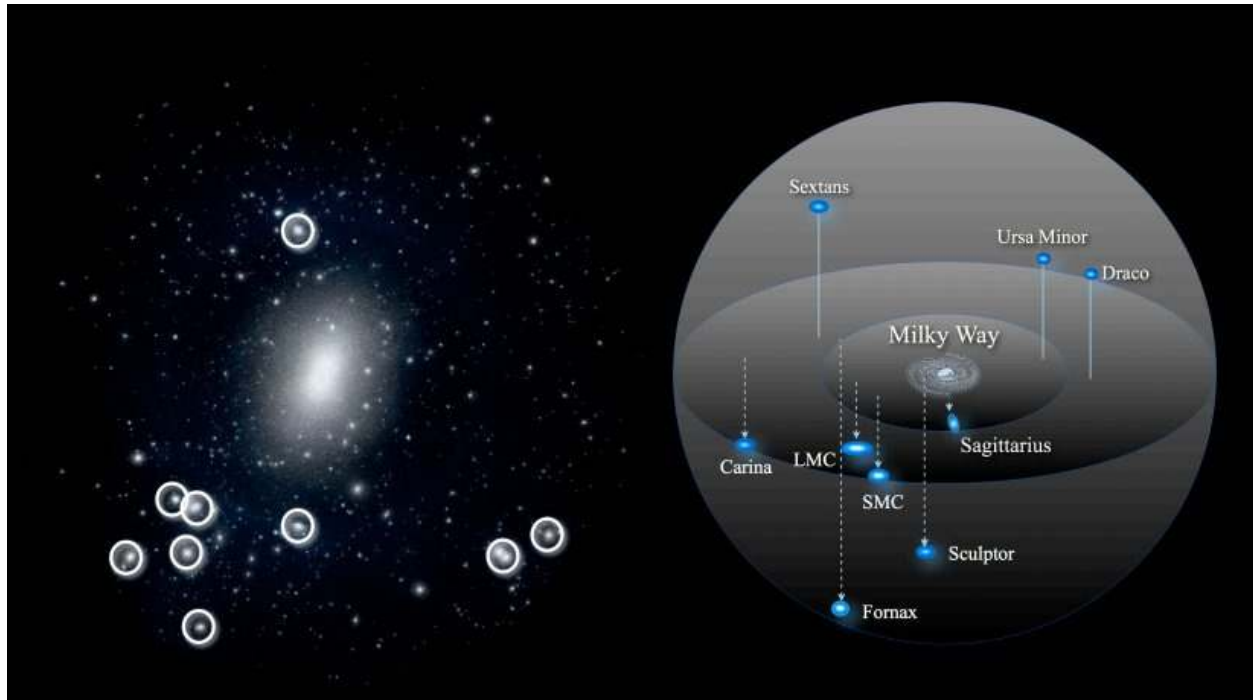


Composite models of strongly interacting dark matter



Jim Cline, McGill University

Invisibles Journal Club, 17 Mar., 2014

Outline

Based on

JC, G.D. Moore, Z. Liu, W. Xue, 1311.6468, 1312.3325

- Hints for strong self-interactions of dark matter
- Self-interactions of atomic dark matter
- Dark mesons
- Dark “baryons”
- Dark glueballs

Hints of DM self-interactions

Standard cold dark matter seems to get structure wrong at small scales.

N-body simulations predict cuspy density profiles, while observations suggest otherwise.

More large satellite galaxies are predicted for the Milky Way than observed.

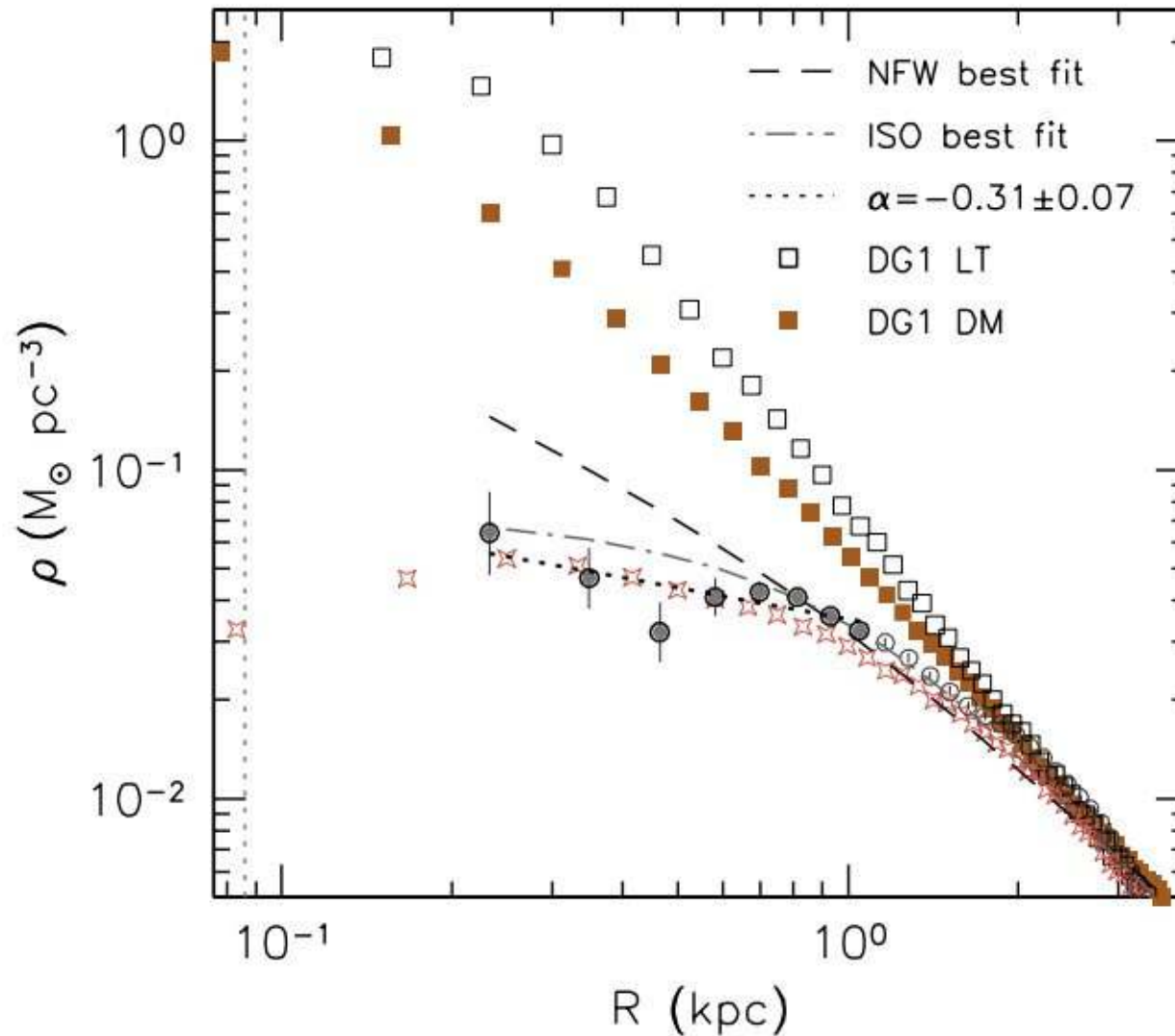
If DM scatters elastically with itself, with

$$\sigma/m \sim 1\text{b}/\text{GeV}$$

these problems are ameliorated. (1b = 100 fm²)

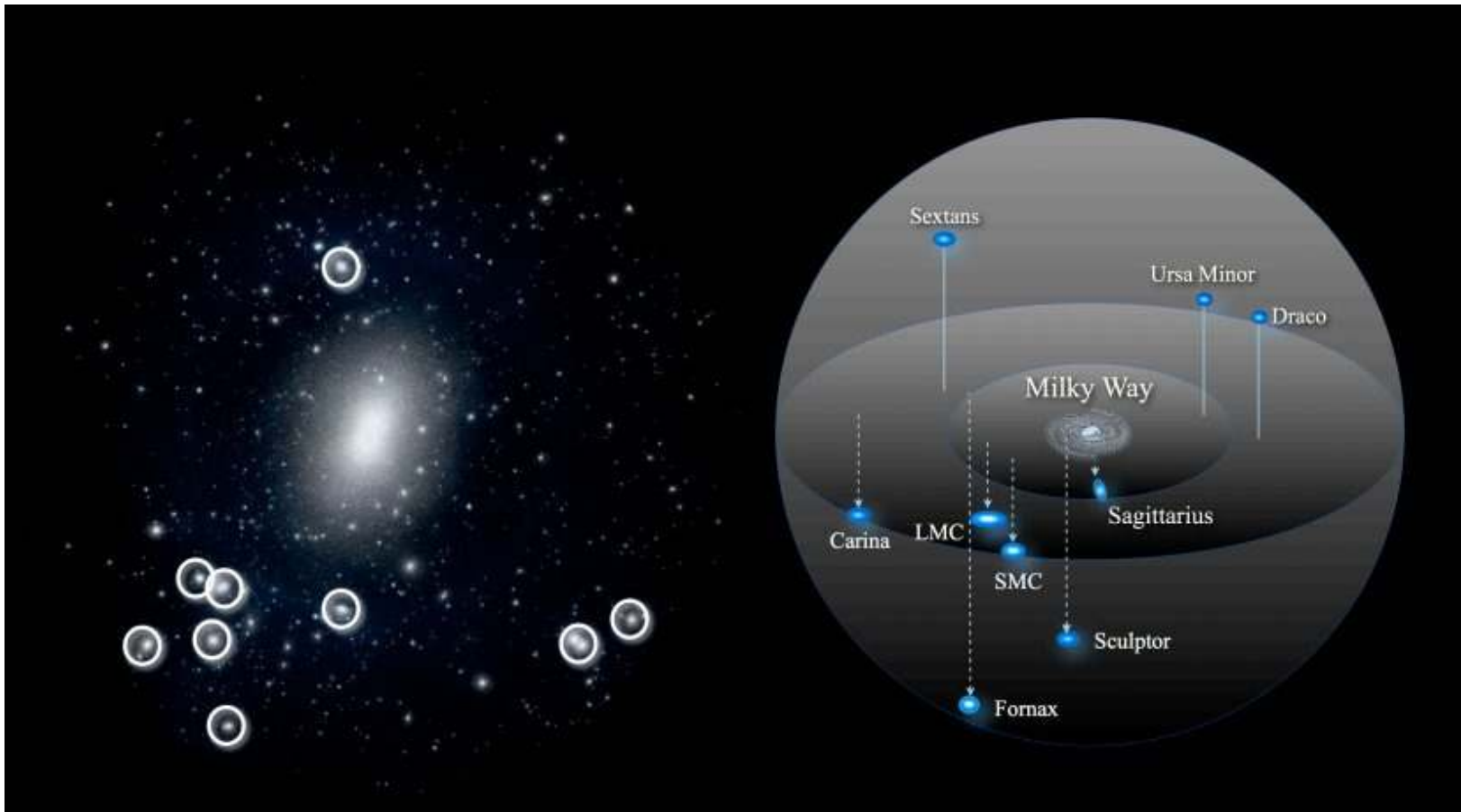
Cusp versus core problem

Oh *et al.*, 1011.2777, compare simulated dwarf galaxies with observed THINGS survey



Too big to fail problem

Garrison-Kimmel, Boylan-Kolchin, Bullock, in preparation



Largest predicted dwarf satellites (left) have too high central densities to match observed ones (right).

How self-interactions help

DM particles at larger radii have larger velocity. They scatter with DM particles at smaller radii, heating them up. Initially cuspy profile gets puffed up.

Simulations ([Zavala *et al.*, 1211.6426](#)) show that

$$\sigma/m \sim 1 \text{ b/GeV}$$

gives the desired effect. Larger values would have too big effect and are ruled out.

E.g., Bullet Cluster simulation requires $\sigma/m < 1.3 \text{ b/GeV}$.
([Randall *et al.*, 0704.0261](#))

How big is 1 b/GeV?

Scalar dark matter with $(\lambda/4!)\phi^4$ interaction and $\lambda = 100$ would need to have $m = 400 \text{ MeV}$ to scatter that strongly.

Normal H atoms have $\sigma/m \sim 30 a_0^2/m_p \sim 10^9 \text{ b/GeV!}$

Cross section is large because atom is large, $a_0 \sim (\alpha m_e)^{-1}$.

Nucleons have $\sigma/m \sim 10 \text{ b/GeV}$ due to residual strong interactions.

→ Composite dark matter naturally has large self-interactions.

Dark atom self-interactions

Atomic physicists know how to compute H-H elastic scattering. We can use their results/methods to generalize to dark atoms.

Three parameters:

$$\{\alpha', m_e, m_p\} \longrightarrow \{\alpha', m_H = m_e + m_p, R = m_p/m_e\}$$

We can scale out two of them by choice of (atomic) units for distance and energy:

$$a_0 = (\alpha' \mu)^{-1}, \quad \epsilon_0 = \alpha'^2 \mu$$

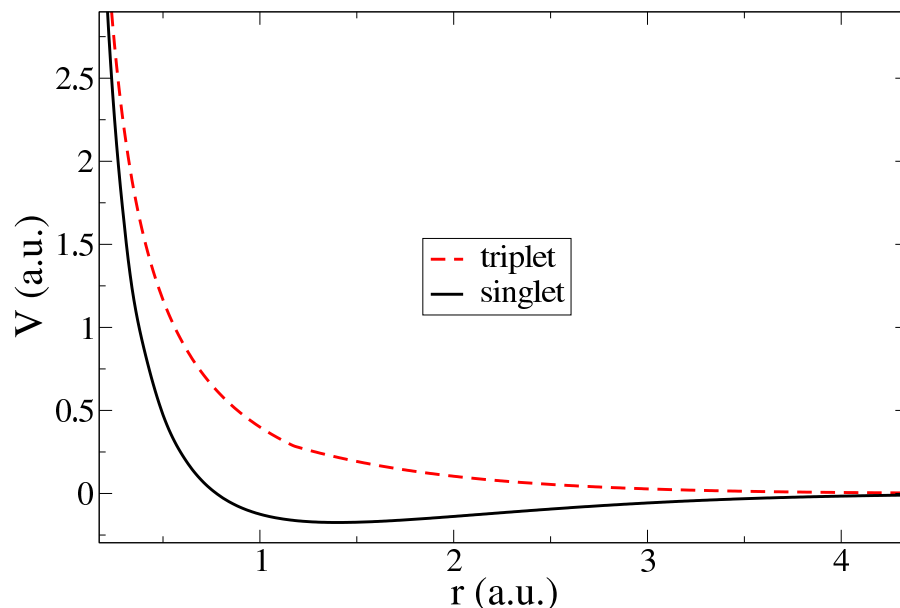
($\mu = \frac{m_e m_p}{m_e + m_p}$ = reduced mass). Only $R \geq 1$ remains as nontrivial parameter.

Partial wave scattering

In atomic units, Schrödinger eq. for partial wave amplitudes is

$$\left(\partial_r^2 - \frac{\ell(\ell+1)}{r^2} + f(R) (E - V_{s,t}) \right) u_\ell^{s,t}(r) = 0$$

where $f(R) = m_H \epsilon_0 = R + 2 + R^{-1}$, and $V_{s,t}$ are potentials for electron spin singlet and triplet channels, determined by atomic physicists:



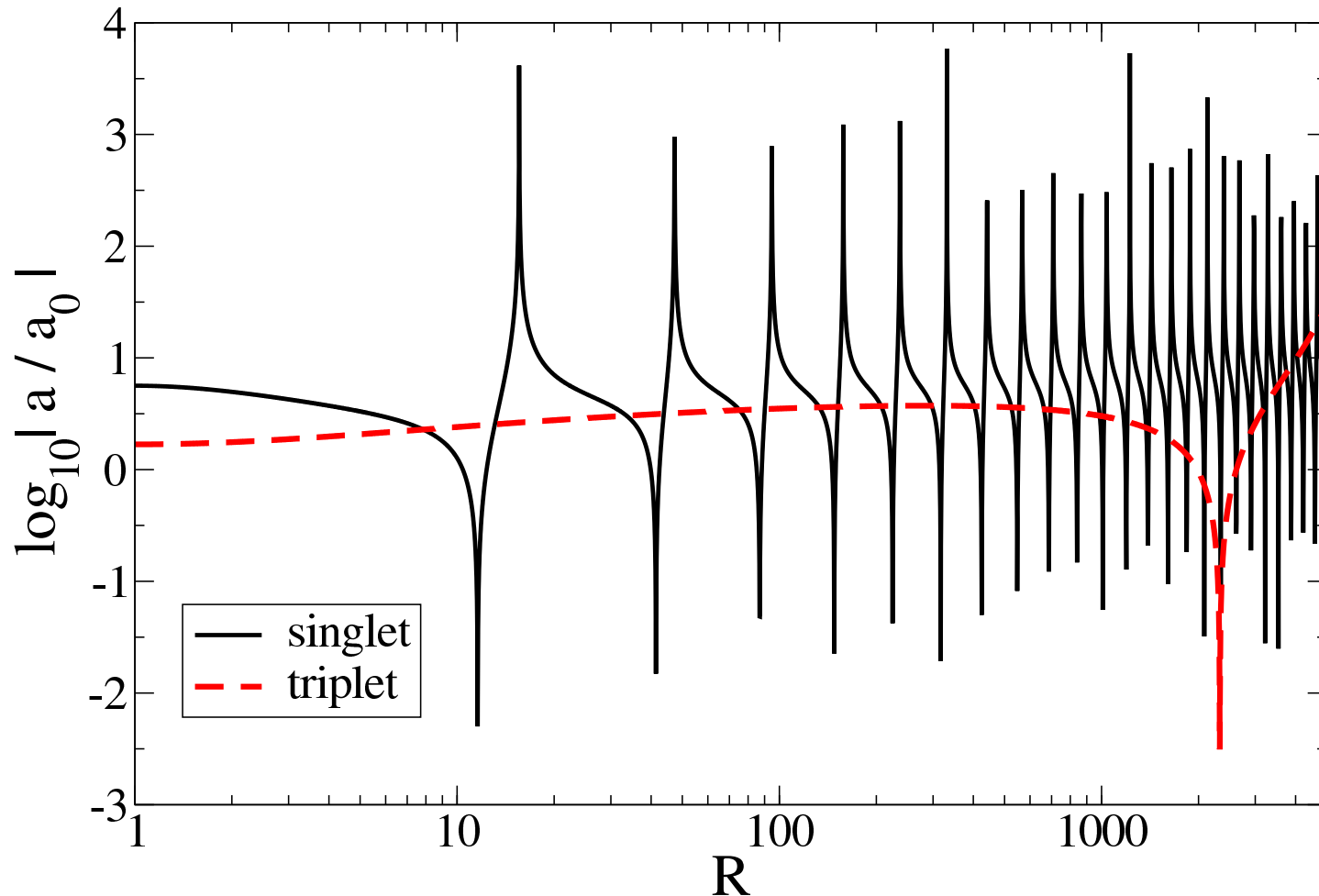
$V_{s,t}$ depend only upon a_0, ϵ_0 , not R .
We can use them directly for dark atoms!

$f(R)$ acts like particle mass

R -dependence of cross section

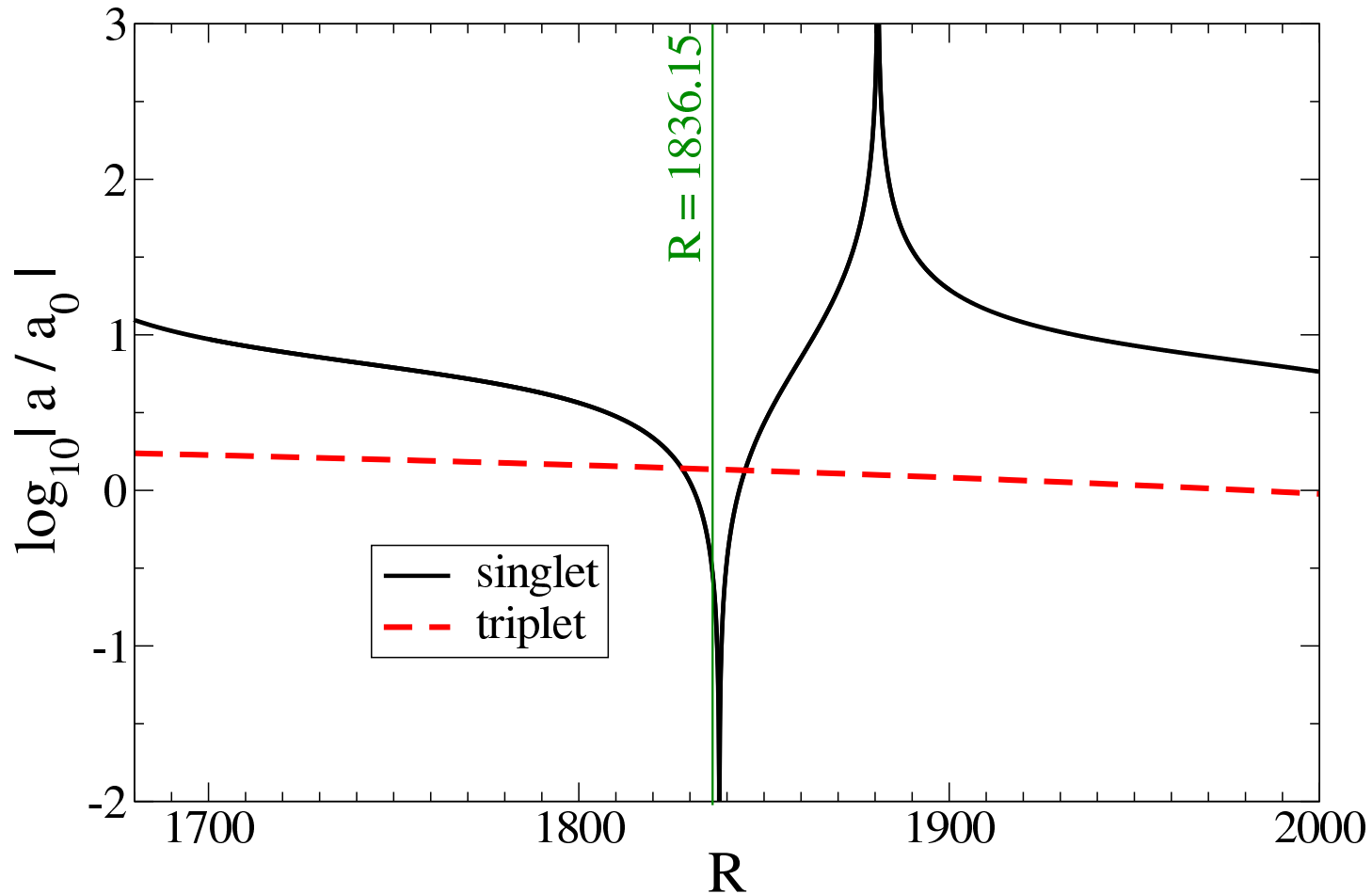
Effective mass increases with R : deeper potential \rightarrow more bound states \rightarrow divergences in scattering length,

$$a = \lim_{k \rightarrow 0} \sqrt{\sigma(k)/4\pi}$$



R -dependence of cross section

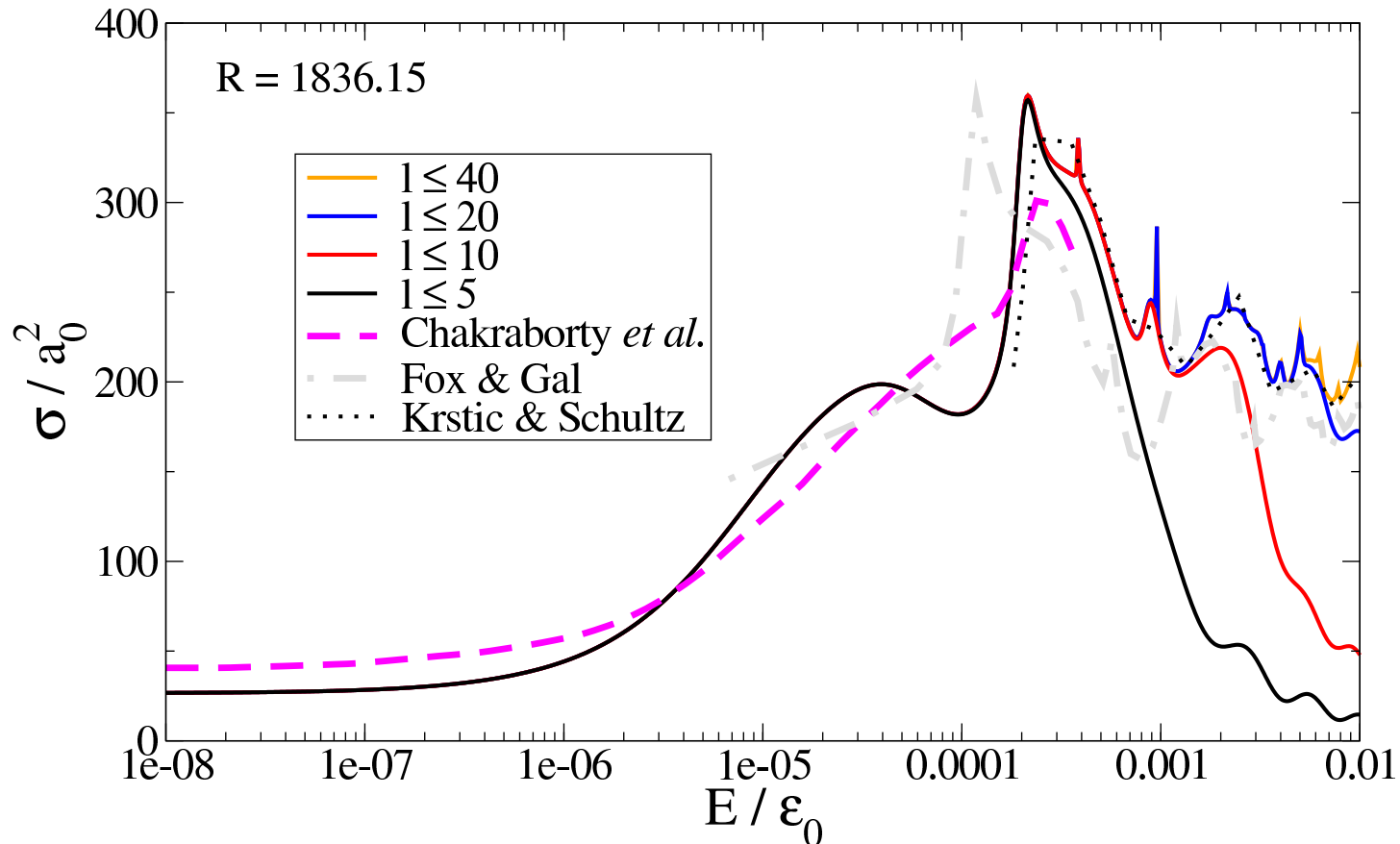
Real world happens to be close to a zero of the singlet channel scattering length:



→ Real-world cross section $\sigma \sim 30 a_0^2$ is atypically small.

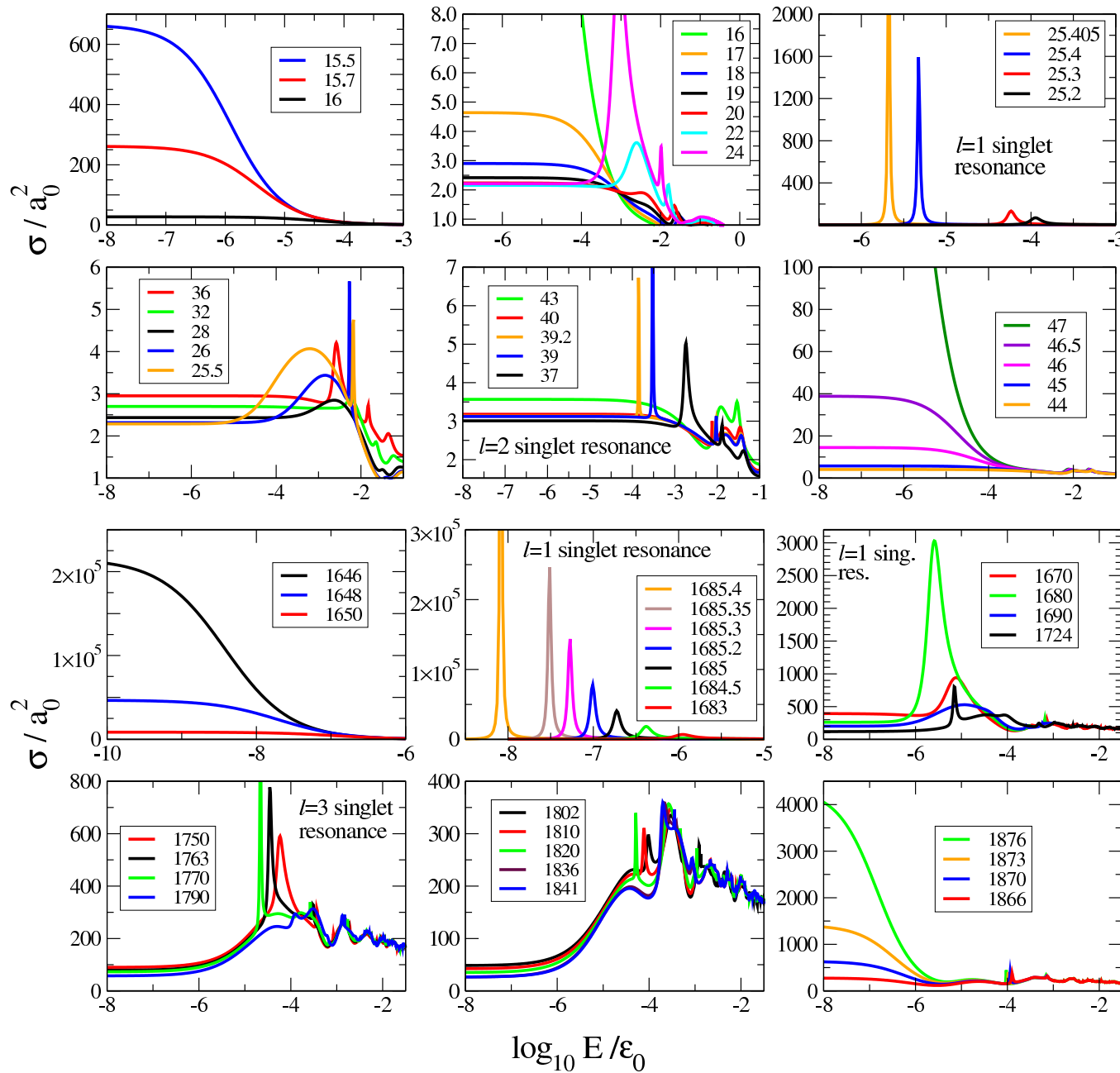
Reproducing known results

We can reproduce the most recent result from the atomic physics literature for $R = 1836.35$:



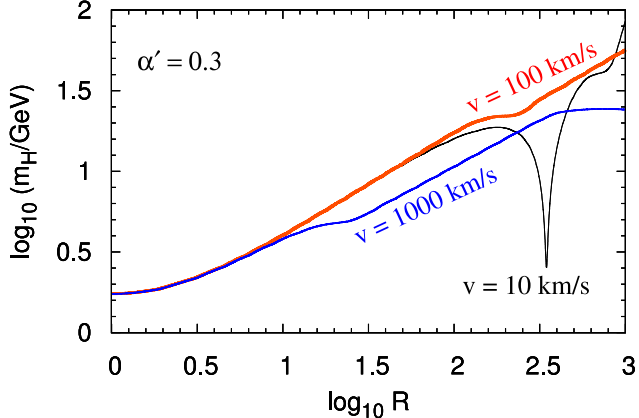
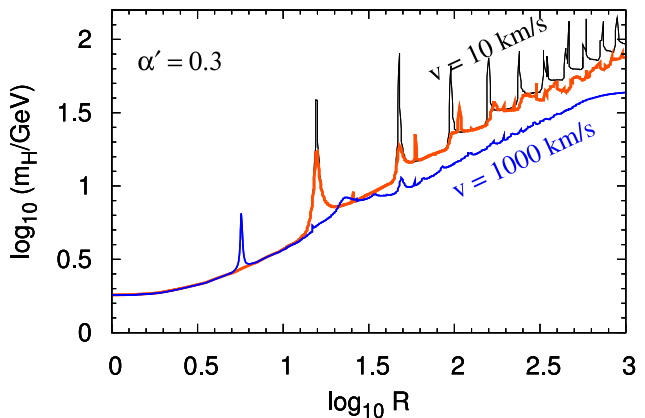
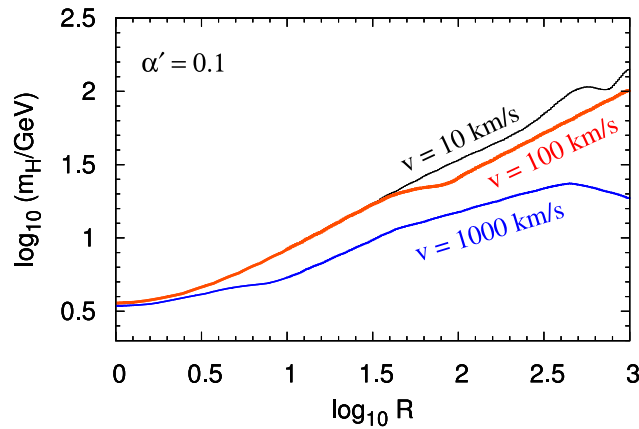
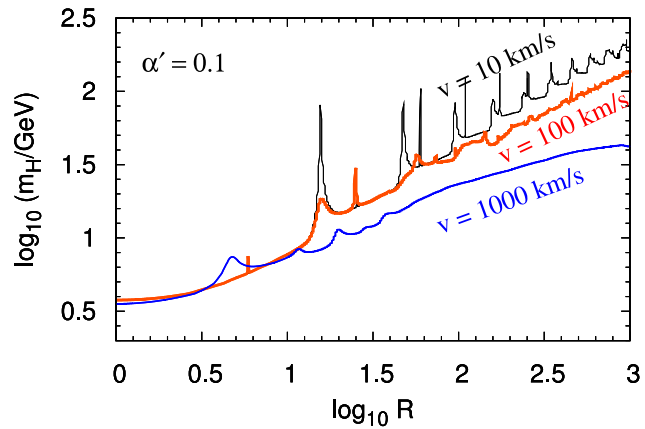
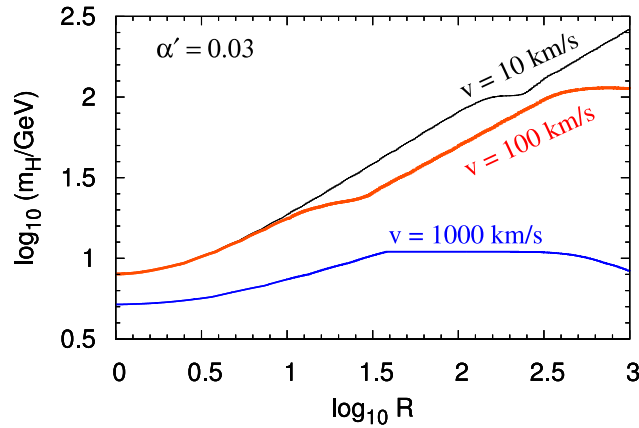
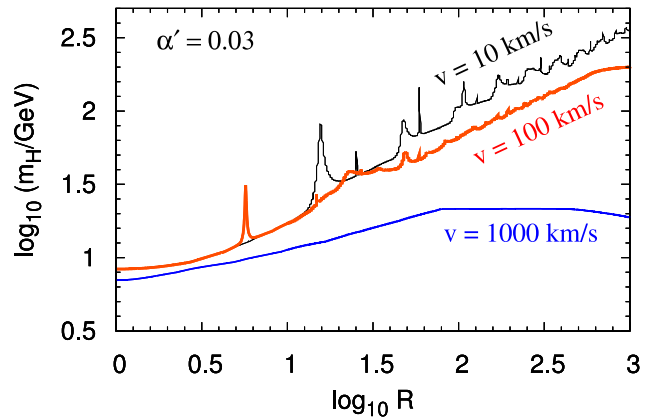
Differences with earlier results are due to refinements in V_s over the years, or some authors' neglect of m_e contribution to m_H .

R -dependence of cross section



We get many intricate features in σ as a function of energy and of R

Preferred regions of parameter space



Left: preferred m_H versus R for different α' and DM velocities.

Right: same for dark H_2 molecules.

Roughly fit by

$$\frac{m_H}{\text{GeV}} \cong \left(\frac{R}{5.3\alpha'} \right)^{2/3}$$

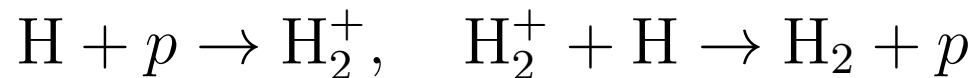
or

$$a_0 \cong 1 \text{ fm} \left(\frac{m_H}{\text{GeV}} \right)$$

Dark molecules?

We can compute scattering of dark H_2 molecules in same way, since intermolecular potential is known.

Could dark atoms bind primarily into H_2 molecules? Residual ionized fraction of dark atoms catalyzes molecule production, *e.g.*,



No dark stars, no ionizing radiation; dark molecules may dominate.

Danger: rotational excitations are too easy if $R \gg 1$, making dark matter too dissipative. We can quantify:

Electric quadrupole transition requires $\ell = 2$ bound state.

For what value of R do we get the first zero-energy $\ell = 2$ bound state?

We find $R = 15.42$

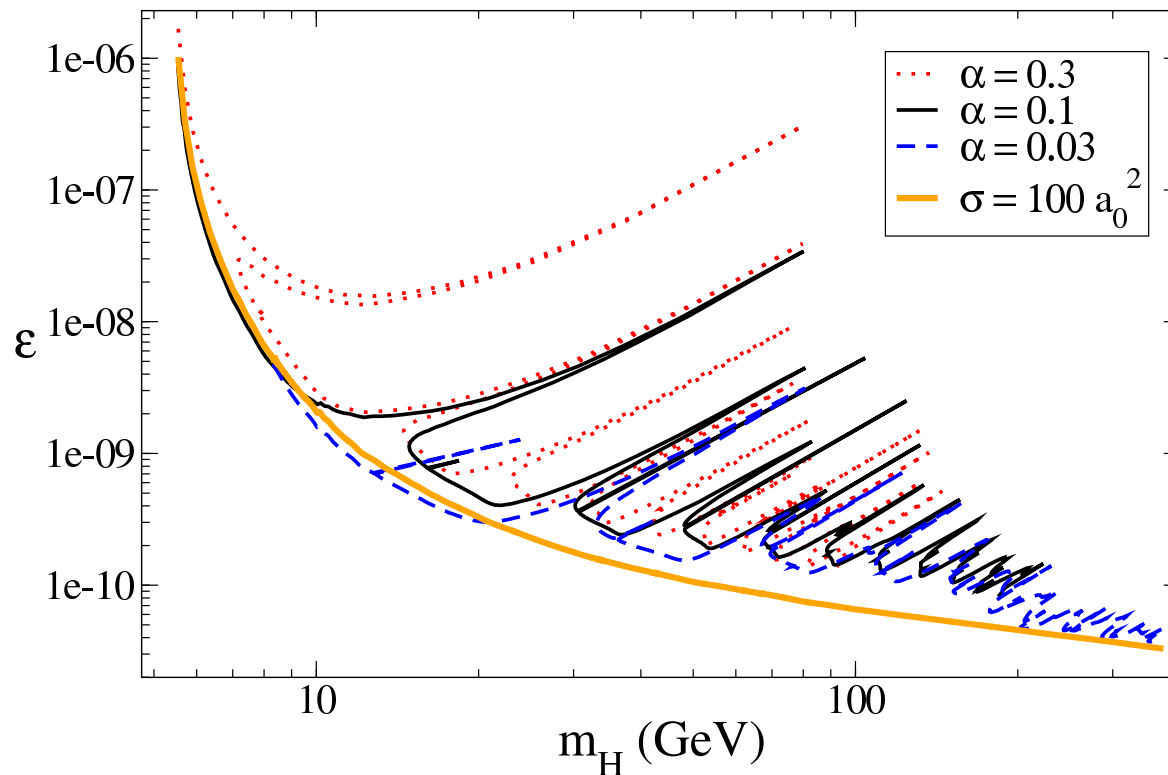
Thus for $R < 15.42$, dark molecules are not dissipative.

Direct detection of dark atoms

If dark photon kinetically mixes with normal photon via

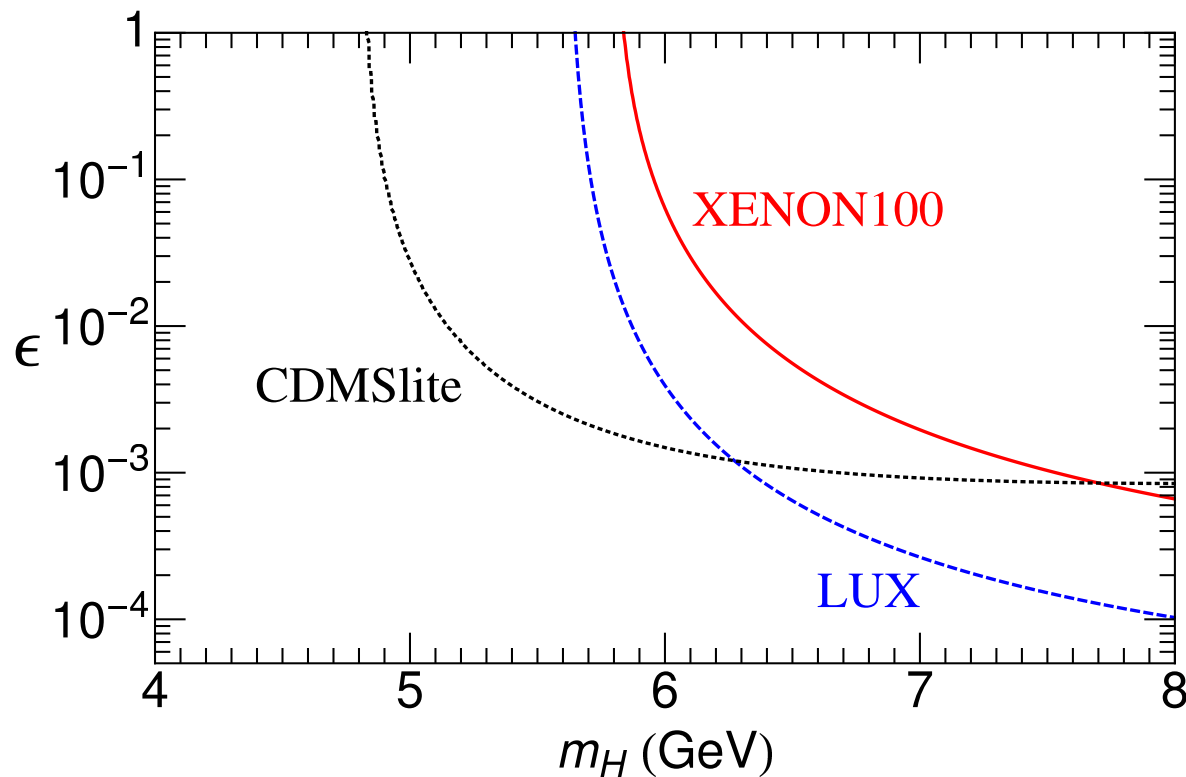
$$\frac{1}{2}\epsilon F^{\mu\nu} F'_{\mu\nu}$$

then dark constituents become millicharged $\pm\epsilon e$ and can scatter on protons with $\sigma_p = 4\pi(\alpha\epsilon\mu_{pH})^2 a_0^4$. Using SIDM constraint to eliminate R , we get LUX upper bound on ϵ :



Direct detection of dark atoms

$R = 1$ ($m_e = m_p$) is a special case. Scattering of H on p is by magnetic dipole transition, much weaker than screened Coulomb interaction:



Dark mesons

Consider QCD-like dark sector with N_f identical flavors of quarks. We can use chiral Lagrangian

$$\frac{F_\pi^2}{4} \text{tr} (\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) + \frac{\xi}{4} F_\pi^3 \text{tr} (M \Sigma + \text{h.c.})$$

to predict $\pi\pi \rightarrow \pi\pi$ scattering cross section, where $\Sigma = e^{2i\pi^a T_a / F_\pi}$. Find

$$\sigma = \frac{m_\pi^2}{32\pi F_\pi^4} C(N_f)$$

where $C(N) = (2N^4 - 25N^2 + 90 - 65/N^2)/(N^2 - 1)$. m_π/F_π is free parameter. If $m_\pi/F_\pi = 1.5$ as in QCD, then $\sigma/m = 1.1 \text{ b/GeV}$ requires

$$m_\pi = 33, 36, 61, 83, 100 \text{ MeV}$$

for

$$N_f = 2, 3, 4, 5, 6$$

Dark meson relic density

If the dark quarks couple to a light $U(1)'$ gauge boson Z' , relic density can be determined by $\pi\pi \rightarrow Z'Z'$. Leading operators are

$$\frac{\lambda_1}{4F_\pi^2} Z'_{\mu\nu} Z'^{\mu\nu} \text{tr}(\partial_\alpha \Sigma^\dagger \partial^\alpha \Sigma) + \frac{\lambda_2}{4F_\pi^2} Z'_{\alpha\mu} Z'^{\nu\alpha} \text{tr}(\partial^\mu \Sigma^\dagger \partial_\nu \Sigma)$$

where we expect $\lambda_i \sim \alpha'/4\pi$. Need $\alpha' \sim 10^{-5}$ to get the right relic density.

Danger: if Z' and photon kinetically mix via $\epsilon F^{\mu\nu} Z'_{\mu\nu}$ (as is natural), we can have $Z' \rightarrow e^+e^-$, hence

$$\pi\pi \rightarrow Z'Z' \rightarrow 2(e^+e^-)$$

in early universe, which is ruled out by CMB for such small m_π . Can avoid this problem if Z' is massless!

Need $\epsilon \lesssim 10^{-3}$ to avoid too much $\pi\pi \rightarrow Z'\gamma$ in CMB, since dark quarks become millicharged.

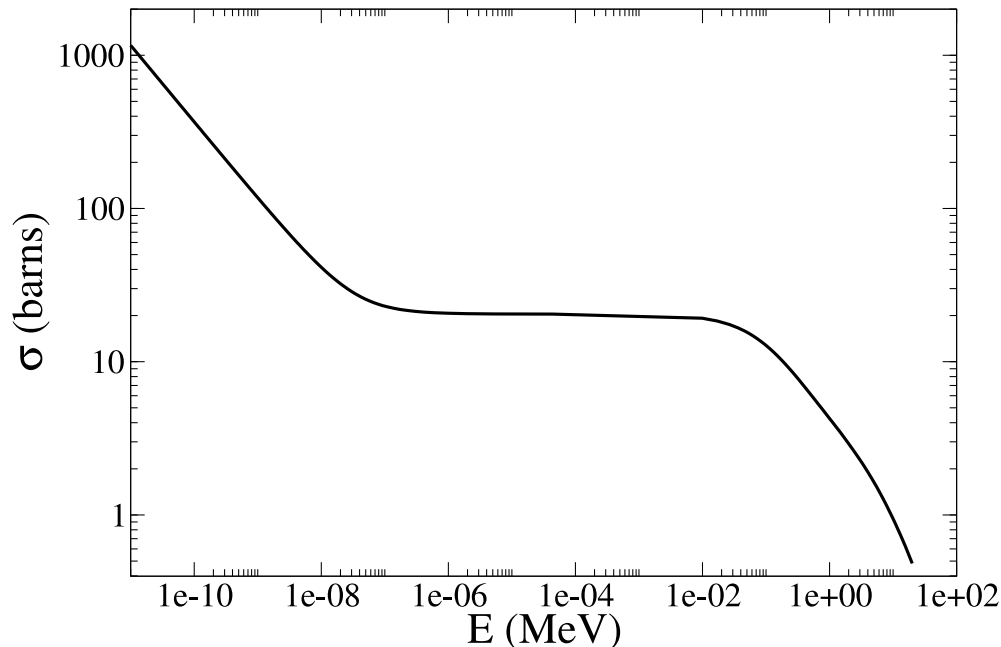
Dark “baryons”

Suppose that nucleons of a strongly-interacting hidden sector are the DM.

How big is σ/m_N for NN scattering? Naive estimate:

$$\sigma \sim 4\pi\Lambda^{-2}, \quad m_N \sim N_c\Lambda$$

for dark confinement scale Λ . Predicts $\sigma/m_N \sim 0.4 \text{ b/GeV}$ for QCD—too low by factor of 50 compared to observed value!



neutron-proton scattering cross section versus energy

(electromagnetic interaction dominates at very low E)

The weakly bound deuteron

p - n scattering is resonantly enhanced by deuteron intermediate state:

$$pn \rightarrow D \rightarrow pn$$

Enhancement due to small binding energy $E_B = 2.2$ MeV of deuteron:

$$\frac{\sigma}{m_N} \rightarrow \frac{2\pi}{N_c \Lambda^2 E_b} \quad \left(\text{c.f. } \frac{4\pi}{N_c \Lambda^3} \right)$$

How to generalize this to other QCD-like theories with different fundamental parameters? How does E_b scale?

Lattice gauge theorists have done it for us! (though not in terms of E_b).

NN scattering lengths

As $E \rightarrow 0$, cross section approaches

$$\sigma = \pi(a_s^2 + 3a_t^2)$$

where $a_{s,t}$ are singlet/triplet scattering lengths (deuteron has spin 1 and so is in triplet channel).

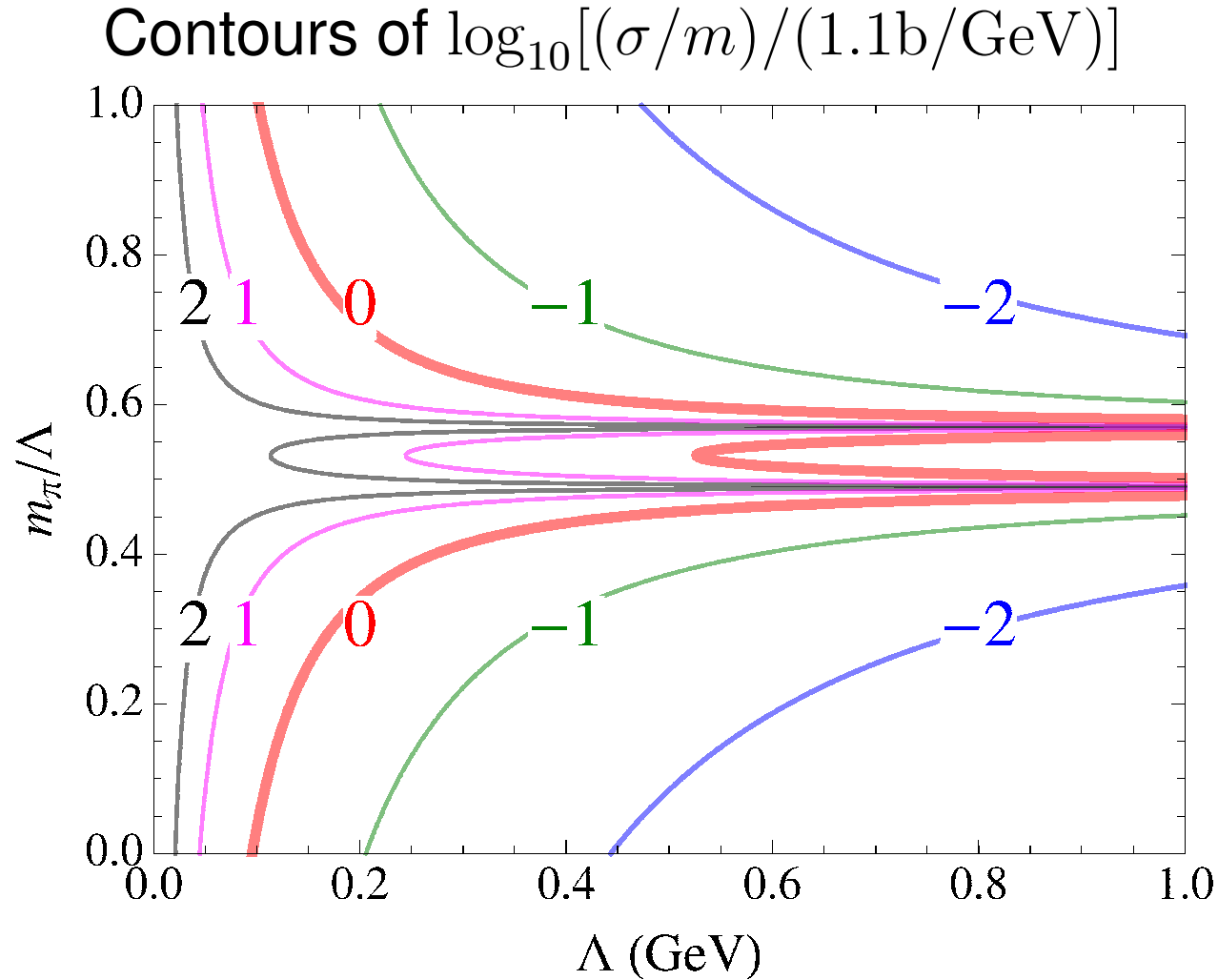
Lattice gauge theorists **Chen *et al.*, 1012.0453** computed $a_{s,t}$ in QCD as function of m_π . We extract

$$a_s = \frac{0.58 \Lambda^{-1}}{m_\pi/\Lambda - 0.57}, \quad a_t = \frac{0.39 \Lambda^{-1}}{m_\pi/\Lambda - 0.49}$$

by dimensional analysis (Λ is only other scale in problem).

We can compute σ/m_N for any m_π , Λ , assuming $m_N = 3.8 \Lambda$ as in QCD.

SIDM prediction for dark baryons



Note that $m_\pi = 0$ is allowed, so that π would contribute only to dark radiation, not dark matter.

Typical dark baryon mass is $O(\text{GeV})$.

Dark baryon relic density

Like normal baryons, dark ones are asymmetric DM, need some baryogenesis mechanism, which we don't provide.

How to ensure that dark pions aren't the DM?

- Could have $m_\pi = 0$
- Could have strong $\pi\pi \rightarrow Z'Z'$ annihilation, followed by $Z' \rightarrow e^+e^-$ decay
- Could have unstable $\pi \rightarrow e^+e^-$ via effective operator

$$c_{ij}\Lambda_h^{-2}(\bar{\mathbf{q}}_i\gamma_5\gamma_\mu\mathbf{q}_j)(\bar{e}\gamma_5\gamma^\mu e)$$

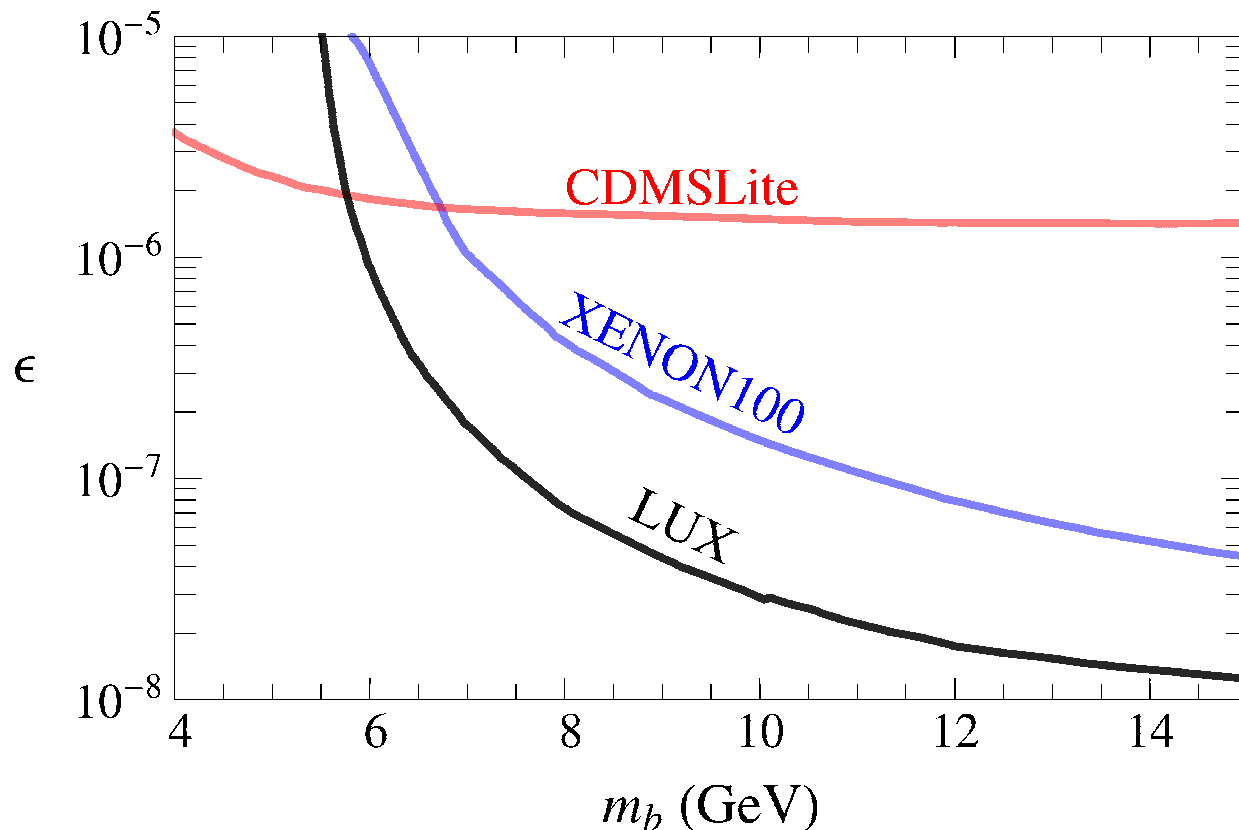
induced by heavy Z' exchange, with $\Lambda_h \lesssim 10$ TeV to avoid BBN constraint on π contribution to energy density.

Dark baryon direct detection

If quarks interact with kinetically mixed, massive Z' , then dark baryons scatter on protons with cross section

$$\sigma_{pb} = 144\pi \alpha \alpha' \epsilon^2 \frac{\mu^2}{m_{Z'}^4}$$

Direct detection constraints on ϵ :



Assuming
 $g' = 1$,
 $m_{Z'} = 1$ GeV.

Bound scales
as $m_{Z'}^2/g'$ for
other values.

Dark glueballs

If dark quarks are very heavy or nonexistent, glueballs of hidden SU(N) can be the dark matter, with mass and cross section

$$m_\phi \sim 5.5 \Lambda, \quad \sigma \sim \frac{4\pi}{\Lambda^2}$$

Strongly interacting DM implies

$$m_\phi \cong 500 \text{ MeV}, \quad \Lambda \cong 90 \text{ MeV}$$

Hard to explain relic density of dark glueballs—may need to rely on initial conditions from reheating after inflation.

Need to assume some interactions between glueballs and SM to say anything more interesting. We assume some effective interaction

$$\frac{1}{\Lambda_h^n} G_{\mu\nu} G^{\mu\nu} \mathcal{O}_{\text{sm}}$$

where G is SU(N) field strength and \mathcal{O}_{sm} has dimension n .

CMB constraint on glueball decays

If \mathcal{O}_{sm} involves e, μ, γ , glueballs can decay into these particles. The CMB will be distorted unless $\tau \gtrsim 10^{24}$ s. For decays into e^+e^- , we find

$$\left(\frac{\Lambda_h}{m_\phi}\right)^n \gtrsim 10^{19}$$

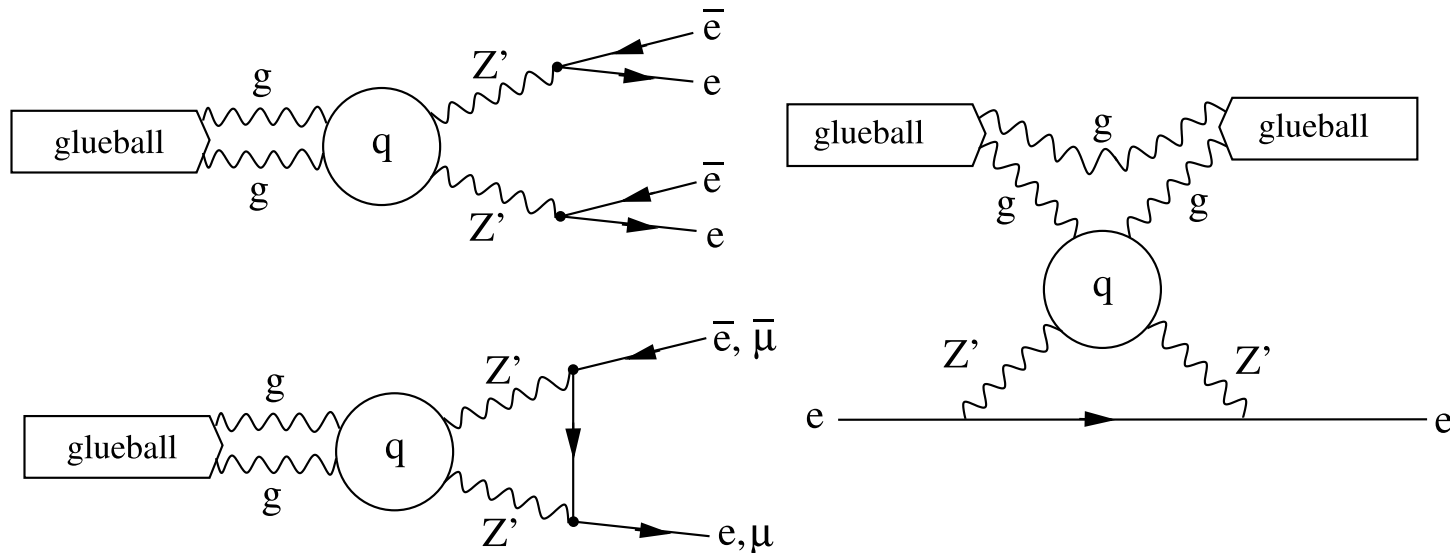
Thus direct detection of glueballs is impossible. *E.g.*,

$$\sigma_{\phi e} \sim m_\phi^{-2} \left(\frac{m_\phi}{\Lambda_h}\right)^{2n} \lesssim 10^{-66} \text{ cm}^2$$

Similarly, thermal origin of glueballs via annihilations does not work since cross section is too small.

Explicit model of glueball interactions

Let heavy dark quarks and visible quarks/leptons couple to massive Z' gauge boson with strength α' . We get diagrams for glueball decay and scattering



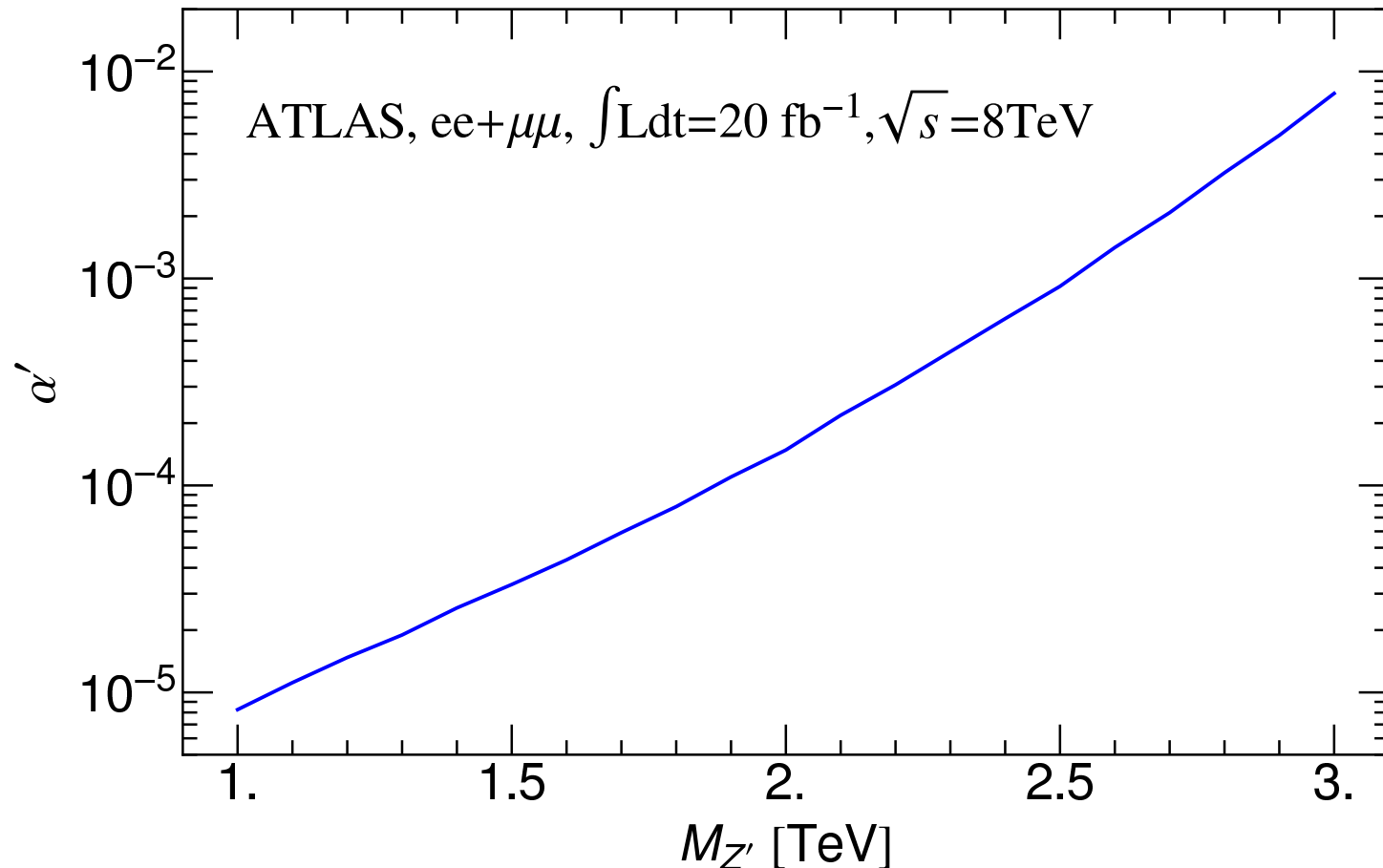
2- and 4-body decays become comparable if $m_q \cong 0.7 \text{ GeV}$.
CMB gives lower bound on $m_{Z'}$:

$$m_{Z'} \gtrsim 2.3 \text{ TeV} \left(\frac{\alpha_N \alpha'^2}{10^{-5}} \right)^{1/4} \begin{cases} x^{-1}, & x < 1 \\ 1, & x > 1 \end{cases}$$

where $x = m_q / (0.7 \text{ GeV})$ and α_N is SU(N) coupling.

LHC bound on $m_{Z'}$

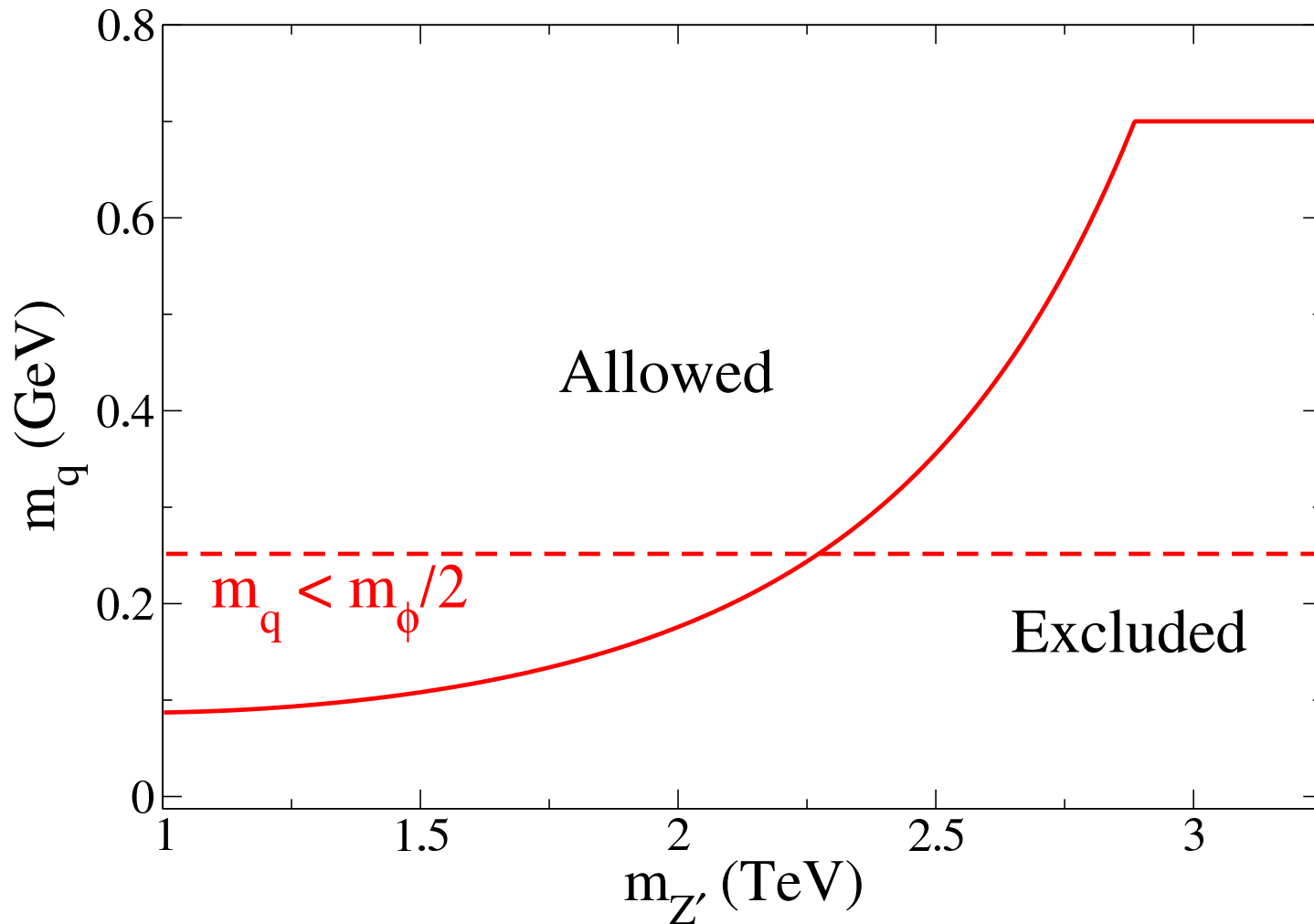
ATLAS also constrains $m_{Z'}$ versus α' from searches for $Z' \rightarrow e^+e^-, \mu^+\mu^-$:



How does this compare to CMB bound (which also depends on m_q)? Suppose the ATLAS constraint is saturated ...

CMB bound on m_q versus $m_{Z'}$

If LHC constraint is saturated, can eliminate α' in favor of $m_{Z'}$.
Then CMB constraint looks like



The two constraints can be of comparable strength (on the verge of discovery) for reasonable parameter values.

Other mediators between glueballs and SM

We also studied Higgs and neutrino portal interactions

$$\frac{\lambda\alpha_N}{m_S^2} G_{\mu\nu}G^{\mu\nu} |H|^2, \quad \Lambda_h^{-5} (LH)^2 G^2 = \Lambda'_h{}^{-3} \bar{\nu}\nu GG$$

induced by heavy scalars S charged under the dark $U(N)$.
CMB bound implies

$$m_S > 10^7 \text{ GeV} \left(\frac{\lambda\alpha_N}{0.01} \right)^{1/2}, \quad \Lambda'_h > 5600 \text{ TeV}$$

beyond reach of LHC or other means of detection.

Z' mediator offers best prospects for independent evidence of dark sector.

Conclusions

- Composite DM models offer natural means of getting strong self-interactions to solve core-cusp and TBTF problems of standard cold DM
- Further interactions between DM and SM are not mandatory, but likely, *e.g.*, through gauge kinetic mixing
- Direct detection gives interesting bounds on kinetic mixing for atomic and nuclear DM: ϵ could generically be larger than the bounds.
- Dark glueballs are unstable if they interact with SM, and so strongly constrained by CMB that scattering on SM particles is unobservably small. But a Z' mediator with $m_{Z'} \gtrsim 2 \text{ TeV}$ could provide indirect evidence.