_{km})$$

static electron: $\partial_0\ \psi^\dagger\sigma_k\psi$ -> 0

$$D(x-y) = \int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\exp(iq\cdot(x-y))$$

$$\int dx^0 D(x-y) = \int \frac{d^4q}{(2\pi)^3}\frac{1}{\vec{q}^2}\exp\left(i\vec{q}\cdot(x-y)\right)\delta(q^0) = \frac{1}{|(\vec{x}-\vec{y})|}$$

$$= -ig\mu_{Bohr}\int d^4x\ \ \vec{S}(\vec{x})\cdot\vec{E}(\vec{x},x^0)\ \theta(x^0)$$

$$+ig\mu_{Bohr}\int d^4y\int d^4x\ \vec{S}(\vec{x})\ \cdot\vec{\nabla}D(x-y)\ \theta(y^0)\ \partial_i E_i(y,y^0)$$

Axionic Induced Oscillating EDM

$$= -\frac{ie}{2m}\int d^4x\ \psi^\dagger \sigma_k \psi\ \theta\ (E_k)$$

"Georgi" Calculation

Adapt Heavy Quark Effective FT to the Electron:

$$\psi \to \left(\frac{1+\not{v}}{2}\right)\psi = \psi_v \qquad \overline{\psi} \to \overline{\psi}\left(\frac{1+\not{v}}{2}\right) = \overline{\psi}_v$$

The Dirac magnetic moment operator takes the form:

$$\frac{ie}{4m}\overline{\psi}_v \sigma_{\mu\nu}\psi_v F^{\mu\nu} = \frac{ie}{2m}\overline{\psi}_v\left(\gamma_5\epsilon_{\alpha\beta\rho\gamma}\gamma^\rho v^\gamma\right)\psi_v\partial^\alpha A^\beta \quad (15)$$

where we use:

$$\left(\frac{1+\not{v}}{2}\right)\sigma_{\alpha\beta}\left(\frac{1+\not{v}}{2}\right) = \left(\frac{1+\not{v}}{2}\right)\left(\gamma_5\epsilon_{\alpha\beta\rho\gamma}\gamma^\rho v^\gamma\right)\left(\frac{1+\not{v}}{2}\right) \quad (16)$$

“Georgi” Calculation

Compute time ordered product:

$$T\,\frac{ie}{2m}\int d^4x\overline{\psi}_v(x)\left(\gamma_5\epsilon_{\alpha\beta\rho\gamma}\gamma^\rho v^\gamma\right)\psi_v(x)\partial^\alpha A^\beta(x)$$
$$\times\frac{1}{2}\int d^4y\theta(y)\epsilon^{\eta\kappa\omega\rho}\partial_\eta A_\kappa(y)\partial_\omega A_\rho(y)$$

Use: $\epsilon_{\alpha\beta\gamma\rho}\epsilon^{\alpha\eta\omega\delta}=-\left(g^\eta_\beta g^\omega_\gamma g^\delta_\rho+(-1)^p\text{permutations})\right)$

Lots of algebra ....

“Georgi” Calculation

Result:

$$= \frac{i}{2}\int d^4x \frac{ie}{2m}\overline{\psi}\left(\frac{1+v}{2}\right)\gamma^5\sigma_{\alpha\beta}\left(\frac{1+v}{2}\right)\psi\ (\theta F^{\alpha\beta})$$

$$+\, i\int d^4y\int d^4x \frac{ie}{2m}\partial^\beta\left(\overline{\psi}\left(\frac{1+v}{2}\right)(\gamma^5\sigma_{\alpha\beta})\left(\frac{1+v}{2}\right)\psi(x)\right)\ D(x-y)\cdot\partial_\gamma(\theta F^{\alpha\gamma}(y))$$

Yields more conventional form of the EDM.
Agrees with Pauli in rest frame, static electron.

Electromagnetic Duality

The result of the previous section is a general low energy theorem. Consider an arbitrary localized magnetic dipole interaction in Coulomb gauge $\nabla \cdot \overrightarrow{A} = 0$, $A_0 = 0$:

$$\int d^4x \, \overrightarrow{M}(\overrightarrow{x}) \cdot \overrightarrow{B} = \int d^4x \, \overrightarrow{M} \cdot \overrightarrow{\nabla} \times \overrightarrow{A}$$

The effect of the axion anomaly, to first order in perturbation theory as in the previous section, schematically produces a term,

$$-g_{a\gamma\gamma} \int d^4x \, \overrightarrow{M}(\overrightarrow{x}) \cdot \overrightarrow{\nabla} \times \left(\frac{1}{\nabla^2} (\partial_t \theta(t)) \overrightarrow{\nabla} \times \overrightarrow{A} \right)$$

$$= g_{a\gamma\gamma} \int d^4x \, \theta(t) \, \overrightarrow{M} \cdot \overrightarrow{E}$$

This result is formally related to duality in electromagnetic theory. Deser and Teitelboim [17] elegantly formulated the continuous electromagnetic dual tranformation, whereby $\overrightarrow{E} \leftrightarrow -\overrightarrow{B}$. This arises from an infinitesimal non-local transformation at the level of the vector potential. In Coulomb gauge the Deser-Teitelboim dual transformation is:

$$\delta\overrightarrow{A} = \frac{\epsilon}{\nabla^2}\overrightarrow{\nabla} \times \partial_t\overrightarrow{A}$$

which implies at the field strength level,

$$\delta\overrightarrow{E} = -\epsilon\delta\overrightarrow{B} \qquad \delta\overrightarrow{B} = \epsilon\delta\overrightarrow{B}$$

We see that the transformation acting on the magnetic source term $\overrightarrow{M} \cdot \overrightarrow{\nabla} \times \overrightarrow{A}$ will produce a dual rotation of the magnetic field into the electric field, provided we can replace the dual rotation angle by $\epsilon \to g_{a\gamma\gamma}\theta(t)$.

$$\delta\overrightarrow{A} = \frac{g_{a\gamma\gamma}\theta(t)}{\nabla^2}\overrightarrow{\nabla} \times \partial_t\overrightarrow{A}$$

We might worry that this affects the kinetic term of the electromagnetic theory (note that the $g_{a\gamma\gamma}\theta(t)\overrightarrow{E}\cdot\overrightarrow{B}$ term is already infinitesimal in this sense and does not transform). However, we can see that the electromagnetic action, $S = \int d^4x(\overrightarrow{E}^2 - \overrightarrow{B}^2)/2$, is invariant under the time dependent transformation. Define $\epsilon(t) = g_{a\gamma\gamma}\theta(t)$ and consider:

$$\begin{aligned}
\delta\overrightarrow{A} &= \frac{1}{\nabla^2}\epsilon(t)\left(\nabla\times\partial_t\overrightarrow{A}\right)\\
\delta\overrightarrow{E} &= -(\partial_t\epsilon)\frac{1}{\nabla^2}\left(\nabla\times\partial_t\overrightarrow{A}\right) - \epsilon\overrightarrow{B}\\
\delta\overrightarrow{B} &= \epsilon\frac{1}{\nabla^2}\overrightarrow{\nabla}\times\left(\nabla\times\partial_t\overrightarrow{A}\right) = \epsilon(t)\overrightarrow{E}
\end{aligned}$$

where we follow DT and use the vector potential equation of motion, $\partial_t^2\overrightarrow{A} = \nabla^2\overrightarrow{A}$. We see that $\delta\overrightarrow{B}$ has the same form as that of Deser and Teitelboim. Hence the magnetic dipole moment will cleanly rotate into an oscillating electric dipole moment.

If we consider the action integral we find:

$$\begin{aligned}
\int \frac{1}{2}\delta \overrightarrow{E}^2 &= - \int \epsilon \overrightarrow{E} \cdot \overrightarrow{B} + \int \partial_t \epsilon \overrightarrow{A} \cdot \left(\nabla \times \overrightarrow{A}\right) \\
\int \frac{1}{2}\delta \overrightarrow{B}^2 &= \int \epsilon \overrightarrow{E} \cdot \overrightarrow{B}
\end{aligned}$$

where we've integrated by parts in space and time and discarded surface terms. Note that, with some manipulation, $\int \epsilon \overrightarrow{E} \cdot \overrightarrow{B} = \frac{1}{2} \int \partial_t \epsilon \ \overrightarrow{A} \cdot \nabla \times \overrightarrow{A}$ + total divergence. We thus find for the shift in the action:

$$\begin{aligned}
\delta S &= -2 \int \epsilon \left(\overrightarrow{E} \cdot \overrightarrow{B}\right) + \int \partial_t \epsilon \overrightarrow{A} \cdot \left(\nabla \times \overrightarrow{A}\right) \\
&= 0
\end{aligned}$$

modulo surface terms.

However, static electric dipole moments *will not* acquire oscillating magnetic moments. The effect of the additional nonlocal term in eq.(16) of $\delta\overrightarrow{E}$ is nontrivial. Given an electric dipole moment term in the action, $\int d^4x\,\overrightarrow{P}\cdot\overrightarrow{E}$ where $\overrightarrow{P}$ is time independent, we find $\delta\int d^4x\,\overrightarrow{P}\cdot\overrightarrow{E}=0$ upon integrating the nonlocal term in $\delta\overrightarrow{E}$ by parts and using $\partial_t^2\overrightarrow{A}-\nabla^2\overrightarrow{A}=0$. The asymmetry between magnetic and electric dipoles in axion electrodynamics is a consequence of the exclusive time dependence in $\epsilon(t)$ (this is modified if $\beta_{axion}\neq 0$).

If we introduce large classical magnetic background fields, the physical dual rotation induced by the axion on these fields generates the solutions to Maxwell's equations. In an RF cavity experiment with a large applied constant magnetic field $\overrightarrow{B}_0 = B_0 \hat{z}$ the Maxwell equations take the form of eq.(10) and above. The rhs of eq.(10) is just the time derivative of the dual rotation of the large applied field B_0. The particular solution of is likewise the infinitesimal dual tranformation of $\overrightarrow{B}_0$, $\overrightarrow{E}_r = -g_{a\gamma\gamma}\theta_0(t)\overrightarrow{B}_0$.

Conclusions

The resulting EDM has been criticized for being proportional to $\theta(t) = \theta_0 \cos(m_a t)$ and nonvanishing as $m_a \to 0$.

The same issue arises in the case of the anomaly. The result is intrinsically oscillatory (the nonlocal makes the source for the vector potential transverse, ie, not Coulombic) . The above Feynman amplitudes can be written as :

$$\vec{A}_r(\partial_t \theta) \qquad\qquad \theta(t)\,\vec{E}_r$$

chiral non g.i.   Gauge inv, non-chiral

Conclusions

A space-time filling coherent oscillating axion field will cause any magnetic source or field to become an electric source or field through the anomalous coupling of the axion to electromagnetic fields.

The electron will develop an oscillating electric dipole moment of strength $2g_{a\gamma\gamma}\theta_0\mu_{Bohr}\vec{s}$ where $a/f = \theta_0 \cos(m_a t)$.

Within a few orders of magnitude of current elctron EDM limit (ACME). This may be measureable in experiment. The electron OEDM is about 10^3x neutron OEDM