Limits on Weak Gauge-Breaking Operators of Effective Field Theories

arXiv:1503.07874, with N.F. Bell, Yi Cai, J.B. Dent, and R.K. Leane

Tom Weiler
Vanderbilt University, Nashville, TN, USA

21 April, 2015

A Concrete Example: Bai and Tait (arXiv:1208.4361) proposed a simple weak gauge-broken operator for dark matter (DM) production at the LHC:

$$\frac{1}{\Lambda^2}(\bar{\chi}\gamma^{\mu}\chi)\,(\bar{u}\gamma_{\mu}u + \xi\,\bar{d}\gamma_{\mu}d)$$

At the parton level, this operator characterizes the process $u(p_1)\bar{d}(p_2) \to \chi(k_1)\bar{\chi}(k_2) + W^+(q)$, as shown in the diagrams

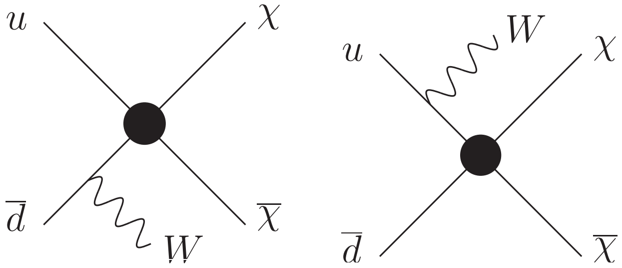

Figure : M_1 and M_2 amplitudes for the mono-W process $u(p_1)\overline{d}(p_2) \to \chi(k_1)\overline{\chi}(k_2)W^+(q)$, in the effective field theory framework.

Bai and Tait note that the mono-W process is unique among the mono-X processes, in its ability to probe different DM couplings to u and d quarks.

Their operator is very popular (many citations), and has been used by ATLAS and CMS in analysis and publication:

• G. Aad et al. (ATLAS Collaboration), Phys.Rev.Lett. 112, 041802 (2014), arXiv:1309.4017 [hep-ex]

• G. Aad et al. (ATLAS Collaboration), JHEP 1409, 037 (2014), arXiv:1407.7494 [hep-ex]

• CMS-Collaboration(2013), CMS-PAS-EXO-13-004

• V. Khachatryan et al. (CMS Collaboration)(2014), arXiv:1408.2745 [hep-ex]

Clearly, the operators is $SU(2)_L$-invariant ($u \leftrightarrow d$) only for $\xi = 1$.
For general ξ, it is NON-invariant.
The claim is that for $\xi < 1$, there is an interference between M_1 and M_2 which can greatly enhance the rate of DM production.
As I will now explain, we see this differently:

1. The $\left(\frac{vev}{\Lambda}\right)^n$ suppression.
2. Goldstone boson equivalence and the Ward identity.
4. The role of the longitudinal mode W_{Long}.
3. Leading s behavior and unitarity.

Our arguments apply to all weak gauge NON-Invariant operators.
Examples are $(\bar{\chi}\Gamma\chi)\times$
(a) for scalar mediators (integrated out),
$(\bar{u_L} u_R + \bar{u_R} u_L)$ independent of $(u \rightarrow d)$;
(b) for vector mediators (integrated),
$(\bar{u_L}\gamma^\mu u_L + \bar{u_R}\gamma^\mu u_R) \neq (u \rightarrow d)$.
(Also, applies to nuclear isospin-breaking models, but these may still apply at direct detection energies (keV)).

1. The $\left(\frac{vev}{\Lambda}\right)^n$ suppression.

Operators which are non-invariant under the weak symmetry must vanish as the weak $vev \to 0$. Thus, there is one or more implicit powers of vev in the operator ("Wilson") coefficient.

Thus, there is a suppression $\left(\frac{vev}{\Lambda}\right)^n$, making the operator magnitude compete with higher dim ops.

The operator is significantly suppressed above the EW scale $\sim vev \sim 250$ GeV.

For example, the $(\bar{u_L} u_R + \bar{u_R} u_L)$ operator is manifestly not $SU(2)_L$ invariant, as u_L is a weak doublet and u_R is a singlet. This costs one power of $\left(\frac{vev}{\Lambda}\right)$.

As another example, $(\bar{u_L}\gamma^\mu u_L + \bar{u_R}\gamma^\mu u_R) \neq (u_L \to d_L)$ costs two powers of $\left(\frac{vev}{\Lambda}\right)$, one for each mismatched $u \leftrightarrow d$.

The vev serves to instill quantum numbers of a weak doublet.

The equivalence of the *vev* as a weak doublet happens because the *vev* is a remnant of the $SU(2)_L$ scalar doublet

$$\Phi \equiv \left(\begin{array}{c} \phi^+ \\ \phi^0 = \frac{1}{\sqrt{2}}(H + vev + i\Im\phi^0) \end{array} \right) . \tag{1}$$

Enough powers of Φ are required to form an $SU(2)_L$-invariant operator. The fields $\phi^\pm$ and $\Im\phi$ are gauged away to become, in unitary gauge, the longitudinal modes of the $W^\pm$ and Z. So, it is the real, neutral field $\frac{1}{\sqrt{2}}(H + vev)$ whose n^{th} power appears in the operator. Commonly, the H part of the expression is omitted, leaving just an implicit vev^n in the coefficient. Dimensionally, the vev^n comes with a Λ^{-n}.

We remark that omission of the H part in the operator may ignore some interesting phenomenology.

2. Goldstone boson equivalence and the Ward identity.

Since the two amplitudes of Figure 1, with $\xi \neq 1$, are not gauge invariant, they will not satisfy the relevant Ward Identity.

At high energy, the Goldstone boson equivalence theorem requires that the amplitude for emission of a longitudinally polarized W_L is equivalent to that for the emission of the corresponding Goldstone boson. Since the Goldstone boson couples to quarks with strength proportional to their mass, these terms are close to zero. The Ward identity for the longitudinal W at high energy therefore takes the form

$$\mathcal{M}^{\alpha}\epsilon^{L}_{\alpha} \approx \frac{q_{\alpha}}{m_W}\mathcal{M}^{\alpha}(q,...) = i\mathcal{M}(\phi^{+}(q)) \simeq 0. \tag{2}$$

For the sum of the mono-W amplitudes of Fig.1 we find

$$q_{\alpha}\mathcal{M}^{\alpha} = \frac{g_W}{\Lambda^2}\left[\bar{v}(p_2)\,(1-\xi)\,\gamma^{\mu}\frac{P_L}{\sqrt{2}}u(p_1)\right]\left[\bar{u}(k_1)\gamma_{\mu}v(k_2)\right], \tag{3}$$

which clearly vanishes only for $\xi = 1$.

3. The role of the longitudinal mode, W_{Long}.

The three polarization vectors for the W or Z bosons are orthogonal to the particle four-momentum, $q^\mu = (q^0; 0, 0, |\vec{q}|)$. We may choose this basis to be $\epsilon^\mu_{Tx} = (0; 1, 0, 0,)$, $\epsilon^\mu_{Ty} = (0; 0, 1, 0,)$, and $\epsilon^\mu_{Long} = (|\vec{q}|; 0, 0, q^0,)/M_V$. The polarization sum for the vector bosons is
$\sum_\lambda \epsilon^\lambda_\alpha \epsilon^{\lambda\,*}_\beta = -g_{\alpha\beta} + \frac{q_\alpha q_\beta}{m_W^2}$. The contribution of the transverse components are straightforward, leaving the subtlety, the $\frac{q_\alpha q_\beta}{m_W^2}$ term, to subtract an unphysical piece from $g_{\alpha\beta}$ that arises from the growing longitudinal mode. In fact, this longitudinal mode may be written as $\epsilon^\mu_{Long} = (\frac{|\vec{q}|; 0, 0, q^0,}{M_V}) = \frac{q^\mu}{M_V} - (\frac{M_V}{q^0 + |\vec{q}|})(1; 0, 0, 1)$
which makes it clear that at high-energy, ϵ^μ_{Long} approaches $(\frac{q^\mu}{M_V})$.
This result leads to two important observations:
First, that no high-energy longitudinal W is included in the previous operator, since current conservation argues against a q^μ insertion onto an external fermion leg, for any value of ξ.
So why does the operator calculation get a large enhancement?
Precisely because the polarization sum $\sum_\lambda \epsilon^\lambda_\alpha \epsilon^{\lambda\,*}_\beta = -g_{\alpha\beta} + \frac{q_\alpha q_\beta}{m_W^2}$

is used, incorrectly, for what should be a sum on just two transverse polarizations, $\sum_{\mathrm{T}} \epsilon_\alpha^{\mathrm{T}} \epsilon_\beta^{\mathrm{T}\,*} = \mathrm{diag}(0,1,1,0)$. For $\xi = 1$, gauge invariance leads to the correct cancellations to allow the previous insertion, but for $\xi \neq 1$, just the sum on transverse polarizations should be used.

The Second observation is that gauge invariance as given by a UV completion would yield a growing longitudinal W mode, but it would come from an internal brehmstrahlung process, as shown in a UV-completion model with an SU(2)-doublet, scalar particle called η, which must carry the same quantum numbers, color triplet, weak doublet, hyper charge $1/6$, as the Q_L doublet:

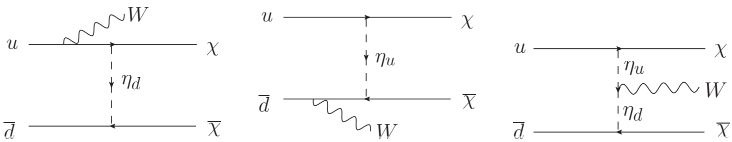

Figure : Contributions to the mono-W process in a UV complete model.

The vector W must couple derivatively to the η, which means it will couple to the mass-squared difference of the two components of the η-doublet, which in turn means proportional to the weak vev^2. It all works out, but now with an additional $\left(\frac{vev}{\Lambda}\right)^2$ suppression. BTW, a mass splitting of the η components is easily arranged in the model, via the term $\lambda_4(\Phi^\dagger\eta)(\eta^\dagger\Phi)$, which upon SSB becomes $\lambda_4\frac{vev^2}{2}\eta_d^2$.

In this UV-completed, renormalizable model, $\xi = (1 + \lambda_4\frac{vev^2}{2})^{-1}$, which shows that negative ξ is not possible in this model, and more importantly, that for $\Lambda \gtrsim 1$ TeV and a perturbative value for λ_4, ξ will not deviate far from 1.

(In progress, a look at mixing of new neutral vector with the (Z, γ) system, in spirit of theorem that in contrast to scalars, vectors can be repulsive as well as attractive; may get negative contribution to ξ, but ....)

3. Leading s behavior and unitarity.

The Optical Theorem (e.g., Itzykson and Zuber) implies that asymptotically, $|M_{elastic}(s)| < 16\pi \frac{t}{t_0}(\ln(s))^2$. Froissart generalized this result to the total cross section. So eventually, an s^2 behavior, coming from the longitudinal W mode in a gauge non-invariant model, cannot be sustained. The issue is, when in energy does the EFT break down. I show some plots relevant to this issue (for simplicity, we have taken $\frac{M_\chi^2}{s}$ to be zero):

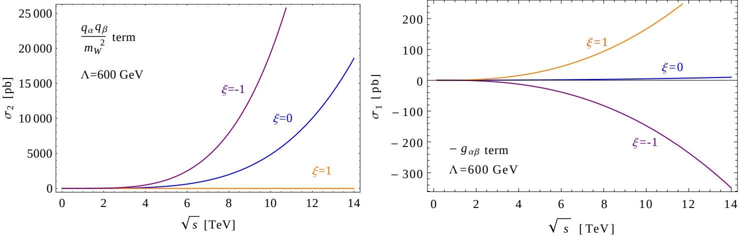

Figure : Total parton-level cross sections versus energy, for $\Lambda = 600$ GeV and various ξ. Upper: contribution from the $+q_\alpha q_\beta/m_W^2$ term in the polarization sum. The cross section scales simply as $(1-\xi)^2$. Lower: contribution from the $-g_{\alpha\beta}$ term. At LHC energies the $q_\alpha q_\beta$ terms dominates unless $\xi \simeq 1$. The separation into σ_1 and σ_2 is accomplished in the unitary gauge.
(Notice the differing vertical scales between the two panels.)

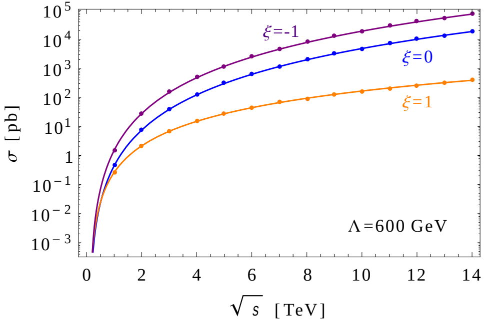

Figure : Total parton-level cross sections for $\Lambda = 600$ GeV, for particular choices of ξ. Solid lines are the analytic calculation and dots are the MadGraph calculation.

We find for the leading behavior in s,
$\sigma_{TOT} = \sigma_1 + \sigma_2 \to \frac{s}{\Lambda^4}(1 + (1-\xi)^2 \frac{s}{M_W^2})$, with the extra power in s coming from σ_2, i.e. from the longitudinal mode of the W. It is this extra power of s that warns us that the weak gauge-violating operator with $\xi \neq 1$ is to be cut off at the weak scale, here, $s \sim M_W^2$. For pp physics at the LHC, this would imply cutting off the parton fractional energies above $x_1 x_2 \sim \frac{M_W^2}{s_{\rm LHC}}$, i.e. at $\sim 10^{-4}$ for the $\sqrt{s} = 7$ TeV run, and at 4 times that for the $\sqrt{s} = 13$ TeV run.

In summary, we have argued that any $SU(2)$-violating difference in the u and d quark couplings must be protected by the EW scale, and therefore cannot be arbitrarily large.

Furthermore, we have shown that spurious terms associated with the longitudinal mode of the W would grow large enough to violate unitarity at high-energies. But even at lower energy, their presence appears to be problematic for the Ward identity. These spurious longitudinal W couplings are avoided only by using a renormalizable UV-completed theory, rather than weak gauge-breaking EFT operators.