Testing GR with Cosmology

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Oxford

Collaborators

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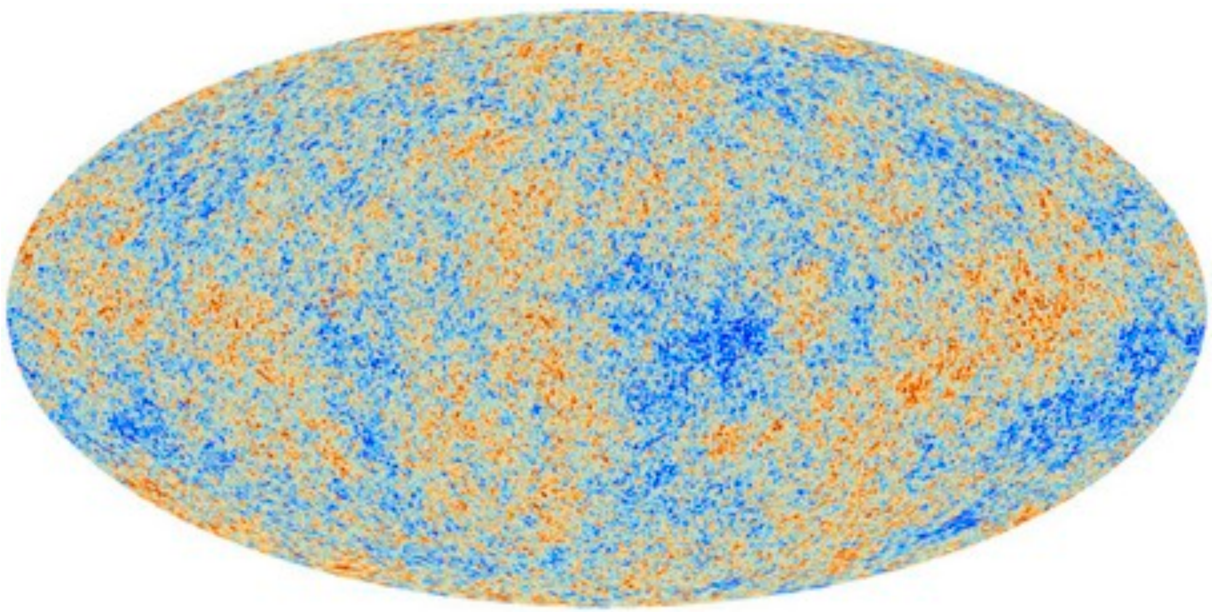
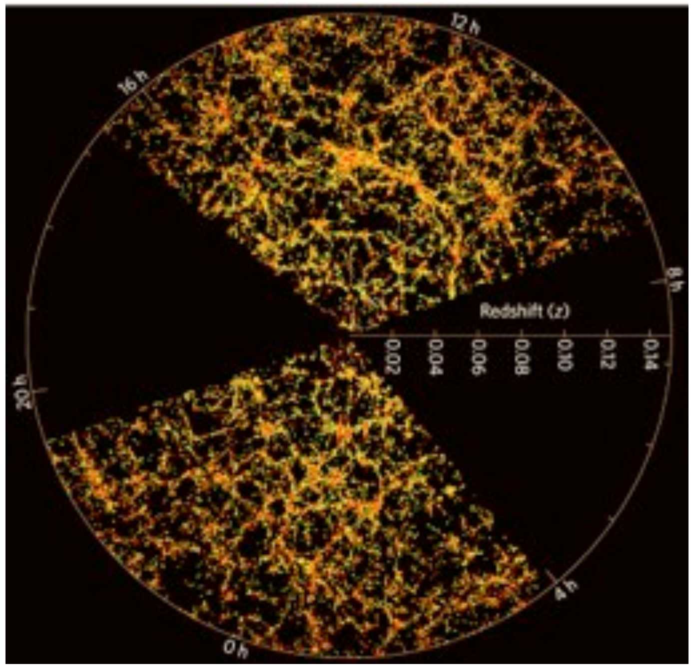
- Stefano Camera (IST)
- Tim Clifton (QMW)
- Ed Macaulay (Oxford/Sussex)
- Tony Padilla (Nottingham)
- Mario Santos (UWC)
- Constantinos Skordis (Nottingham)
- Joseph Zuntz (Manchester)

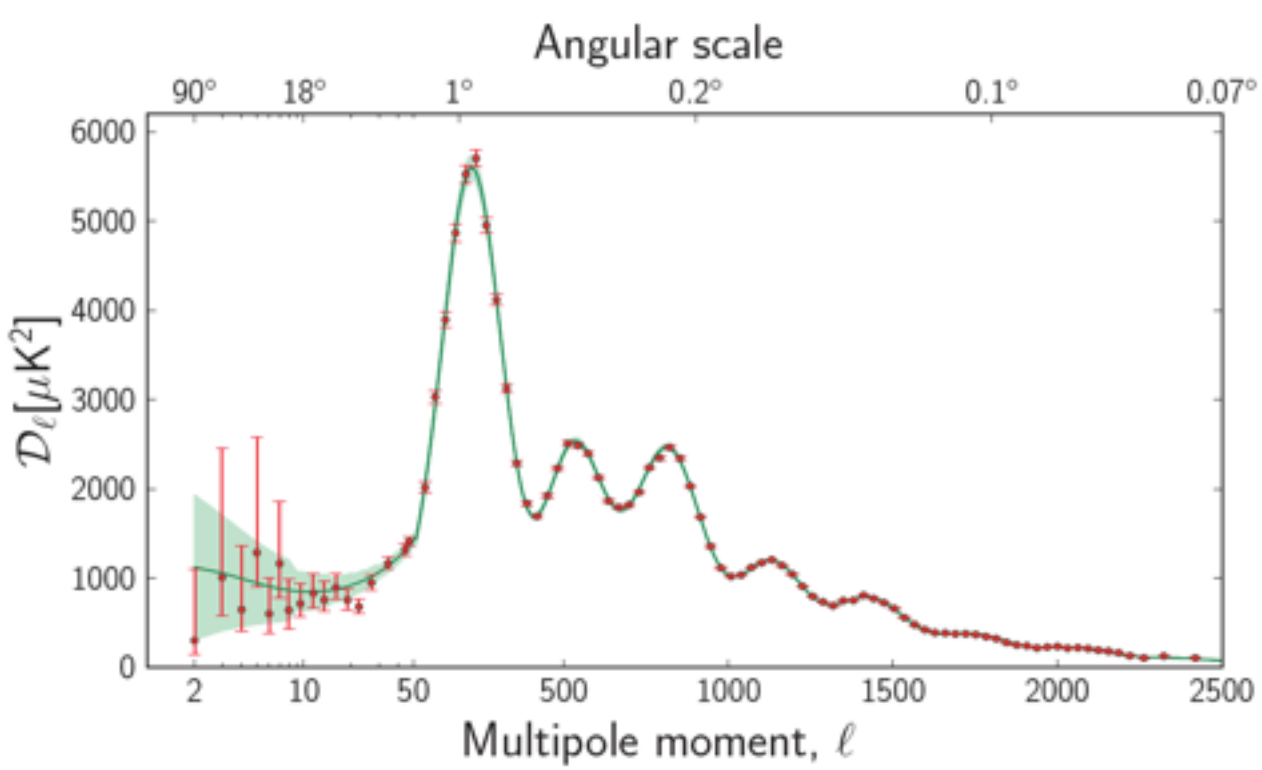
The Large Scale Structure of the Universe

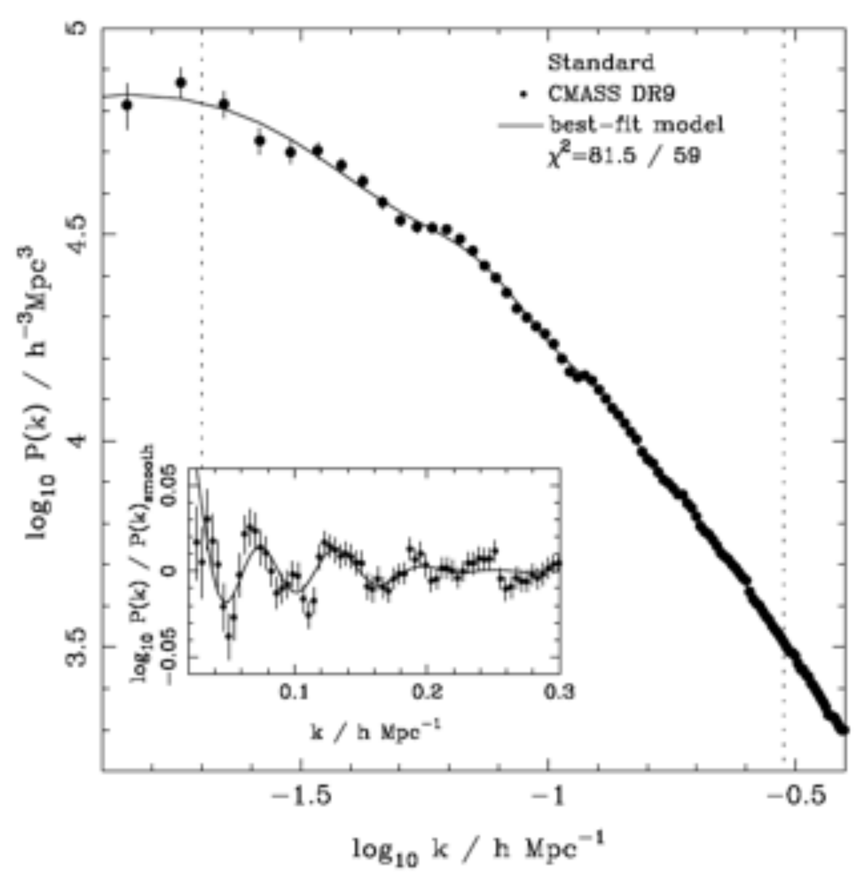
Planck



SDSS

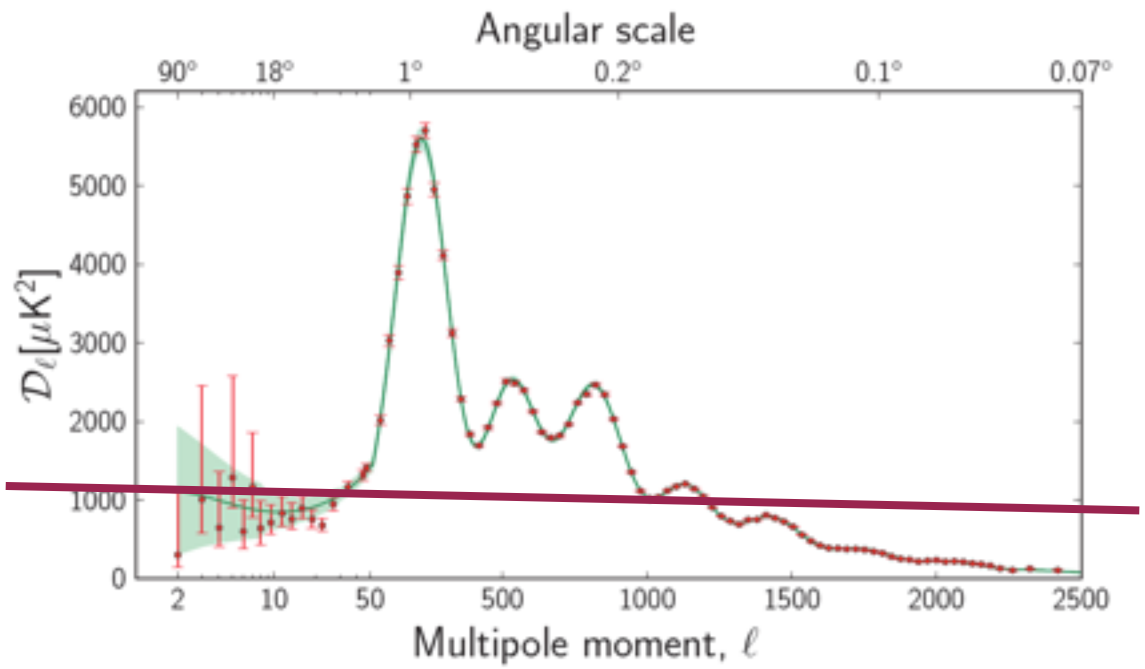






n_s

Primordial Tilt



1992 (COBE): $n_s = 1 \pm 0.6$

2001 (Max+Boom): $n_s = 1.03 \pm 0.09$

2009 (WMAP5): $n_s = 0.963 \pm 0.014$

2013 (Planck+): $n_s = 0.9603 \pm 0.0073$

Outline

- the panorama of gravitation
- the cosmological arena
- cosmological linear perturbations
- what data to look at
- the future

Einstein Gravity

Curvature

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}R(g) + \int d^4x\sqrt{-g}\mathcal{L}(g,\mathrm{matter})$$

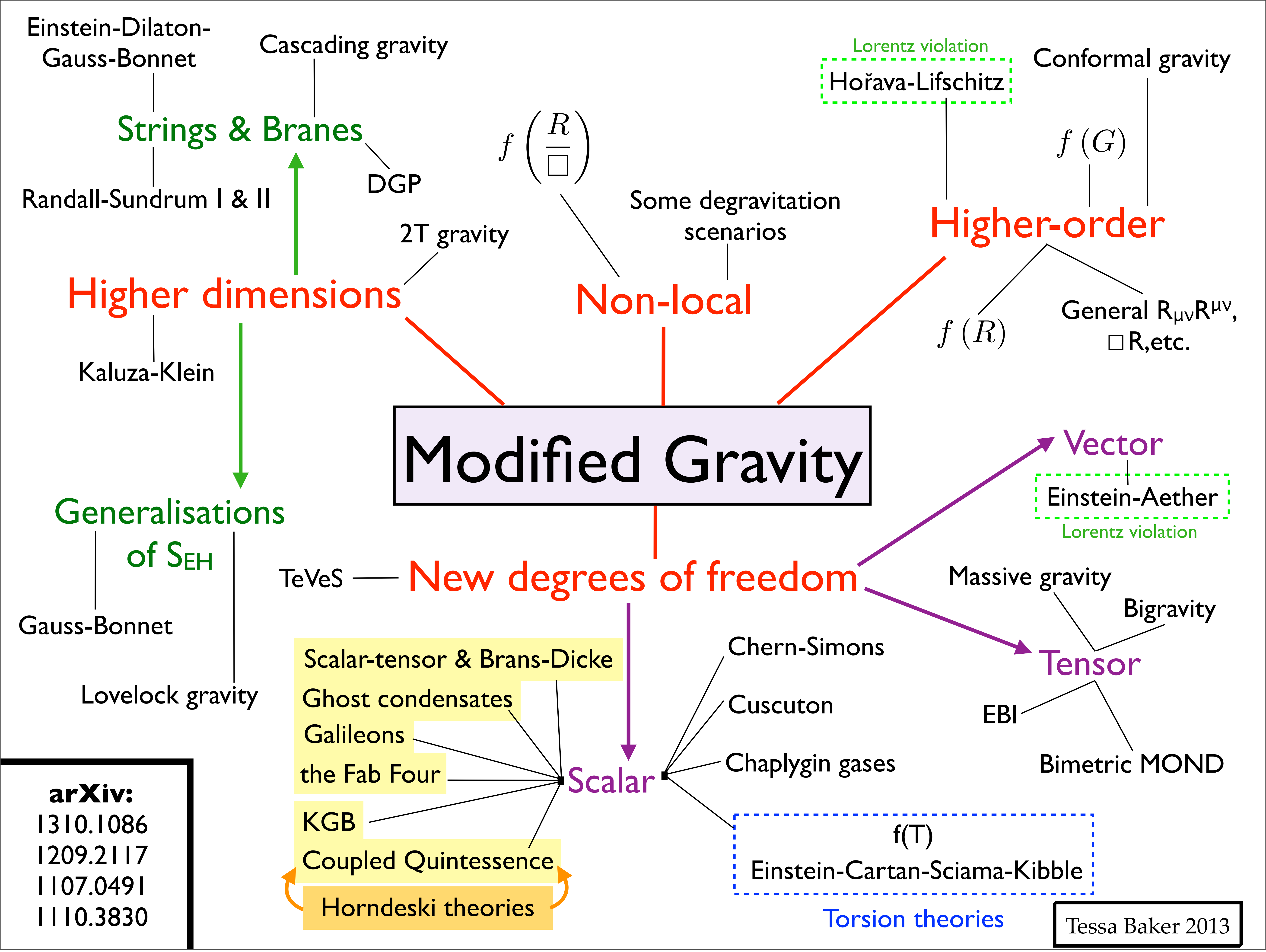
Metric of space time

Lovelock's theorem (1971) :*"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."*

"I think the best viewpoint is to pretend there are experiments and calculate. In this field we are not pushed by experiments- we must be pulled by imagination"

R. Feynman

GR1: Chapel Hill 1957


Modified Gravity
Higher dimensions
Strings & Branes
Einstein-Dilaton-Gauss-Bonnet
Cascading gravity
DGP
Randall-Sundrum I & II
2T gravity
Kaluza-Klein
Generalisations of S_{EH}
Gauss-Bonnet
Lovelock gravity
Non-local
$f\left(\frac{R}{\Box}\right)$
Some degravitation scenarios
Higher-order
Lorentz violation
Hořava-Lifschitz
Conformal gravity
$f(G)$
$f(R)$
General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.
New degrees of freedom
TeVeS
Vector
Einstein-Aether
Lorentz violation
Tensor
Massive gravity
Bigravity
EBI
Bimetric MOND
Scalar
Scalar-tensor & Brans-Dicke
Ghost condensates
Galileons
the Fab Four
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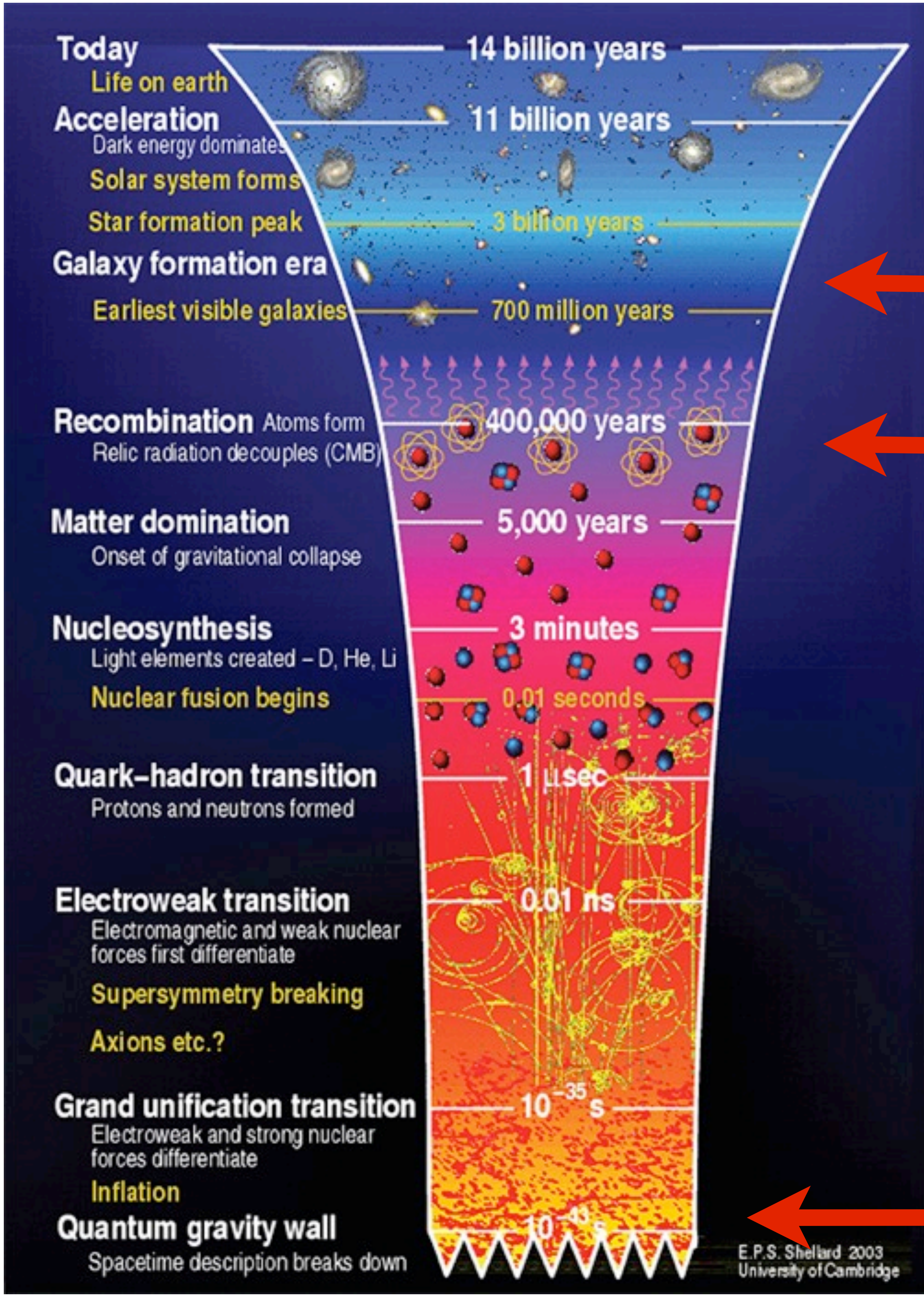
Torsion theories
arXiv:
1310.1086
1209.2117
1107.0491
1110.3830
Tessa Baker 2013

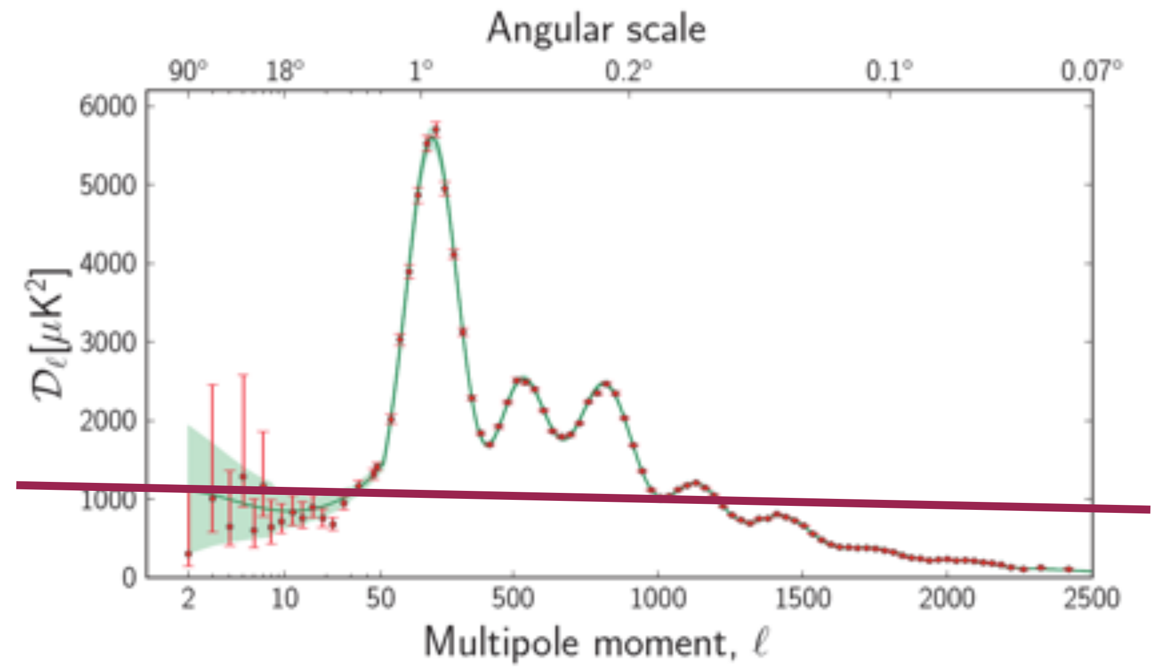


Reionization ("EoR")
Dark ages

Recombination

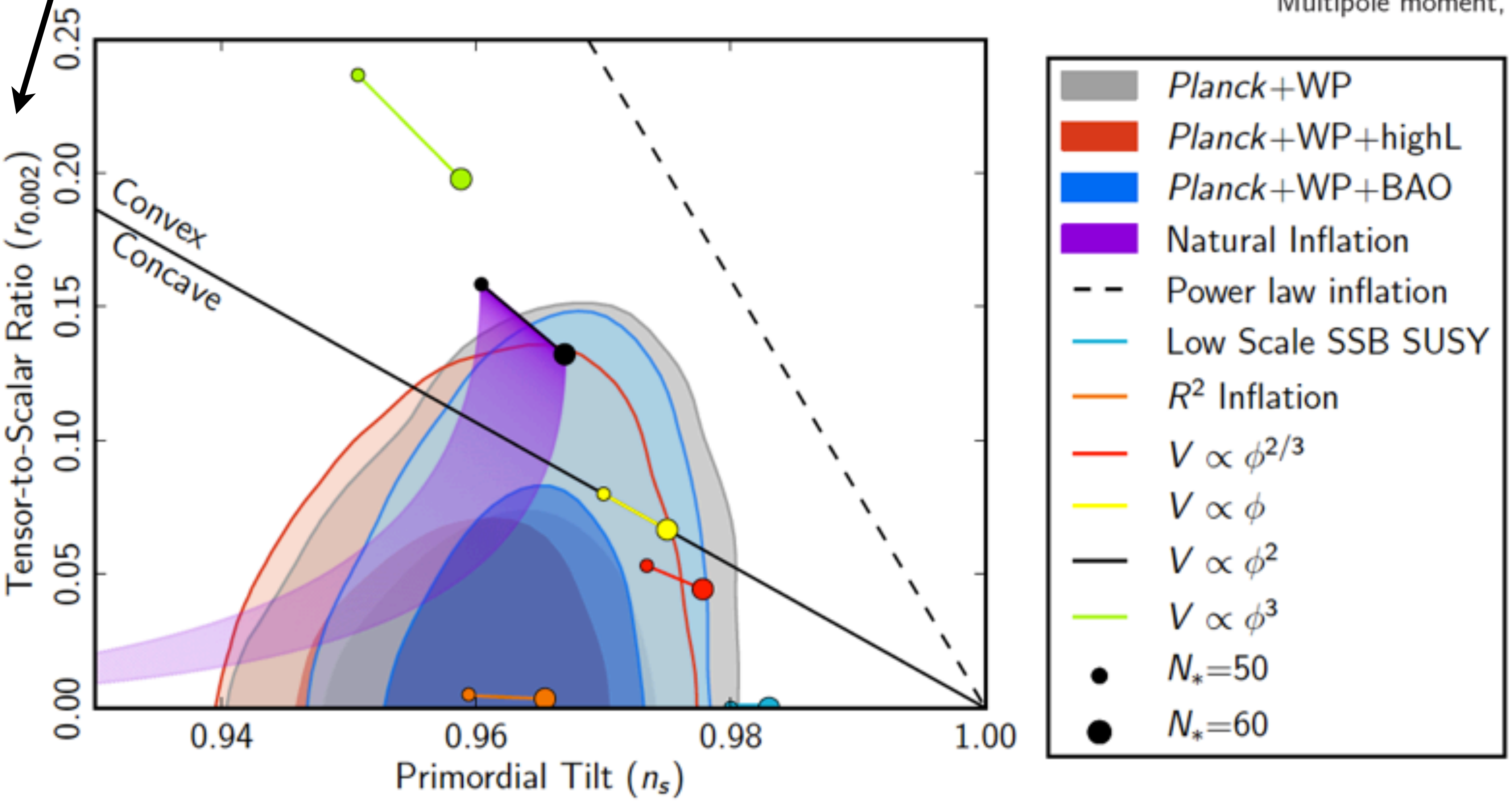
Initial Conditions

Initial Conditions



Primordial Tilt

Primordial Gravitational Waves

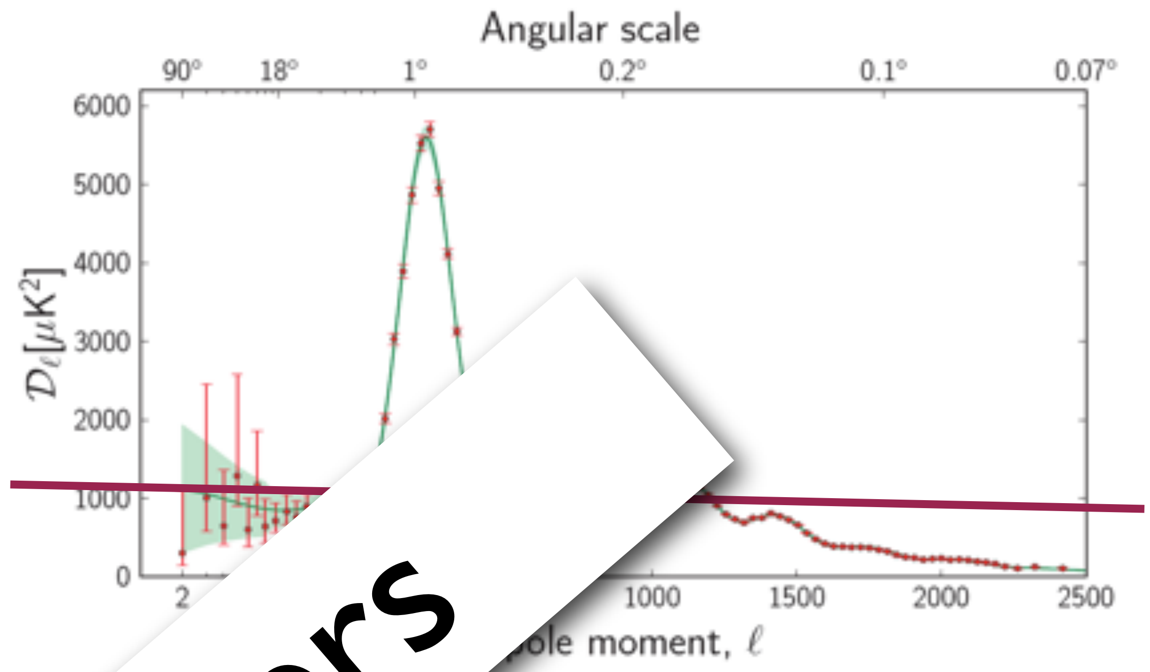


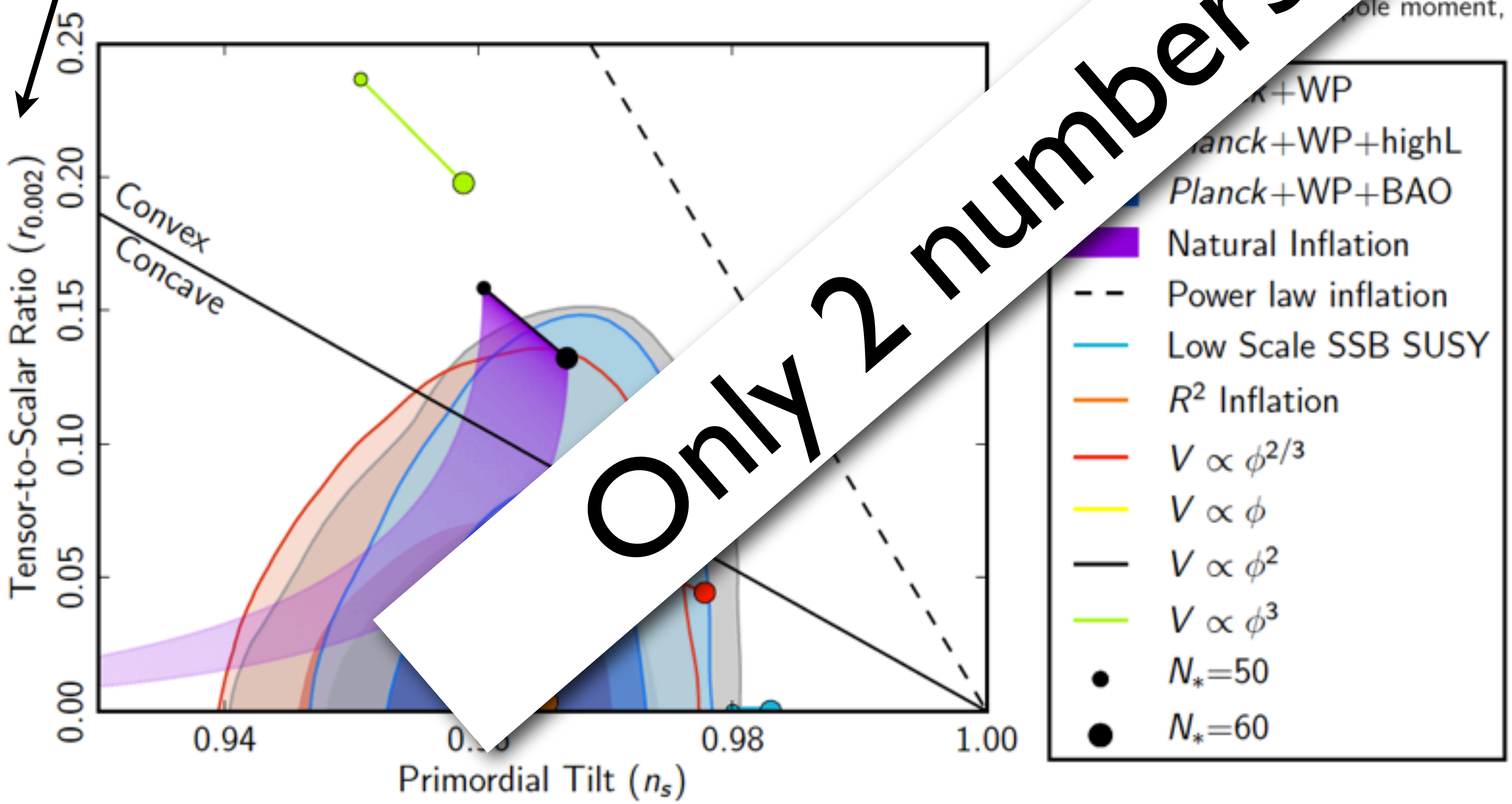
Planck XXII

Initial Conditions

Primordial Gravitational Waves

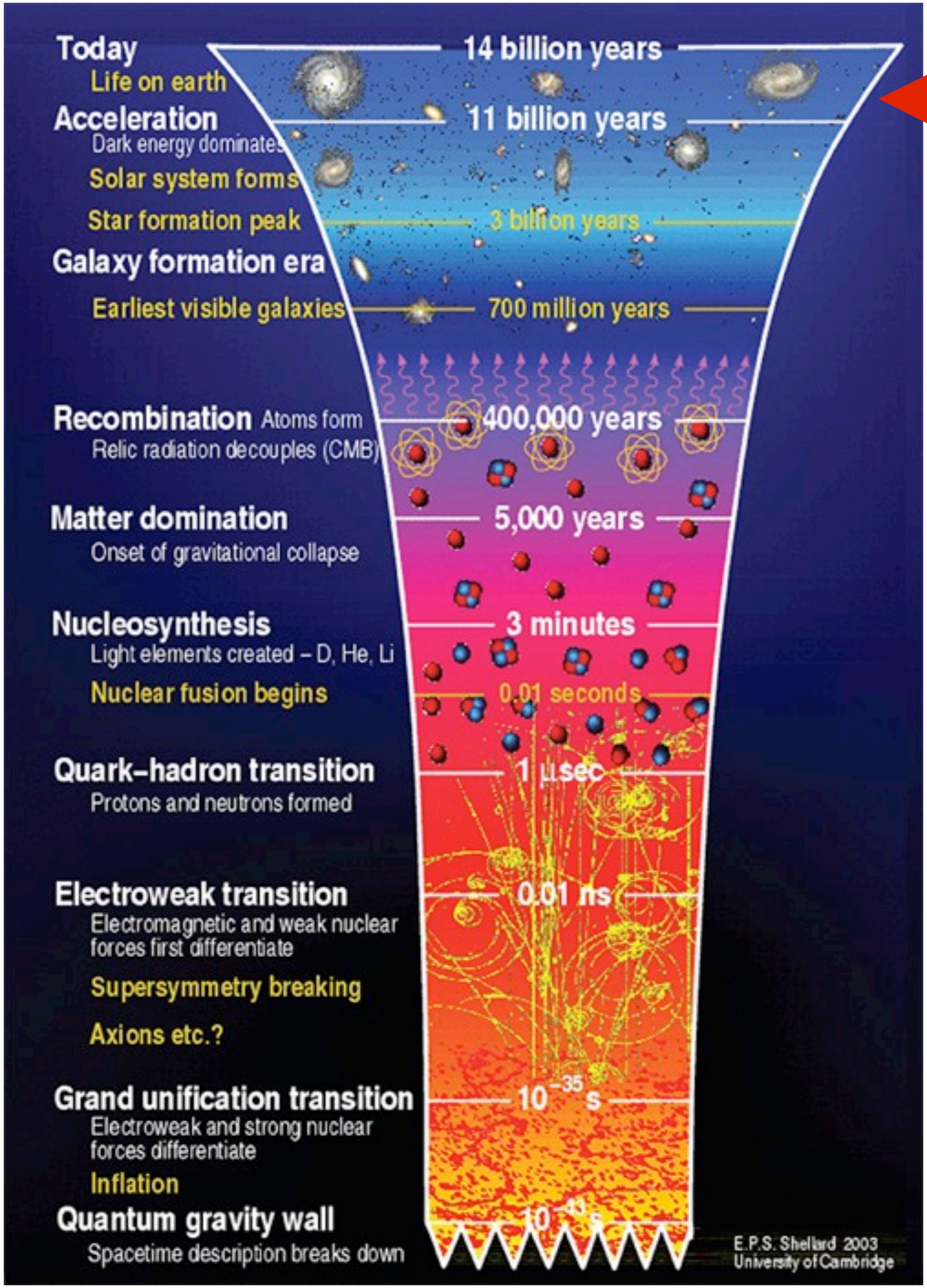
Primordial Tilt





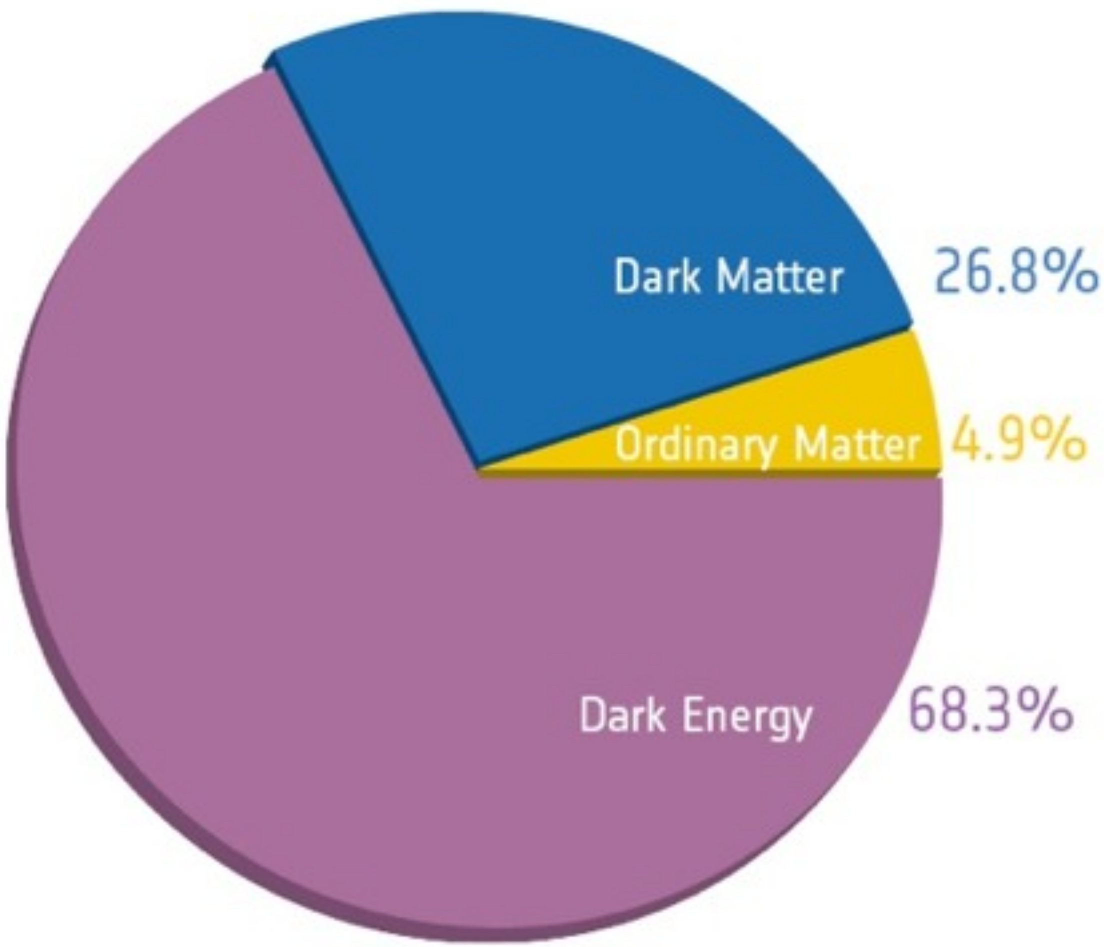
Only 2 numbers

Planck XXII

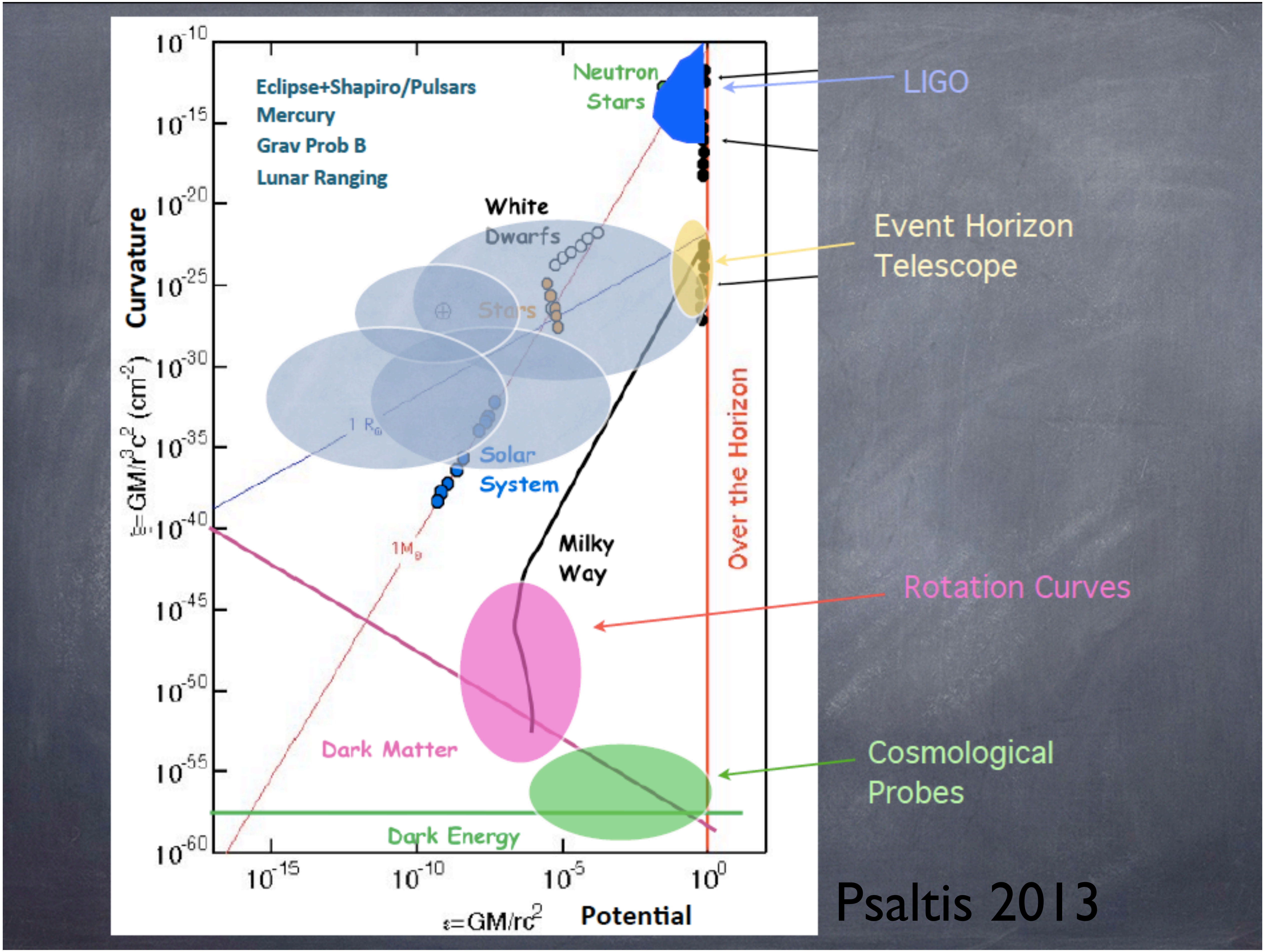


Acceleration

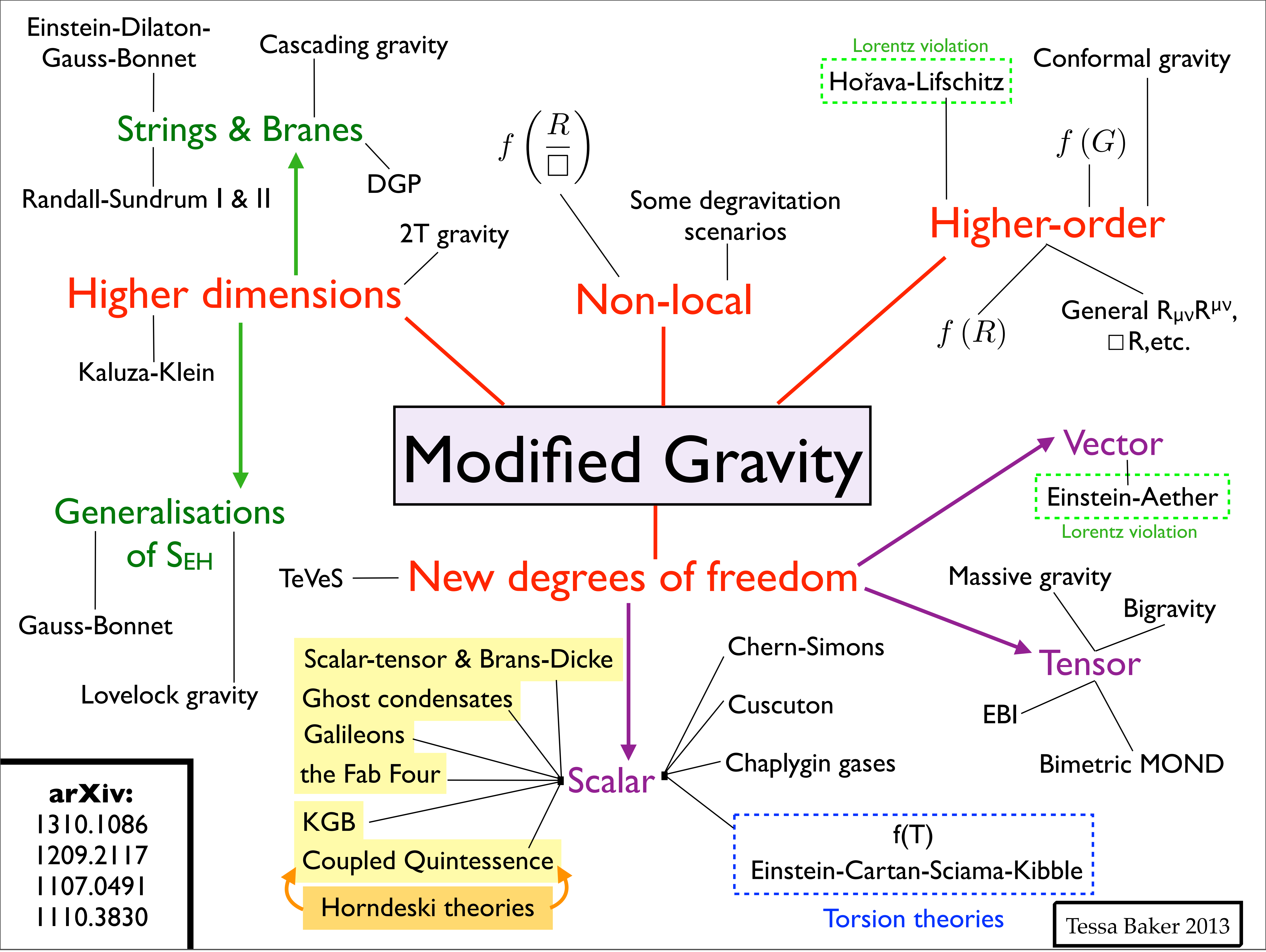
Where strange things *do* happen...



Planck XVIII



Psaltis 2013



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Tessa Baker 2013

An agnostic view: lessons from PPN

`Parameterized Post-Newtonian'

(Will, Nordvedt & Thorne)

Expand around weak-field metric in a set of 10 parameters:
γ, β, ξ, α_1, α_2, α_3, ζ_1, ζ_2, ζ_3, ζ_4

$$-g_{00}(r) = 1 - \frac{2GM}{c^2 r} + 2(\beta - \gamma)\left(\frac{2GM}{c^2 r}\right)^2 \qquad g_{rr}(r) = 1 + \gamma \frac{2GM}{c^2 r}$$

Perform similar expansion in non-GR theory.
Map theory onto parameters.

E.g. in Brans-Dicke theory: $\gamma = \dfrac{1 + \omega_{BD}}{2 + \omega_{BD}}$

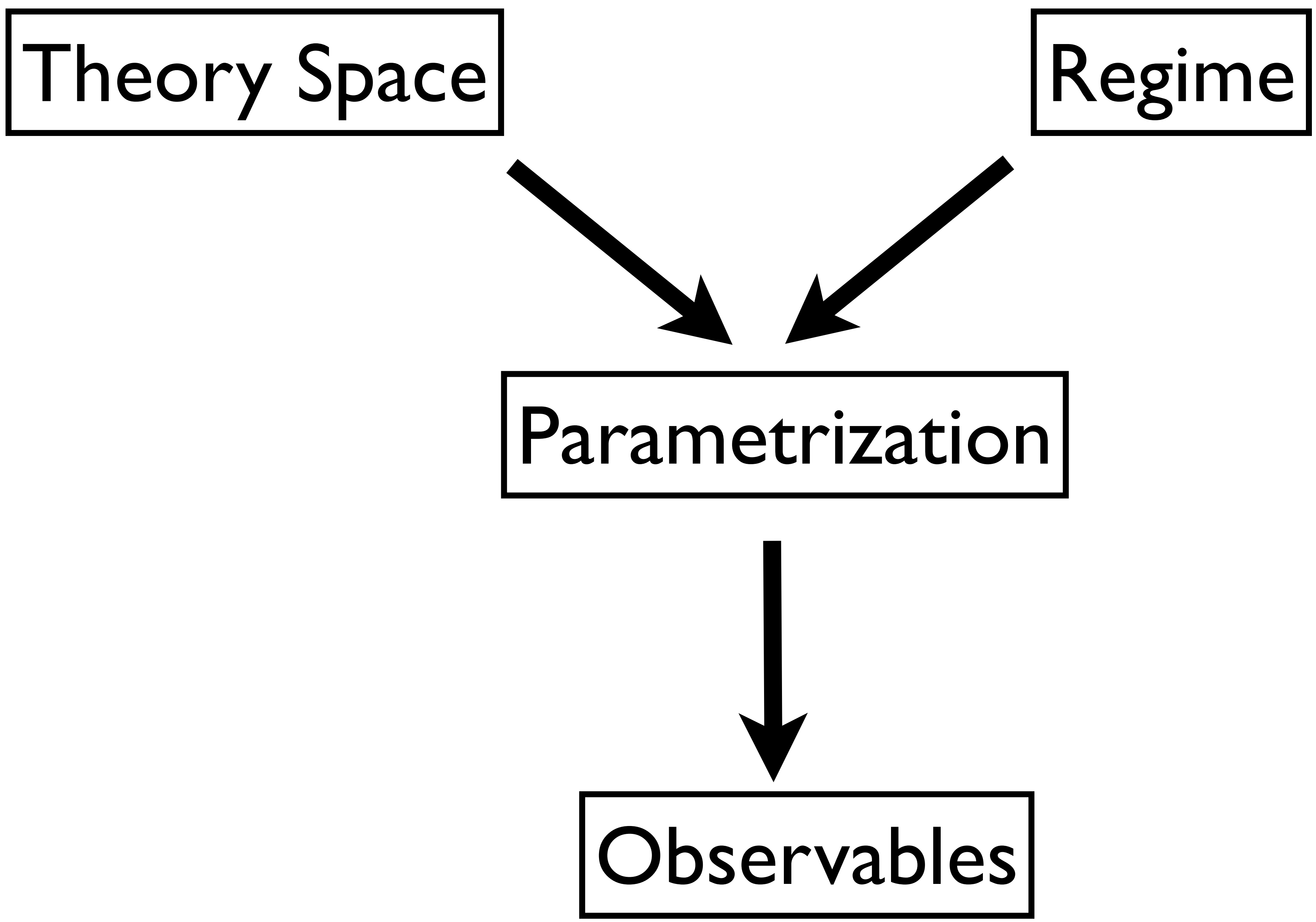
Lessons from PPN

		γ	β	ξ	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
	Einstein (1916) GR	1	1	0	0	0	0	0	0	0	0
Scalar-Tensor	Bergmann (1968), Wagoner (1970)	γ	β	0	0	0	0	0	0	0	0
	Nordtvedt (1970), Bekenstein (1977)	γ	β	0	0	0	0	0	0	0	0
	Brans-Dicke (1961)	γ	1	0	0	0	0	0	0	0	0
Vector-Tensor	Hellings-Nordtvedt (1973)	γ	β	0	$\alpha1$	α_2	0	0	0	0	0
	Will-Nordtvedt (1972)	1	1	0	0	α_2	0	0	0	0	0
Bimetric	Rosen (1975)	1	1	0	0	$c_0 / c_1 - 1$	0	0	0	0	0
	Rastall (1979)	1	1	0	0	$\alpha2$	0	0	0	0	0
	Lightman-Lee (1973)	γ	β	0	α_1	$\alpha2$	0	0	0	0	0
Stratified	Lee-Lightman-Ni (1974)	ac_0 / c_1	β	ξ	α_1	$\alpha2$	0	0	0	0	0
	Ni (1973)	ac_0 / c_1	bc_0	0	α_1	$\alpha2$	0	0	0	0	0
Scalar Field Theories	Einstein (1912) {Not GR}	0	0	0	-4	0	-2	0	-1	0	0
	Whitrow-Morduch (1965)	0	-1	0	-4	0	0	0	-3	0	0
	Rosen (1971)	λ		0	$-4-4\lambda$	0	-4	0	-1	0	0
	Papetrou (1954a, 1954b)	1	1	0	-8	-4	0	0	2	0	0
	Ni (1972) (stratified)	1	1	0	-8	0	0	0	2	0	0
	Yilmaz (1958, 1962)	1	1	0	-8	0	-4	0	-2	0	-1
	Page-Tupper (1968)	γ	β	0	$-4-4\gamma$	0	$-2-2\gamma$	0	ζ_2	0	ζ_4
	Nordström (1912)	-1	β	0	0	0	0	0	0	0	0
	Nordström (1913), Einstein-Fokker (1914)	-1	1	0	0	0	0	0	0	0	0
	Ni (1972) (flat)	-1	$1-q$	0	0	0	0	0	ζ_2	0	0
	Whitrow-Morduch (1960)	-1	$1-q$	0	0	0	0	0	q	0	0
	Littlewood (1953), Bergman(1956)	-1	β	0	0	0	0	0	-1	0	0

Lessons from PPN

Parameter	Bound	Effects	Experiment
$\gamma - 1$	2.3×10^{-5}	Time delay, light deflection	Cassini tracking
$\beta - 1$	2.3×10^{-4}	Nordtvedt effect, Perihelion shift	Nordtvedt effect
ξ	0.001	Earth tides	Gravimeter data
α_1	10^{-4}	Orbit polarization	Lunar laser ranging
α_2	4×10^{-7}	Spin precession	Solar alignment with ecliptic
α_3	4×10^{-20}	Self-acceleration	Pulsar spin-down statistics
ζ_1	0.02	-	Combined PPN bounds
ζ_2	4×10^{-5}	Binary pulsar acceleration	PSR 1913+16
ζ_3	10^{-8}	Newton's 3rd law	Lunar acceleration
ζ_4	0.006	-	Usually not independent

The Process



The Universe: background cosmology

$$ds^2 = a^2\gamma_{\mu\nu}dx^\mu dx^\nu$$

FRW equations

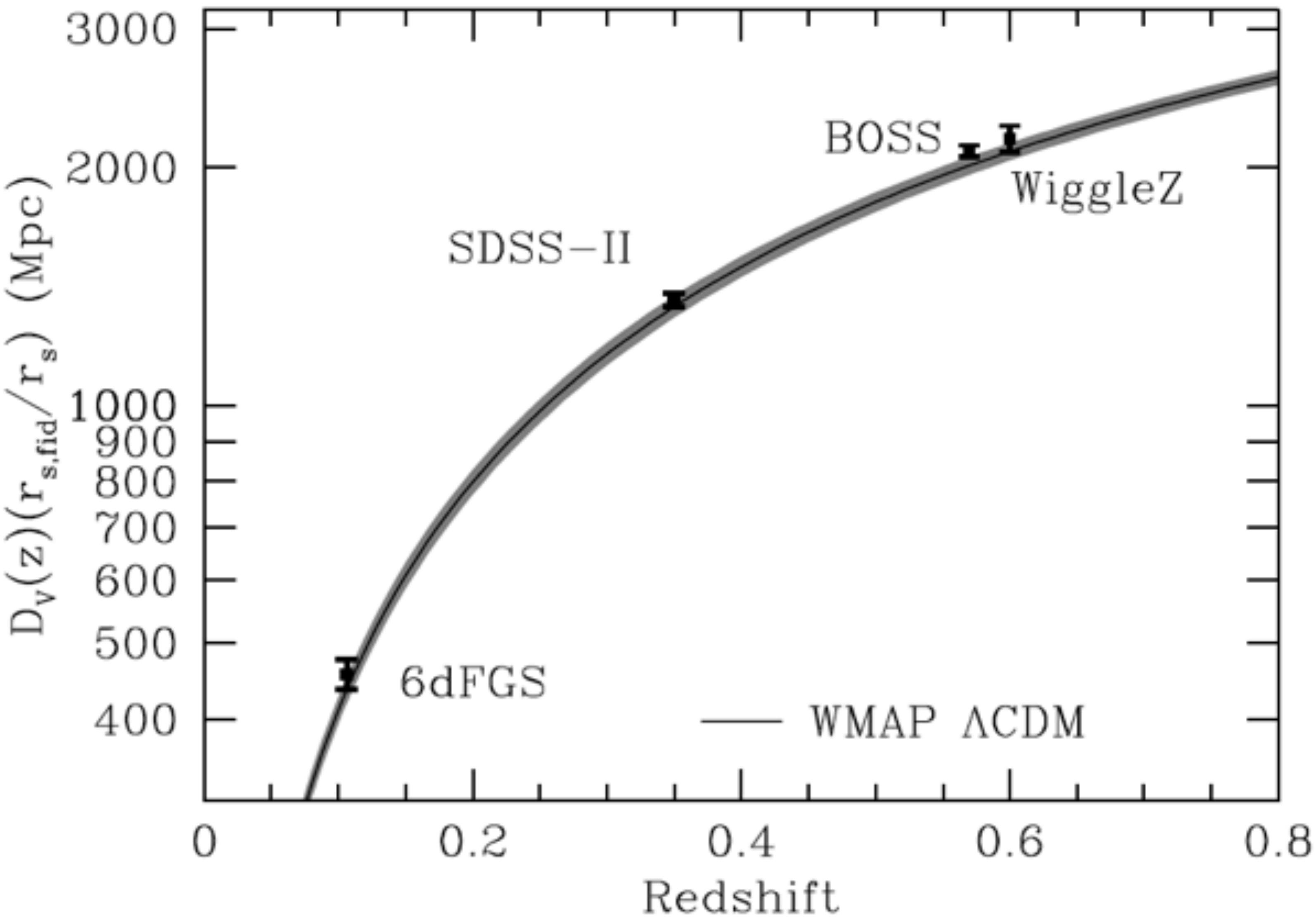
$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta} \longrightarrow \mathcal{H}^2 = \frac{8\pi G}{3}a^2\rho$$

<u>Any</u> theory (modified gravity or otherwise)

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta} + U_{\alpha\beta}$$

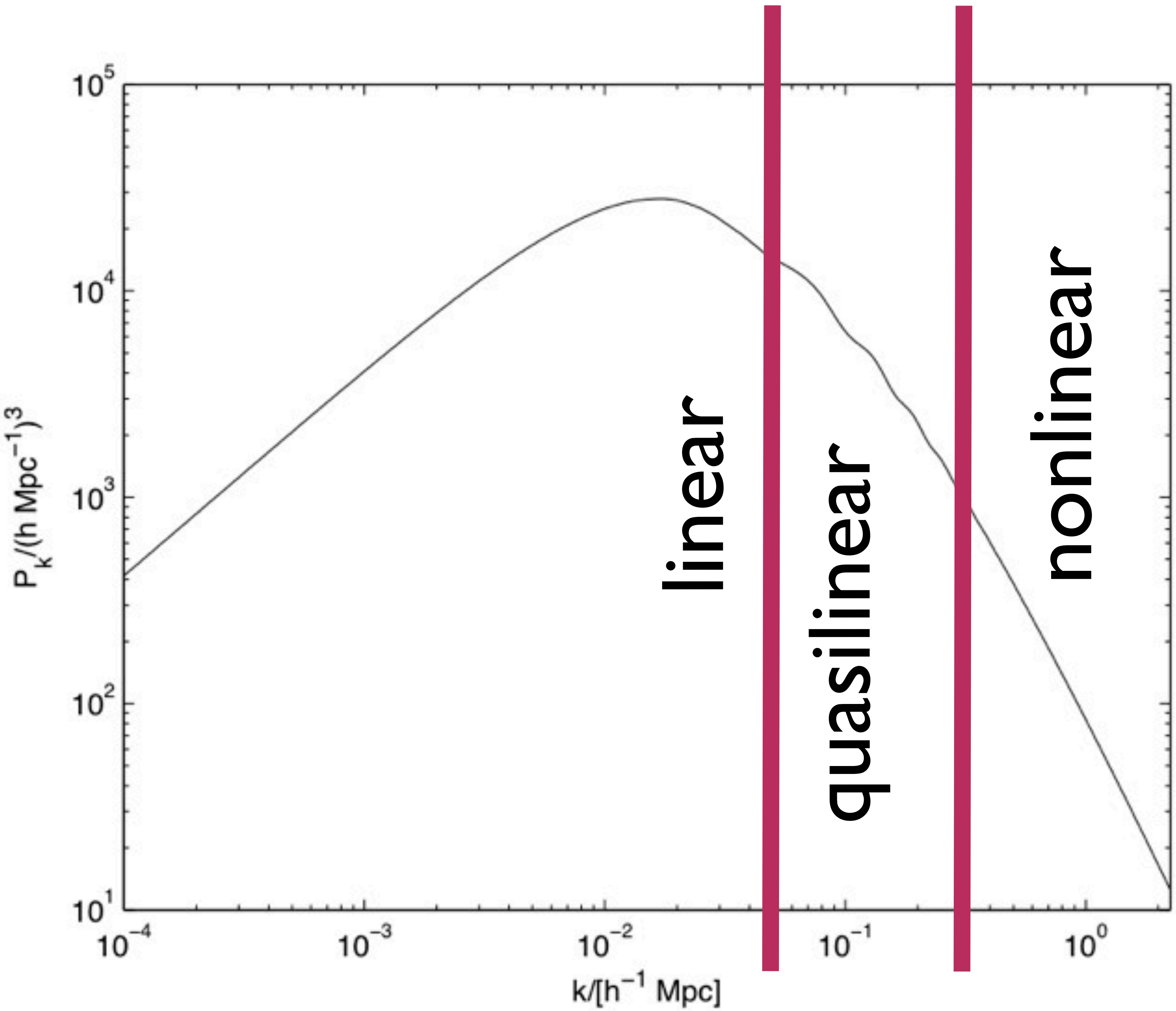
$$\longrightarrow \rho_X(\tau), P_X(\tau)$$

The Universe: background cosmology



Only measure $H(z)$ and Ω_K

The Universe: large scale structure



Linear Perturbation Theory $(10 - 10,000h^{-1}Mpc)$

$$ds^2 = a^2(\gamma_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu$$

Diffeomorphism invariance $\longrightarrow$ Gauge invariant Newtonian potentials

$$(\hat{\Phi}, \hat{\Psi})$$

$$\rho \to \rho(\tau)[1 + \delta(\tau, \mathbf{r})]$$

$$\hat{\Gamma} = \frac{1}{k}\left(\dot{\hat{\Phi}} + \mathcal{H}\hat{\Psi}\right)$$

$$\delta G_{\alpha\beta} = 8\pi G \delta T_{\alpha\beta}$$

$$\delta G_{00}^{(gi)} : \; 2\vec{\nabla}^2\hat{\Phi} - 6\mathcal{H}k\hat{\Gamma} = 8\pi G a^2 \rho \delta^{(gi)}$$

$$\delta G_{0i}^{(gi)} : \; 2k\hat{\Gamma} = 8\pi G(\rho + P)\theta^{(gi)}$$

$$\delta G_{ij}^{(gi)} : \; \hat{\Phi} - \hat{\Psi} = 8\pi G a^2(\rho + P)\Sigma^{(gi)}$$

(+ $\delta G_{ii}^{(gi)}$ equation)

Extending Einstein's equations

$$\delta G_{\mu\nu} = 8\pi G \delta T^{M}_{\mu\nu} + \delta U_{\mu\nu}$$

$$\downarrow$$

Linear in $\hat{\Phi}, \hat{\Gamma}, \hat{\chi}, \dot{\hat{\chi}}$

Skordis 2010
Baker, Ferreira, Skordis 2012
Bloomfield, Flanagan, Park, Watson 2012
Gleyzes, Gubitosi, Piazza, Vernizzi 2013
Pearson, Battye 2011

ArXiv:1209.2117

Extending Einstein's equations

Key: Matter + Metric + New degree of freedom

$$-a^2\delta G_0^{0\,(gi)} = \kappa a^2 G\,\rho_M \delta_M^{(gi)} \quad +\alpha_0 k^2\hat{\chi} + \alpha_1 k\dot{\hat{\chi}}$$
$$+A_0 k^2\hat{\Phi} \quad +F_0 k^2\hat{\Gamma}$$

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$$-a^2\delta G_0^{0\,(gi)} = \kappa a^2 G\,\rho_M \delta_M^{(gi)} + \alpha_0 k^2 \hat{\chi} + \alpha_1 k \dot{\hat{\chi}} + A_0 k^2 \hat{\Phi} + F_0 k^2 \hat{\Gamma}$$

Functions of time (and scale).

$$-a^2\delta G_i^{0\,(gi)} = \nabla_i \left[\kappa a^2 G\,\rho_M (1+\omega_M)\theta_M^{(gi)} + \beta_0 k \hat{\chi} + \beta_1 \dot{\hat{\chi}} + B_0 k \hat{\Phi} + I_0 k \hat{\Gamma}\right]$$

$$a^2\delta G_i^{i\,(gi)} = 3\,\kappa a^2 G\,\rho_M \Pi_M^{(gi)} + \gamma_0 k^2 \hat{\chi} + \gamma_1 k \dot{\hat{\chi}} + \gamma_2 \ddot{\hat{\chi}} + C_0 k^2 \hat{\Phi} + C_1 k \dot{\hat{\Phi}} + J_0 k^2 \hat{\Gamma} + J_1 k \dot{\hat{\Gamma}}$$

$$a^2\delta G_j^i = D_j^i \left[\kappa a^2 G\,\rho_M (1+\omega_M)\Sigma_M + \epsilon_0 \hat{\chi} + \frac{\epsilon_1}{k}\dot{\hat{\chi}} + \frac{\epsilon_2}{k^2}\ddot{\hat{\chi}} + D_0 \hat{\Phi} + \frac{D_1}{k}\dot{\hat{\Phi}} + K_0 \hat{\Gamma} + \frac{K_1}{k}\dot{\hat{\Gamma}}\right]$$

ArXiv:1209.2117

Extending Einstein's ...uations

Key: Matter + Metric ... dom

$$-a^2\delta G_0^{0\,(gi)} = \kappa a^2 G \ldots$$

$$+ \ldots$$

ime

$$-a^2\delta G_i^{0\,(gi)} \ldots \beta_1\dot{\hat{\chi}}]$$

$$\ldots \hat{\chi} + \gamma_1 k\dot{\hat{\chi}} + \gamma_2\ddot{\hat{\chi}}$$

$$\ldots J_0 k^2\hat{\Gamma} + J_1 k\dot{\hat{\Gamma}}$$

$$a^2\delta G_j^i = \ldots + \omega_M)\Sigma_M \quad +\epsilon_0\hat{\chi} + \frac{\epsilon_1}{k}\dot{\hat{\chi}} + \frac{\epsilon_2}{k^2}\ddot{\hat{\chi}}]$$

$$\ldots + \frac{D_1}{k}\dot{\hat{\Phi}} \quad +K_0\hat{\Gamma} + \frac{K_1}{k}\dot{\hat{\Gamma}}$$

Integrability

↓

7 to 9 free functions of time

ArXiv:1209.2117

Integrability

Most general action with 1 d.o.f. (use unitary gauge)

$$S=\int d^4x\sqrt{-g}\,L(N,K^\mu_{\ \mu},K_{\mu\nu}K^{\mu\nu},{}^{(3)}\!R,{}^{(3)}\!R_{\mu\nu}{}^{(3)}\!R^{\mu\nu},\dots;t)\ .$$

Expand to 2nd order

$$\begin{aligned}L(N,K,\mathcal{S},\mathcal{R},\mathcal{Z})&=\bar{L}-\dot{\mathcal{F}}-3H\mathcal{F}+(\dot{\mathcal{F}}+L_N)\,\delta N+L_{\mathcal{R}}\,\delta\mathcal{R}\\&+\frac{\mathcal{A}}{2}\,\delta K^2+L_{\mathcal{S}}\,\delta K^\mu_{\ \nu}\delta K^\nu_{\ \mu}+\left(\frac{1}{2}L_{NN}-\dot{\mathcal{F}}\right)\delta N^2\\&+\frac{1}{2}L_{\mathcal{R}\mathcal{R}}\,\delta\mathcal{R}^2+\mathcal{B}\,\delta K\delta N+\mathcal{C}\,\delta K\delta\mathcal{R}+L_{N\mathcal{R}}\,\delta N\delta\mathcal{R}+L_{\mathcal{Z}}\delta\mathcal{Z}+\mathcal{O}(3)\end{aligned}$$

where:

$$\begin{aligned}\mathcal{F}&\equiv 2HL_{\mathcal{S}}+L_K\ ,\\\mathcal{A}&\equiv 4H^2L_{\mathcal{S}\mathcal{S}}+4HL_{\mathcal{S}K}+L_{KK}\ ,\\\mathcal{B}&\equiv 2HL_{\mathcal{S}N}+L_{KN}\ ,\\\mathcal{C}&\equiv 2HL_{\mathcal{S}\mathcal{R}}+L_{K\mathcal{R}}\ .\end{aligned}$$

The $L_X,\,L_{XY}$ are functions of time only.

Baker, Gleyzes, Ferreira, Vernizzi in prep

Extending Einstein's equations

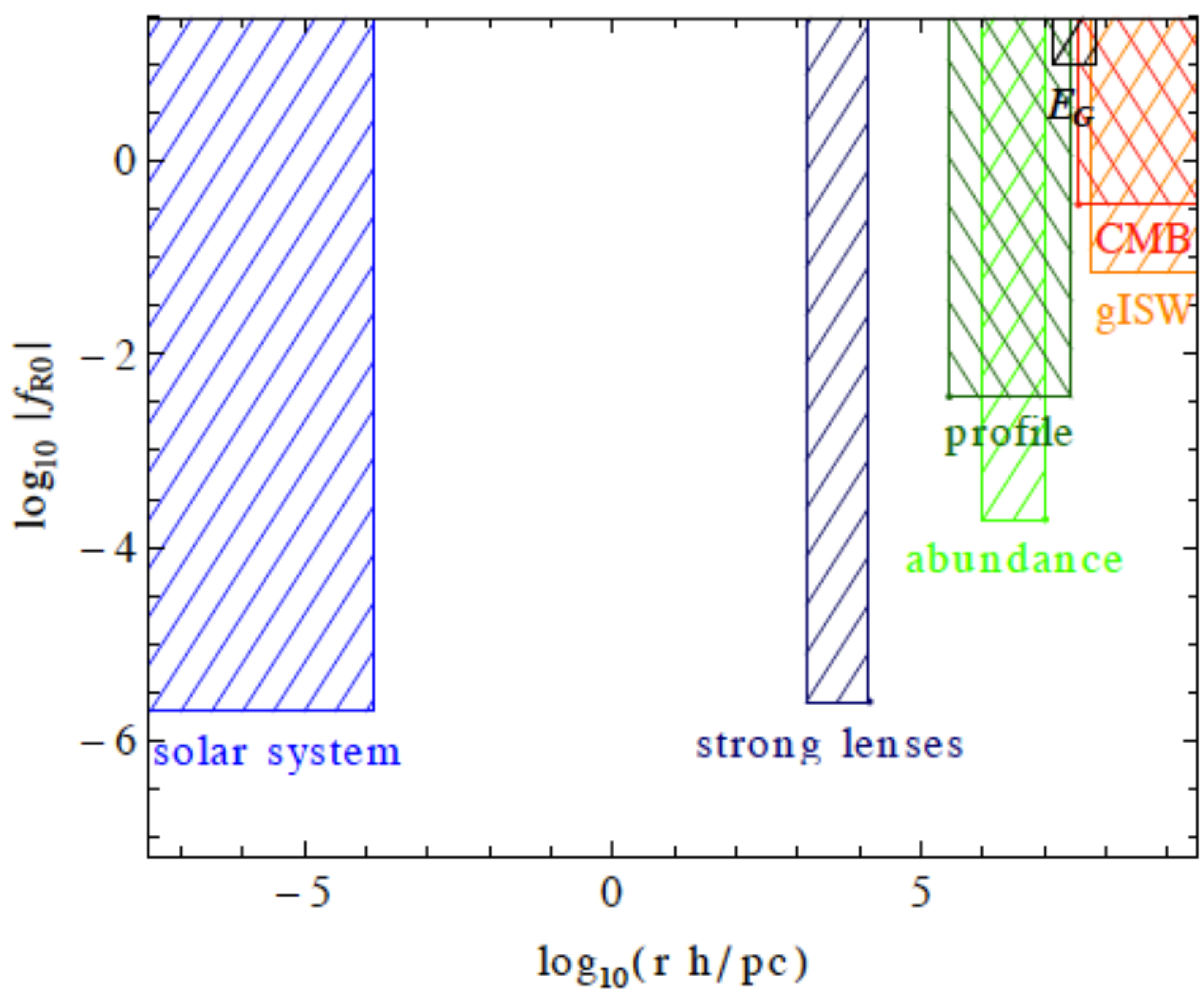
Scalar-Tensor	Galileons	K.G.B.	DGP	Einstein-Aether
f(R) gravity	The Fab Four	Quintessence	EBI	Horava-Lifschitz
f(G) theories	K-essence	Dark fluids	TeVeS	G-inflation

ArXiv:1209.2117

What about the non-linear regime?

Pros: Much better sampling of density field $N_{modes} \propto k^3$

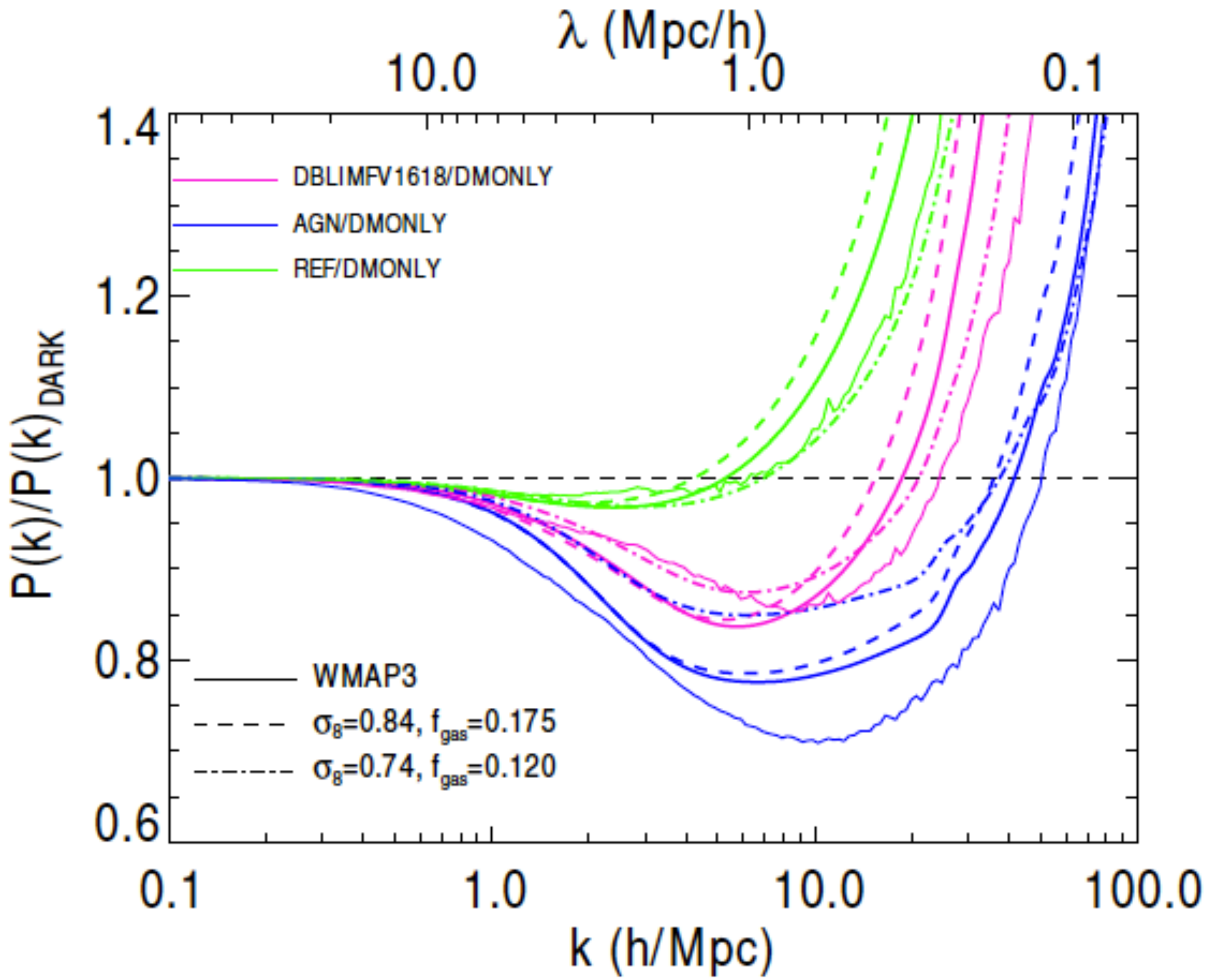
$$F(R) = 1 + f(R)$$



Lombriser et al 2012

What about the non-linear regime?

Baryon, feedback and bias



Semboloni et al 2012

And now to what we observe: Light vs Matter

- For a perturbed line element of the form:

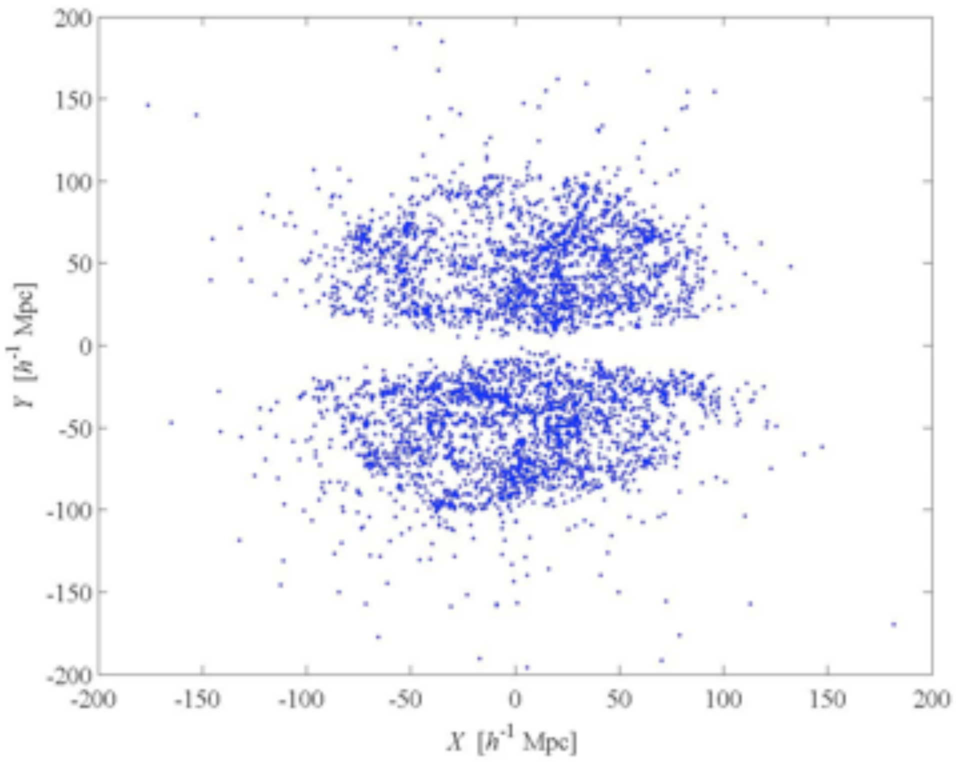
$$ds^2 = a^2(\tau)[-(1+2\Phi)d\tau^2 + (1-2\Psi)\gamma_{ij}dx^i dx^j]$$

the equations of motion are:

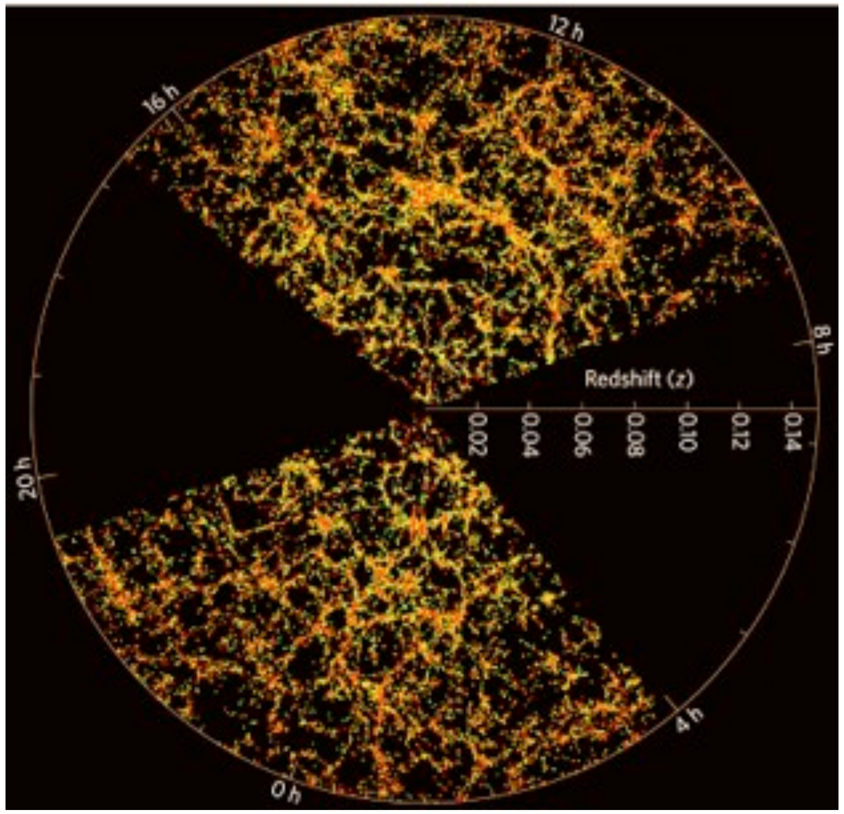
$$\frac{1}{a}\frac{d(a\mathbf{v})}{d\tau} = -\nabla\Phi$$ (non-relativistic particles)

$$\frac{d\mathbf{v}}{d\tau} = -\nabla_\perp(\Phi+\Psi)$$ (relativistic particles)

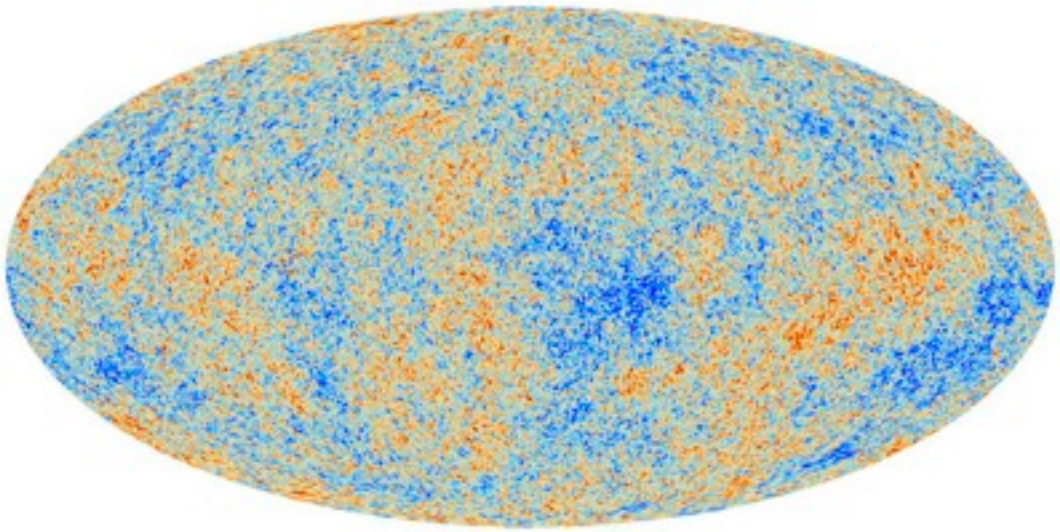
What we observe.



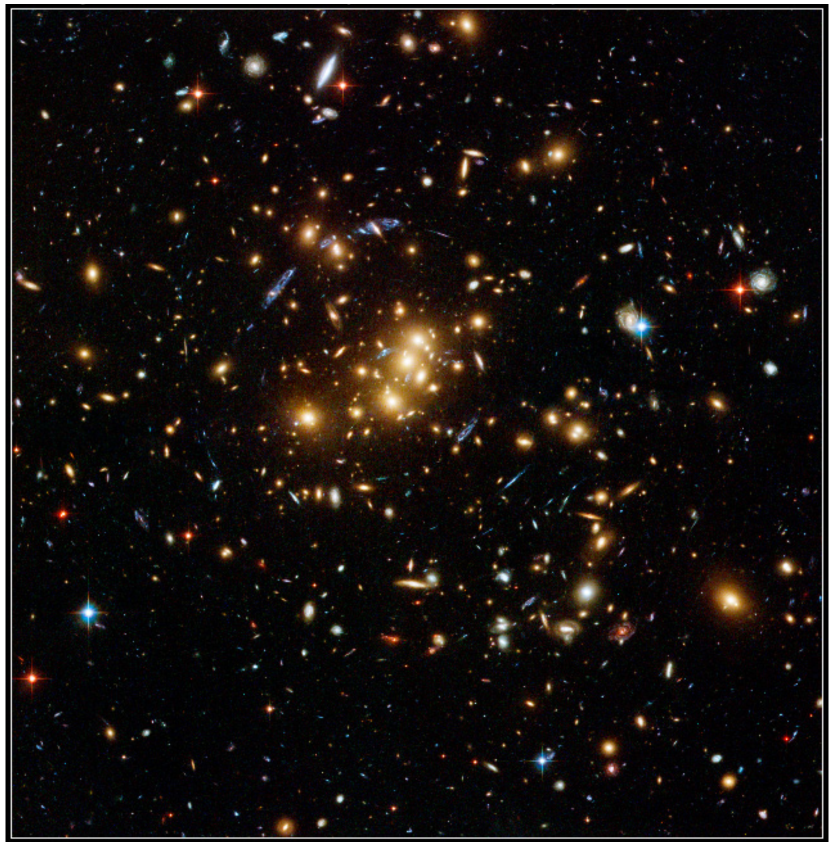
$\vec{v}$



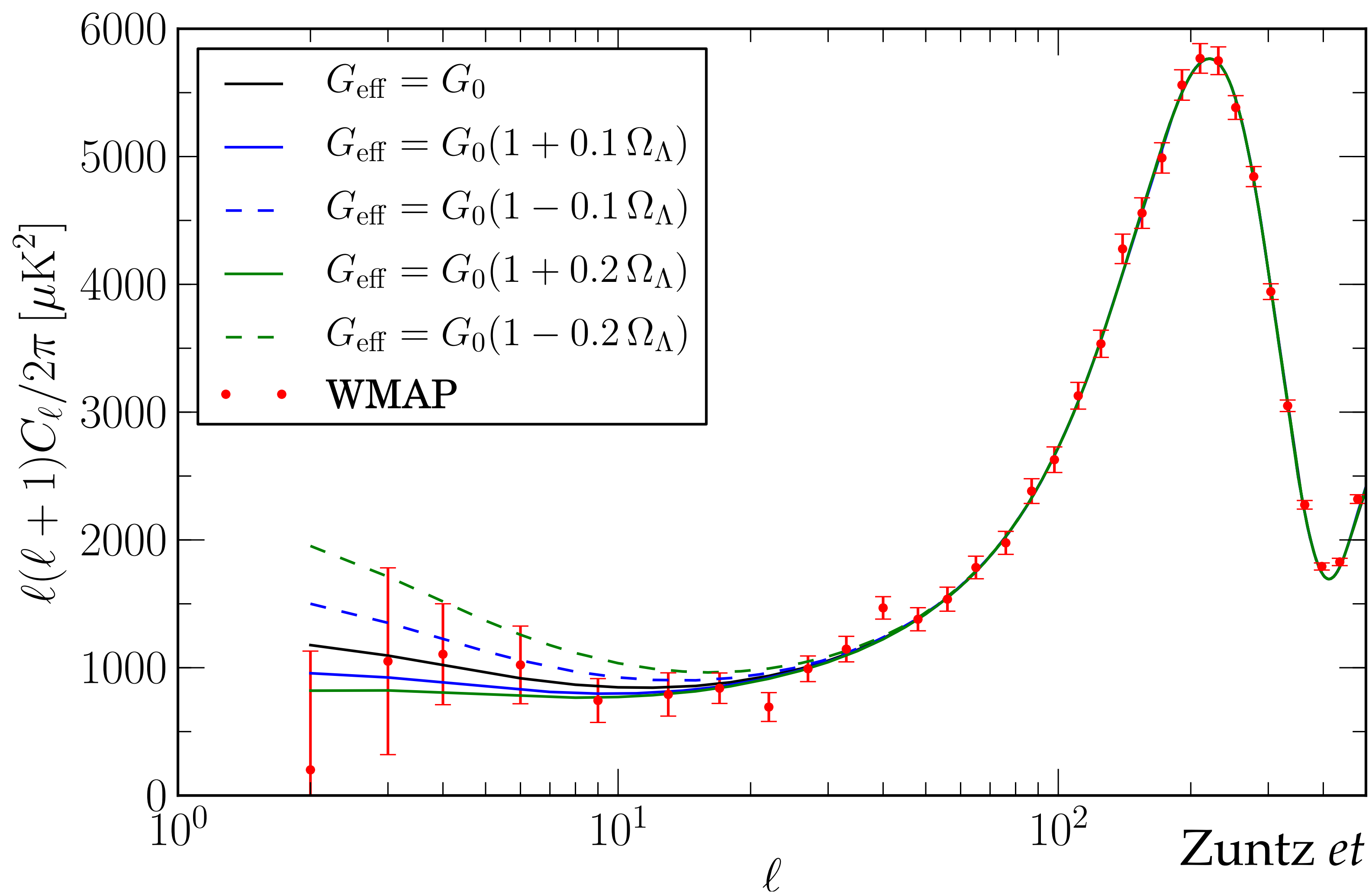
$\delta, \vec{v}$



Φ, Ψ



Large Scales: the problem with cosmic variance

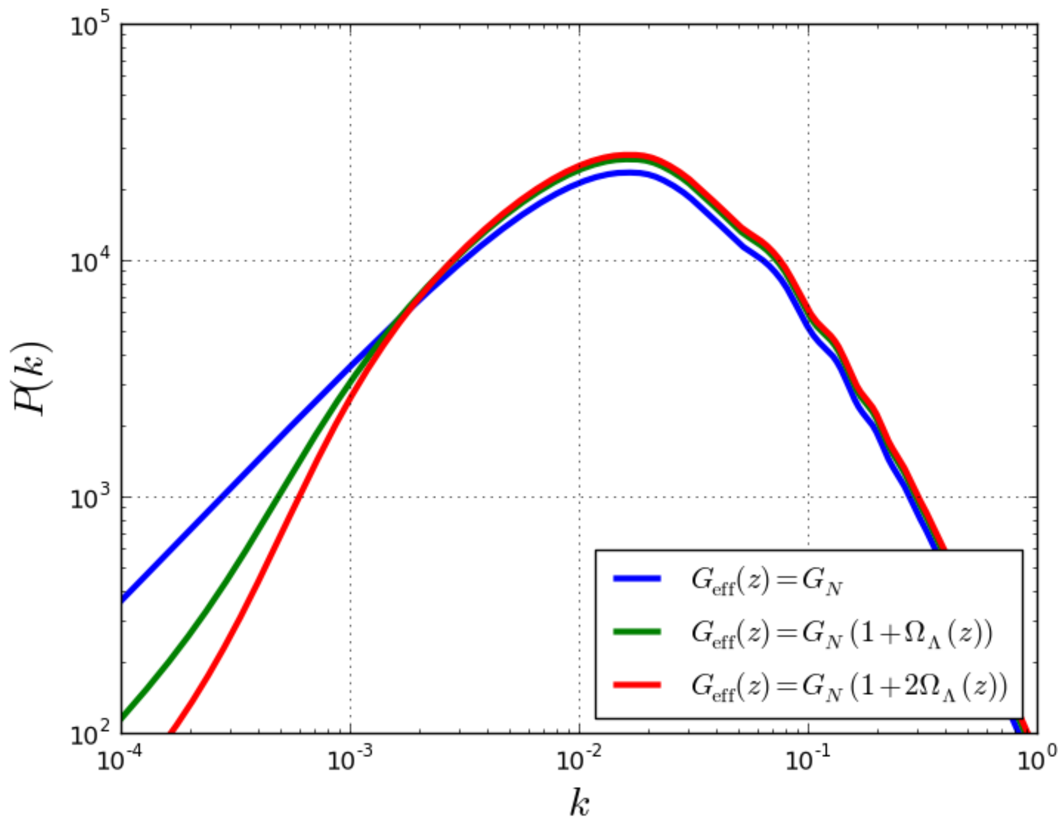


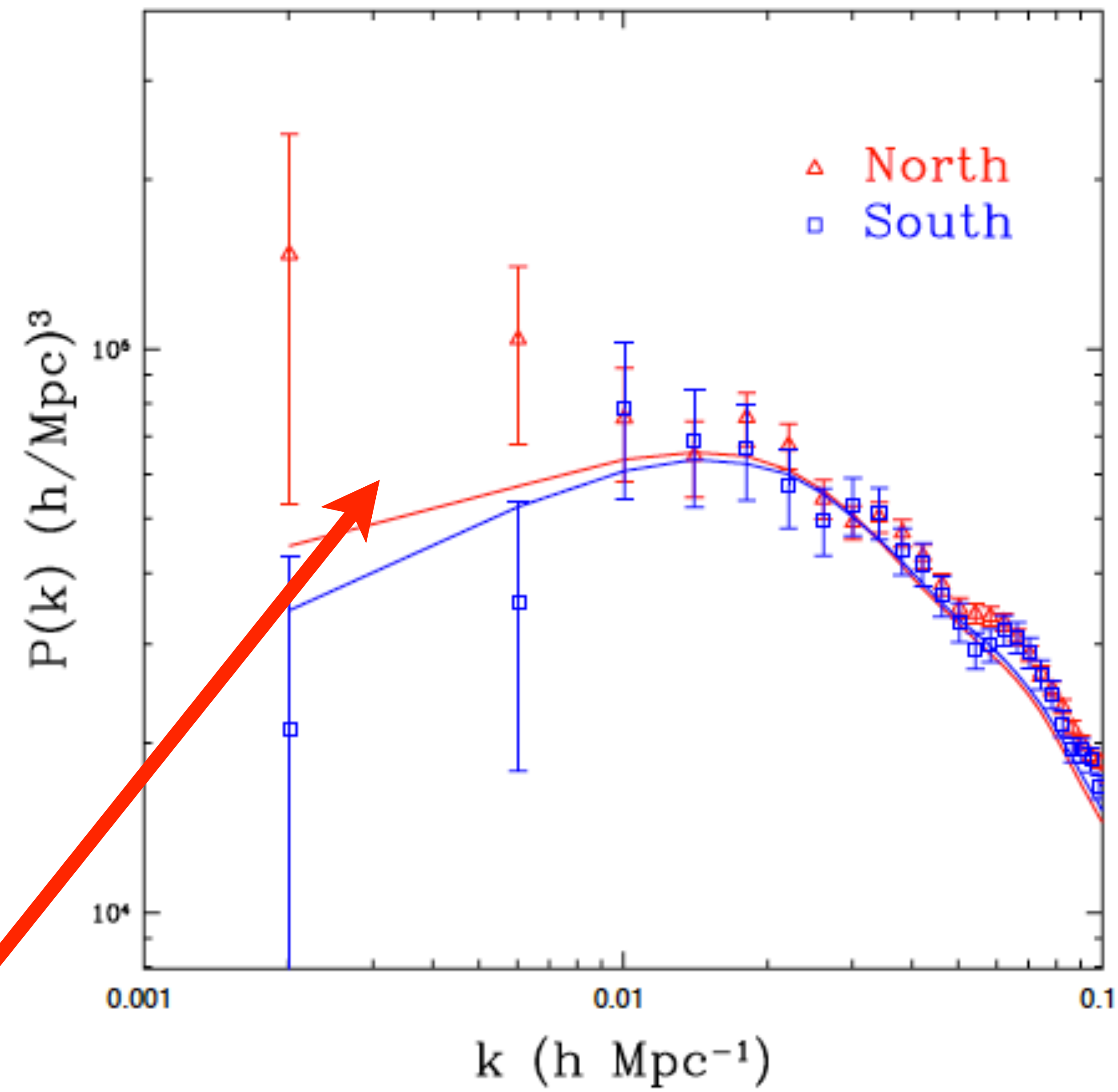
Zuntz *et al.* 2011

ISW- late time effects on large scales

$$\propto \int (\dot{\Phi} + \dot{\Psi})\, d\eta$$

Large scales: the problem with the Galaxy

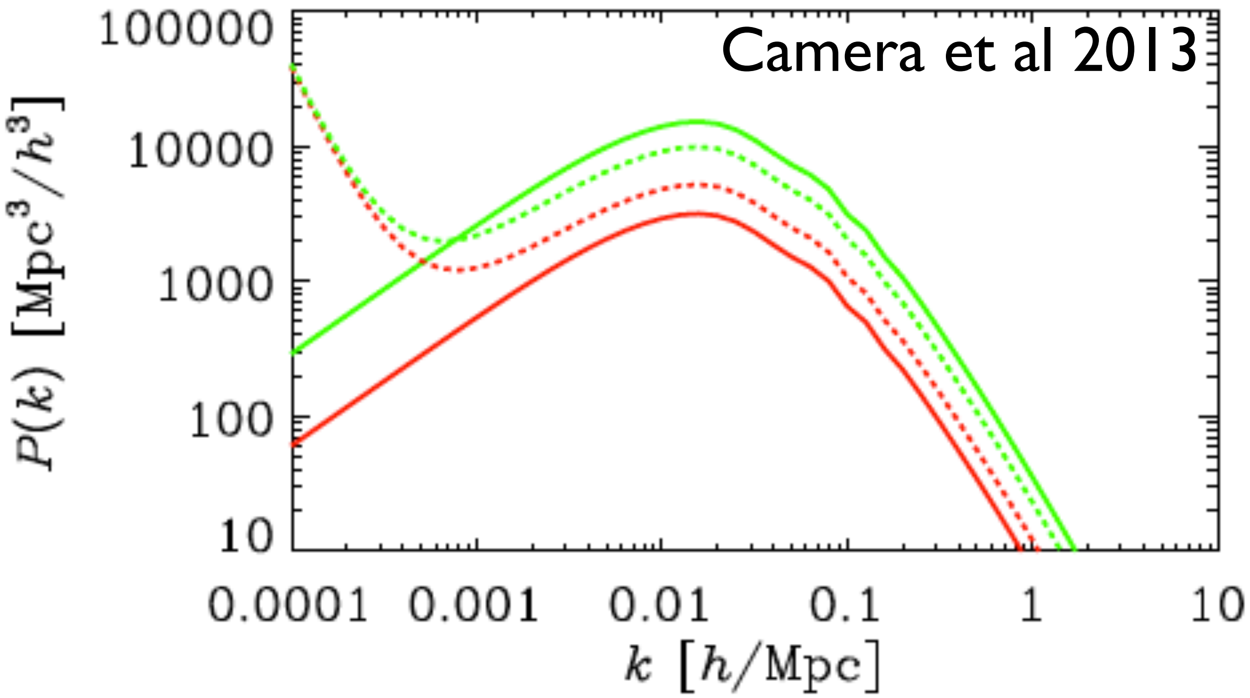


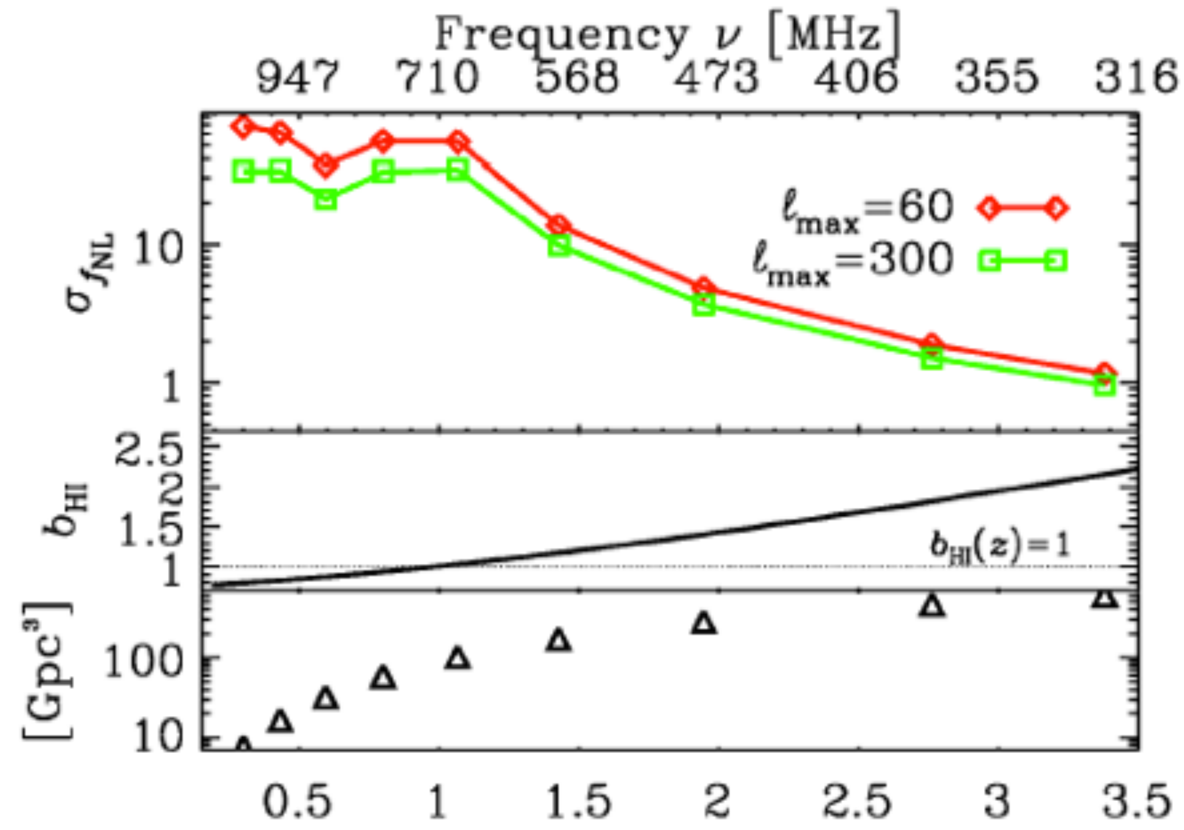


Systematic effect due to stellar densities

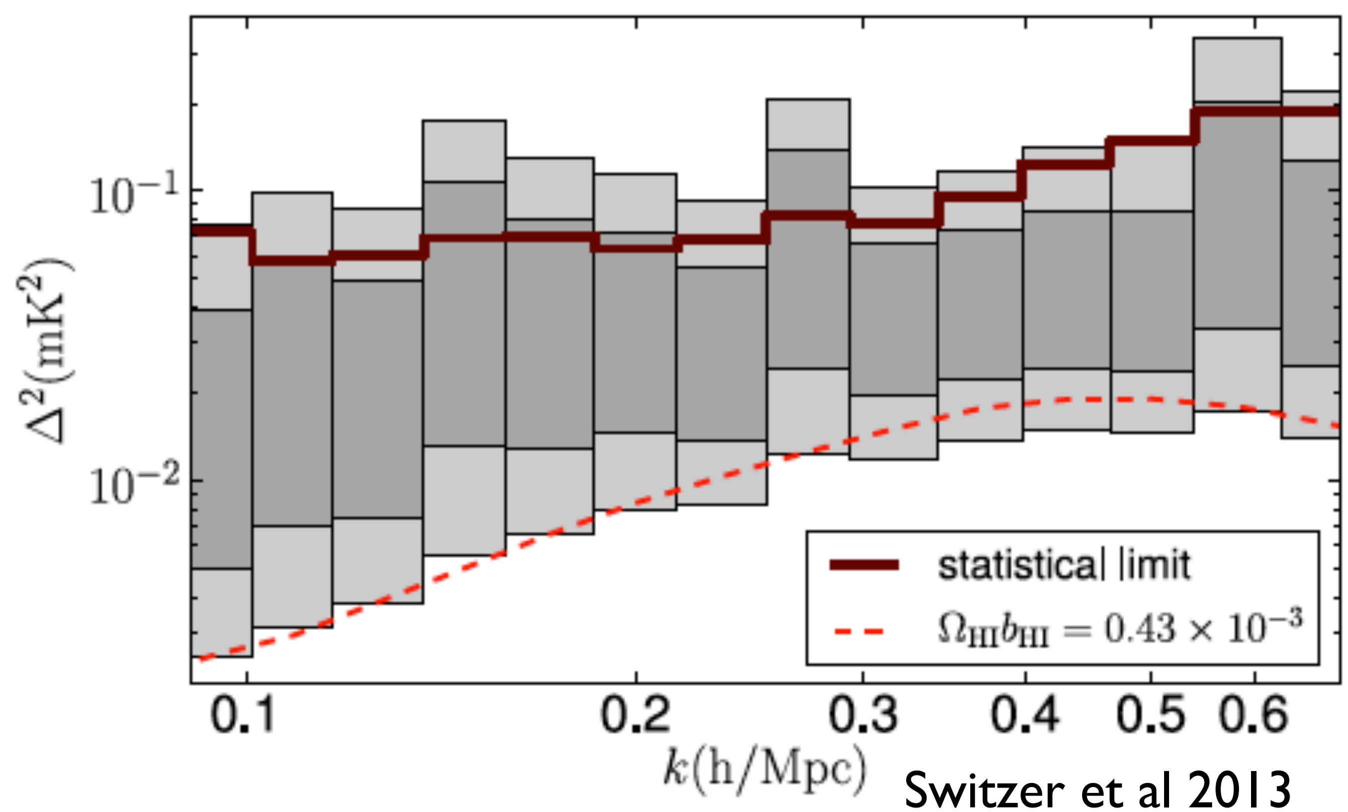
Ross et al (BOSS) 2012

Large Scales: Tomography of Neutral Hydrogen





First attempts: the GBT



Switzer et al 2013

Not so large scale: "quasi-static" regime

A preferred length scale- the horizon

$$\mathcal{H}^{-1} \equiv \left(\frac{\dot{a}}{a}\right)^{-1} \propto \tau \simeq 3000 h^{-1}\mathrm{Mpc}$$

Focus on scales such that $k\tau \gg 1$

Most surveys $\leq 300 h^{-1}\mathrm{Mpc}$

Caldwell, Cooray, Melchiorri, Amendola, Kunz, Sapone, Bertschinger, Zukin, Amin, Blandford, Wagoner, Linder, Pogosian, Silvestri, Koyama, Zhao, Zhang, Liguori, Bean, Dodelson

$$-k^2\Phi = 4\pi G\mu a^2\rho\Delta$$

$$\gamma\Psi = \Phi$$

Note: not applicable to CMB!

Not so large scale: "quasi-static" regime

The "quasi-static" functions reduce to a simple form

$$\gamma = \frac{p_1(a) + p_2(a)k^2}{1 + p_3(a)k^2} \ ,$$

$$\mu = \frac{1 + p_3(a)k^2}{p_4(a) + p_5(a)k^2} \ .$$

Baker et al 2012
Silvestri et al 2013

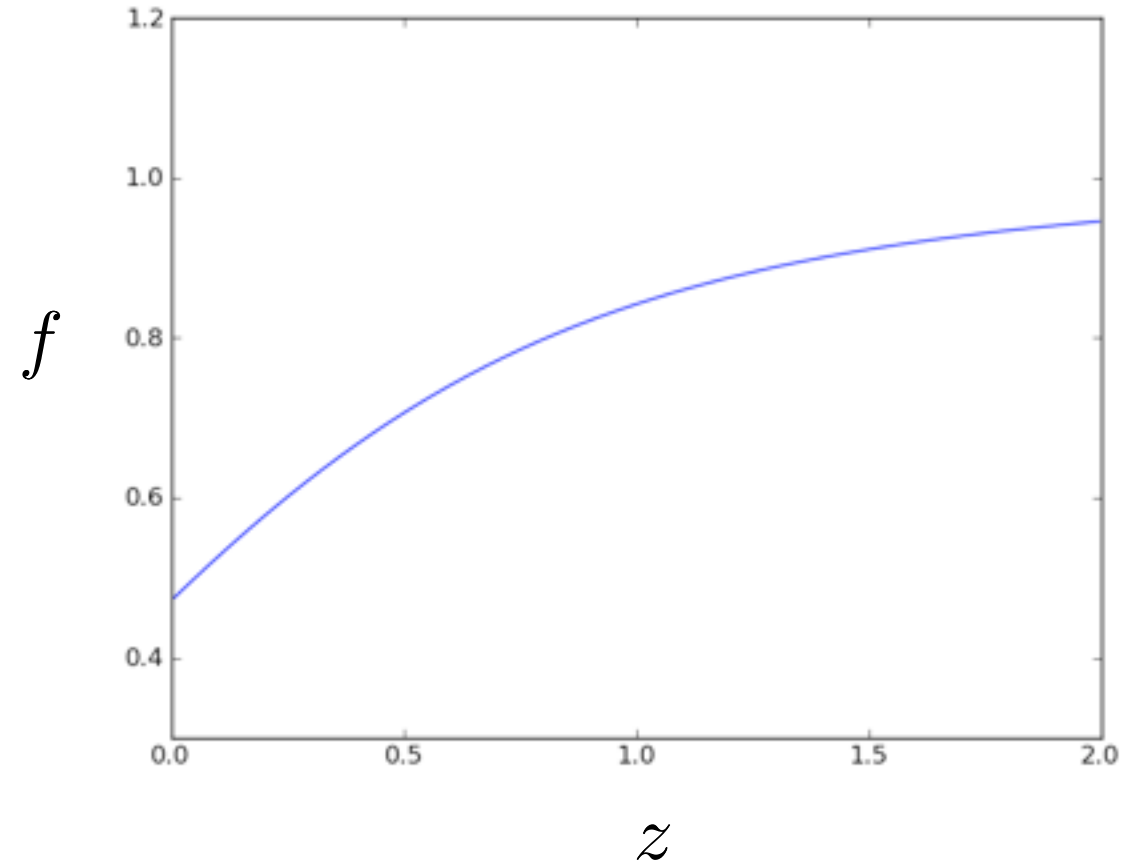
where $p_i = p_i[L_K, L_{KK}, \cdots]$

Goal: to use k and z dependent measurements of (γ, μ) to constrain PPF functions

Growth of Structure

Growth rate

$$f(k,a) = \frac{d\ln\delta_M(k,a)}{d\ln a}$$

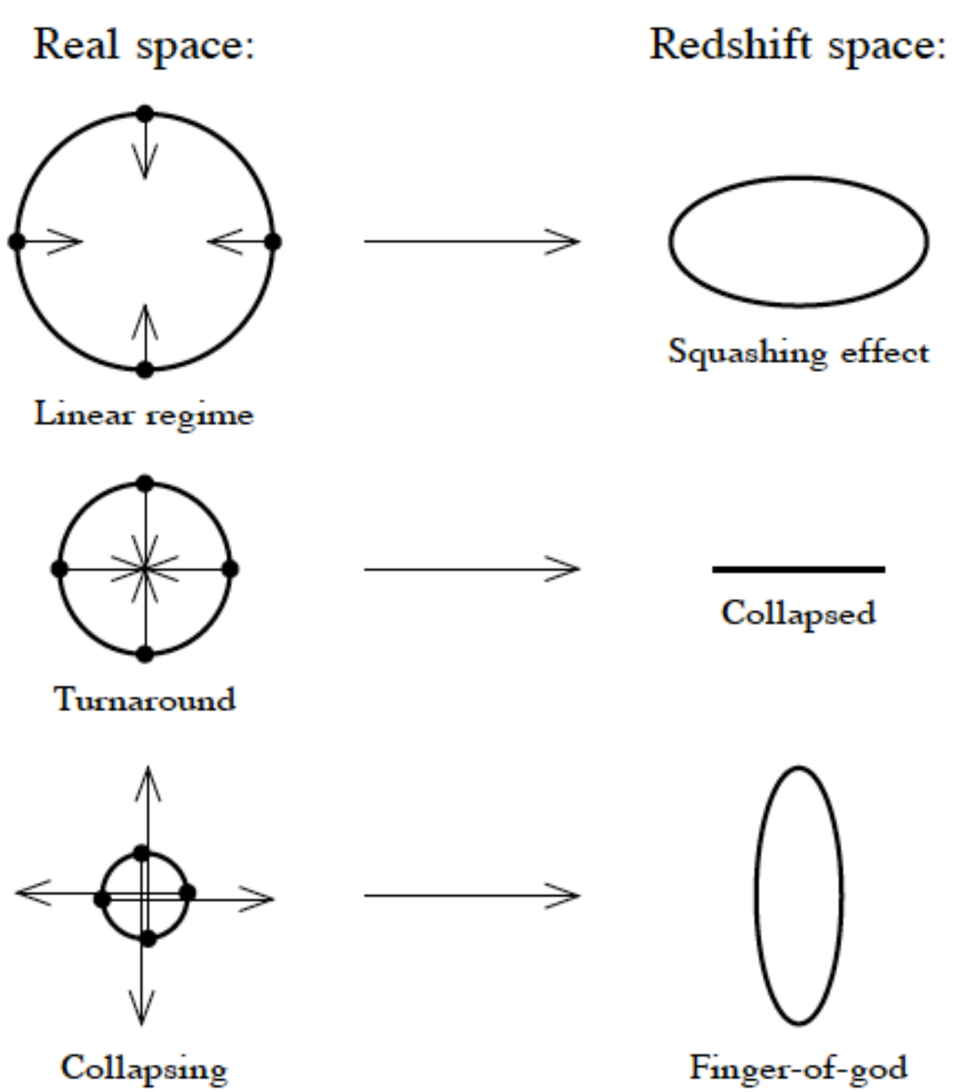


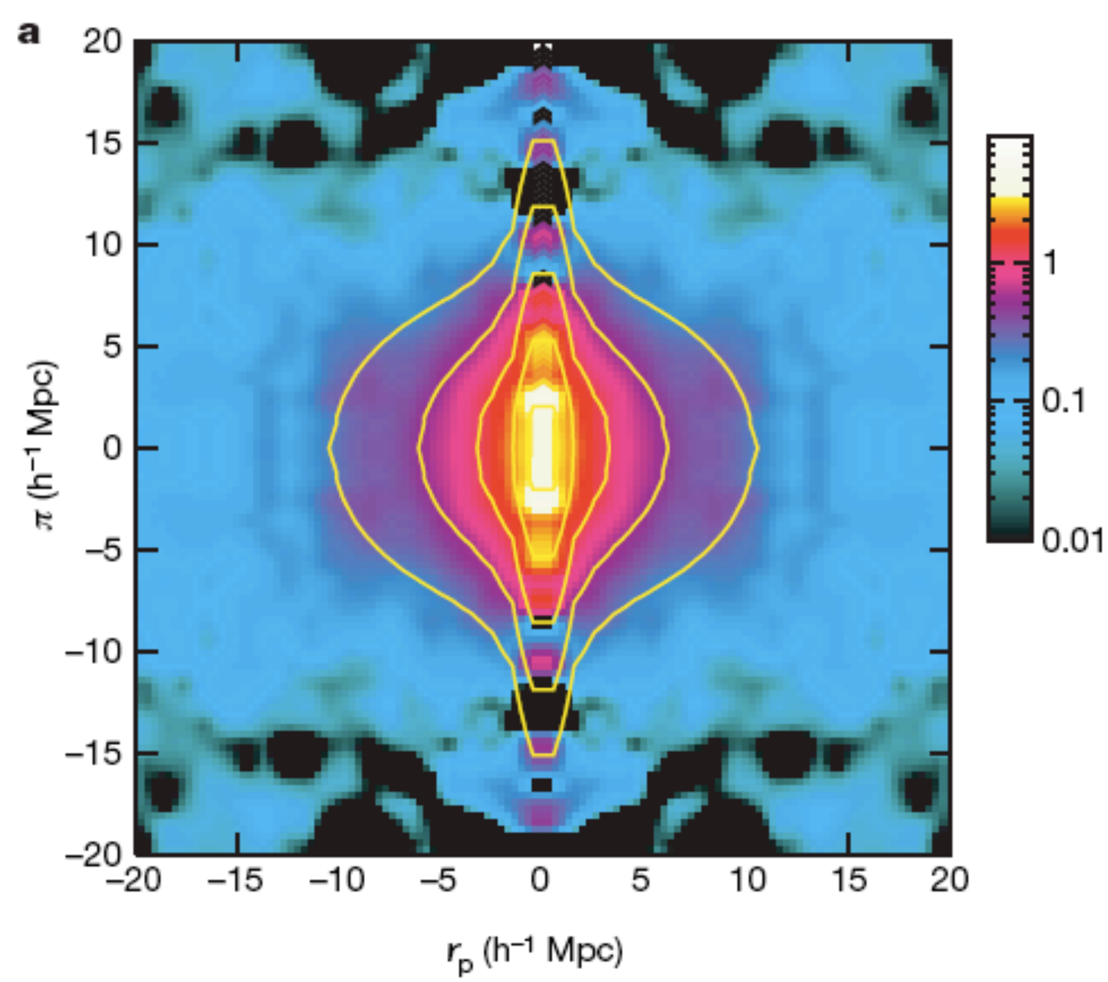
f satisfies a simple ODE

$$\frac{df}{d\ln a} + qf + f^2 = \frac{3}{2}\Omega_M\xi$$

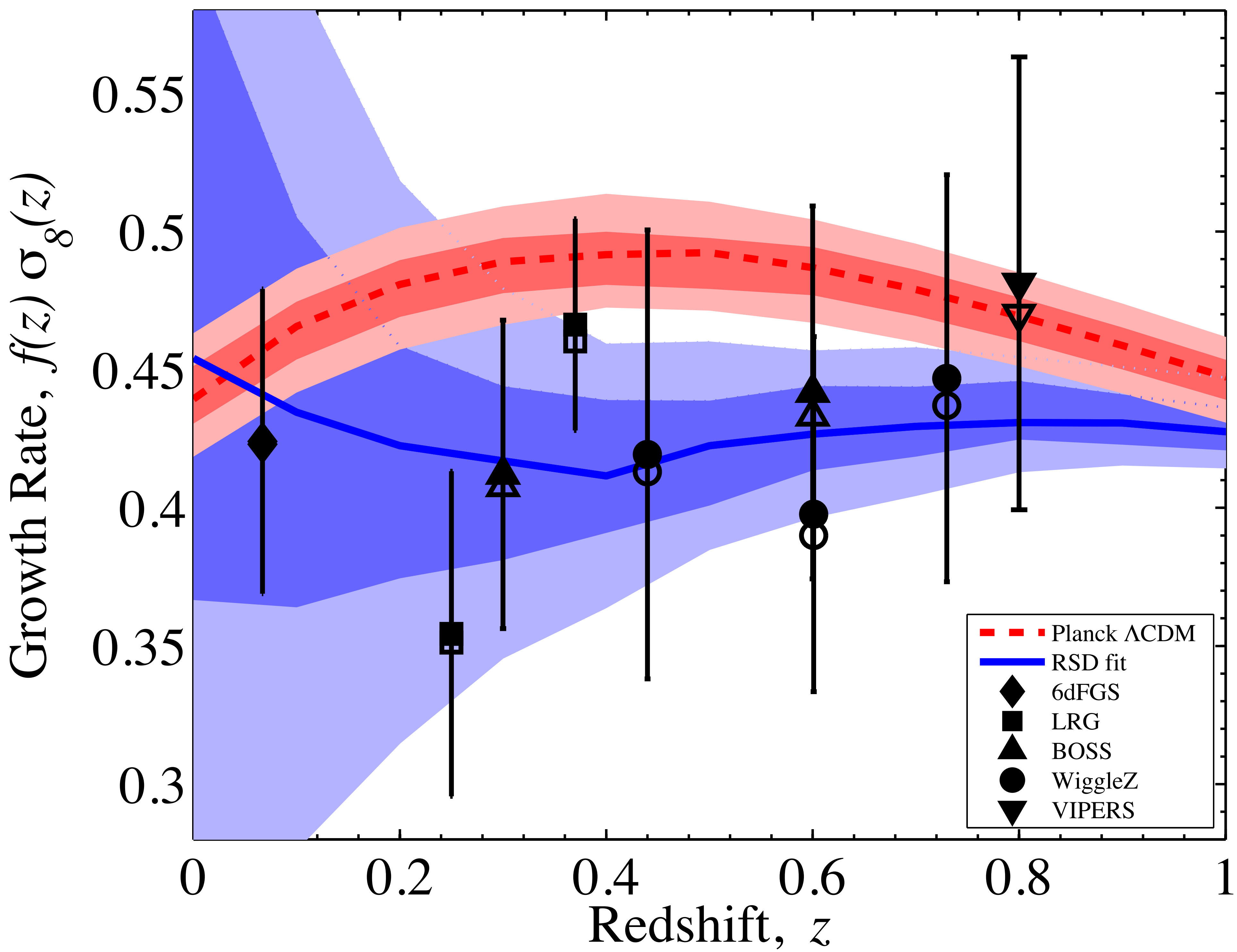
with $q = \frac{1}{2}[1 - 3w(1-\Omega_M)]$ and $\xi = \frac{\mu}{\gamma}$

Growth of structure: Redshift Space Distortions





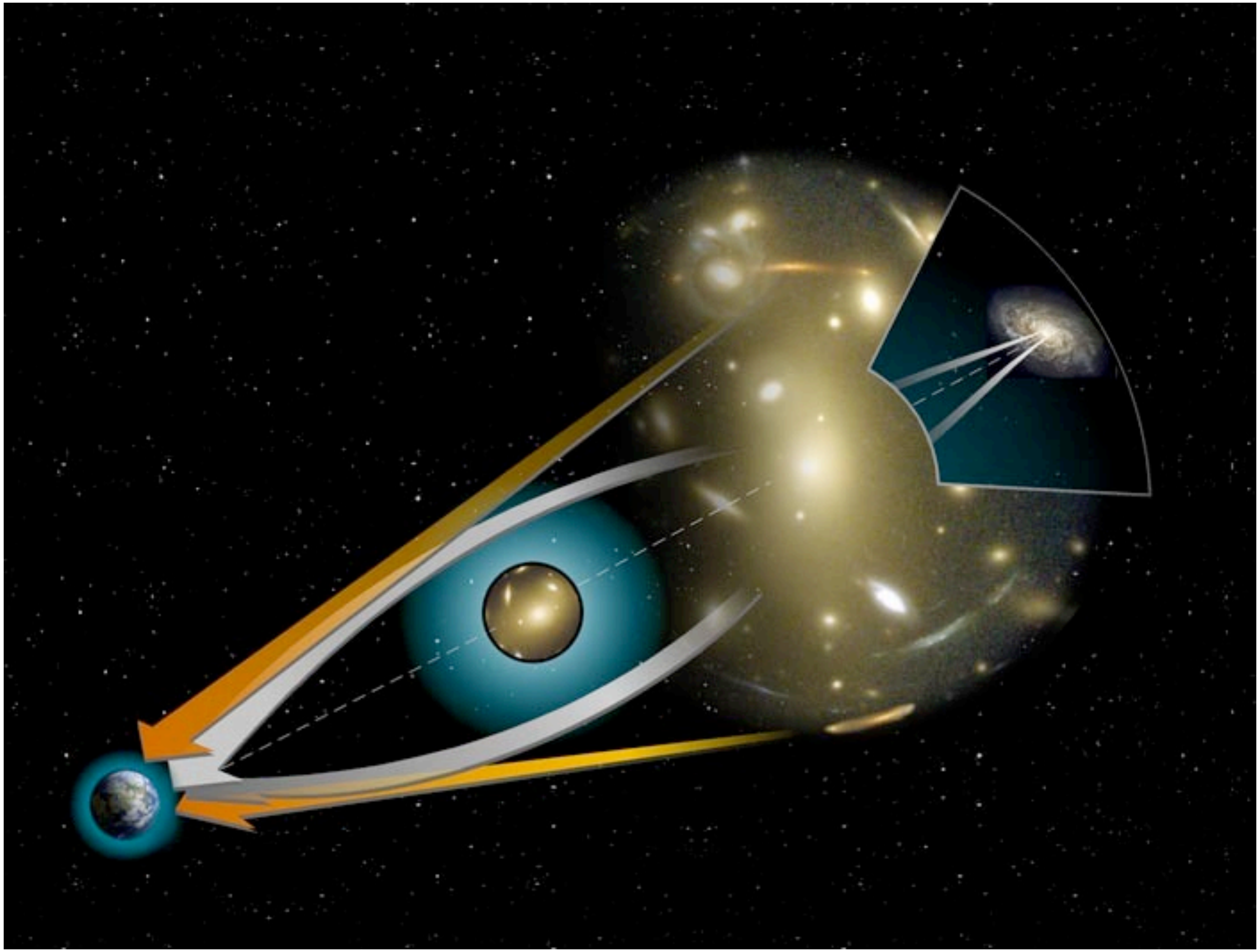
Guzzo et al 2008



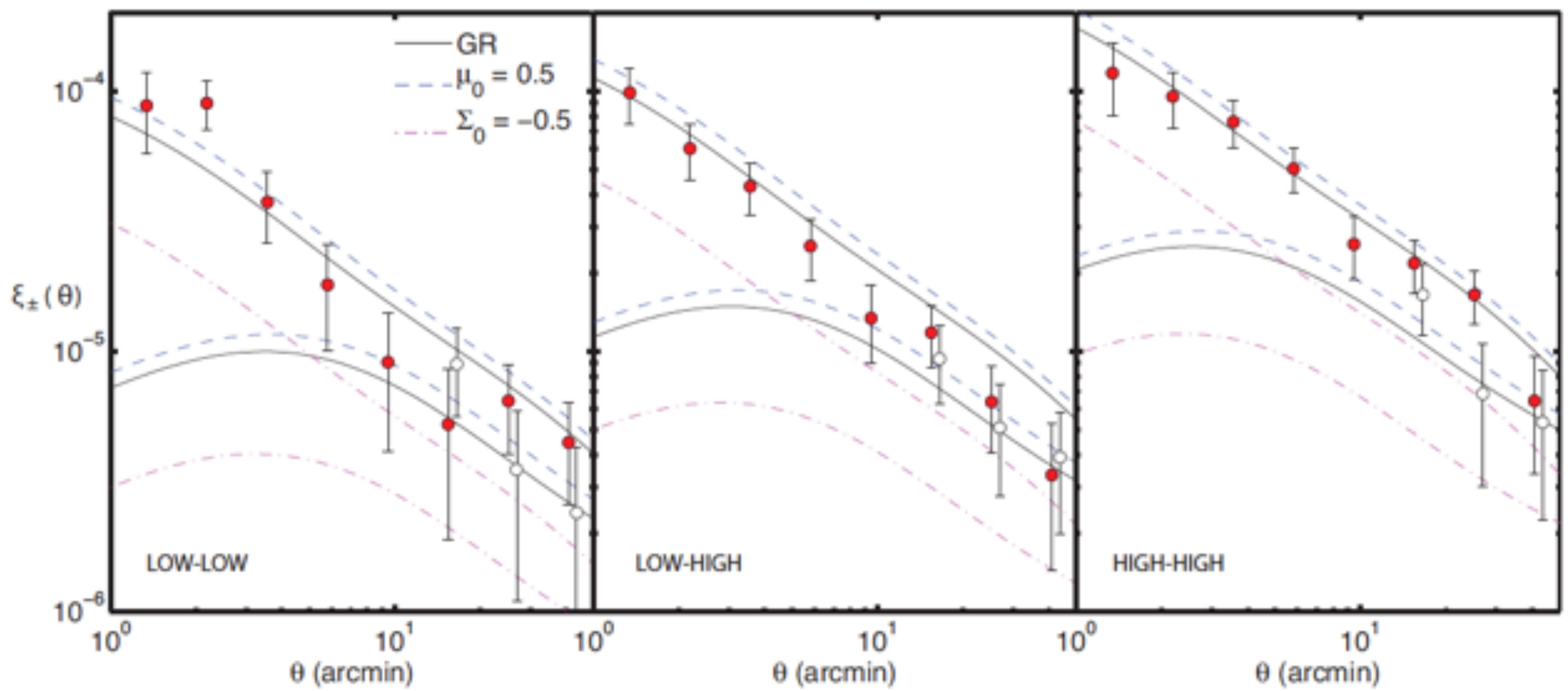
$$f(k,a) = \frac{d\ln\delta_M(k,a)}{d\ln a}$$

Macaulay et al
ArXiv:1303:6583

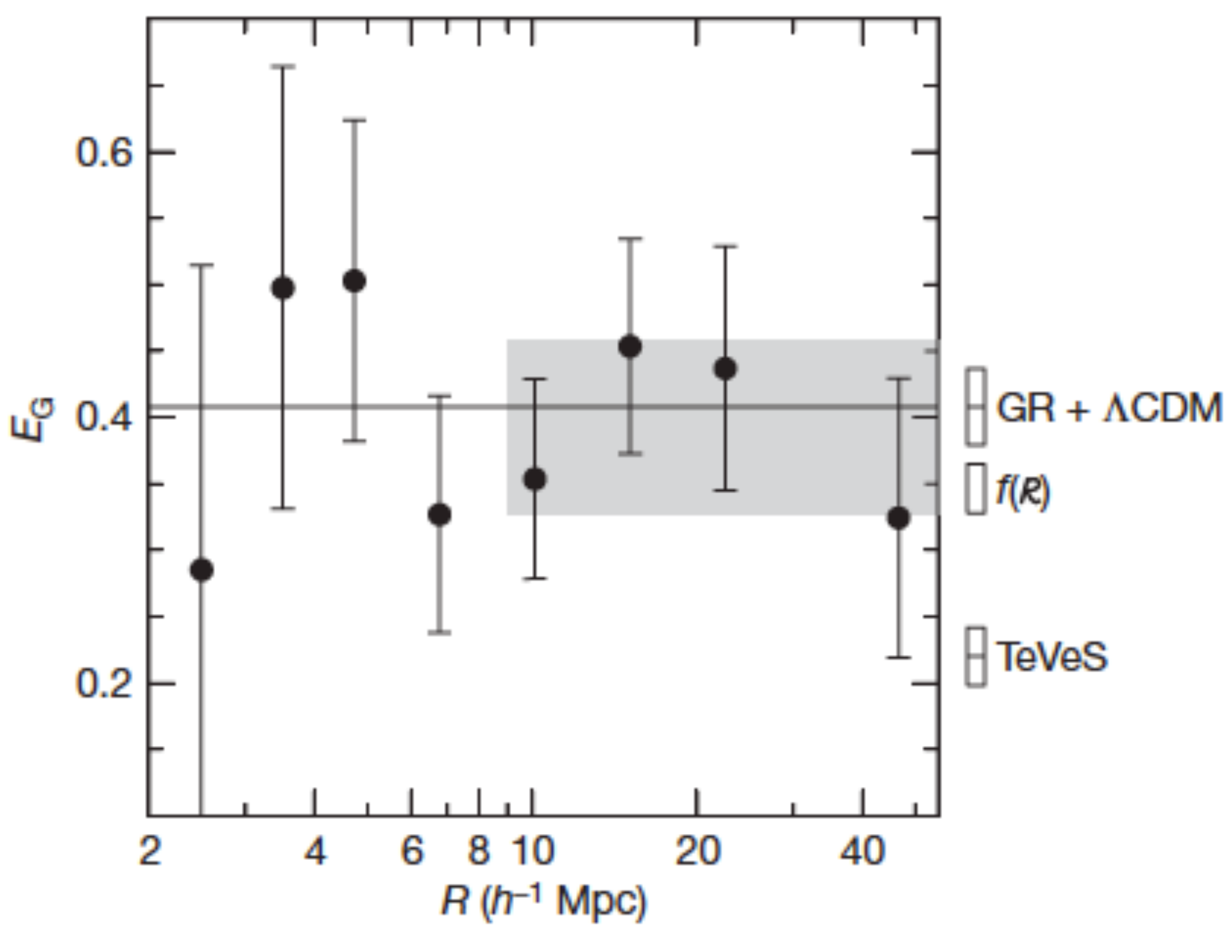
Weak Lensing



Galaxy Weak Lensing



Simpson et al 2012
(CFHTLens)



Reyes et al 2010

State of Play in 2014

no constraints on GR

however...

The Future

EUCLID
MAPPING THE GEOMETRY OF THE DARK UNIVERSE

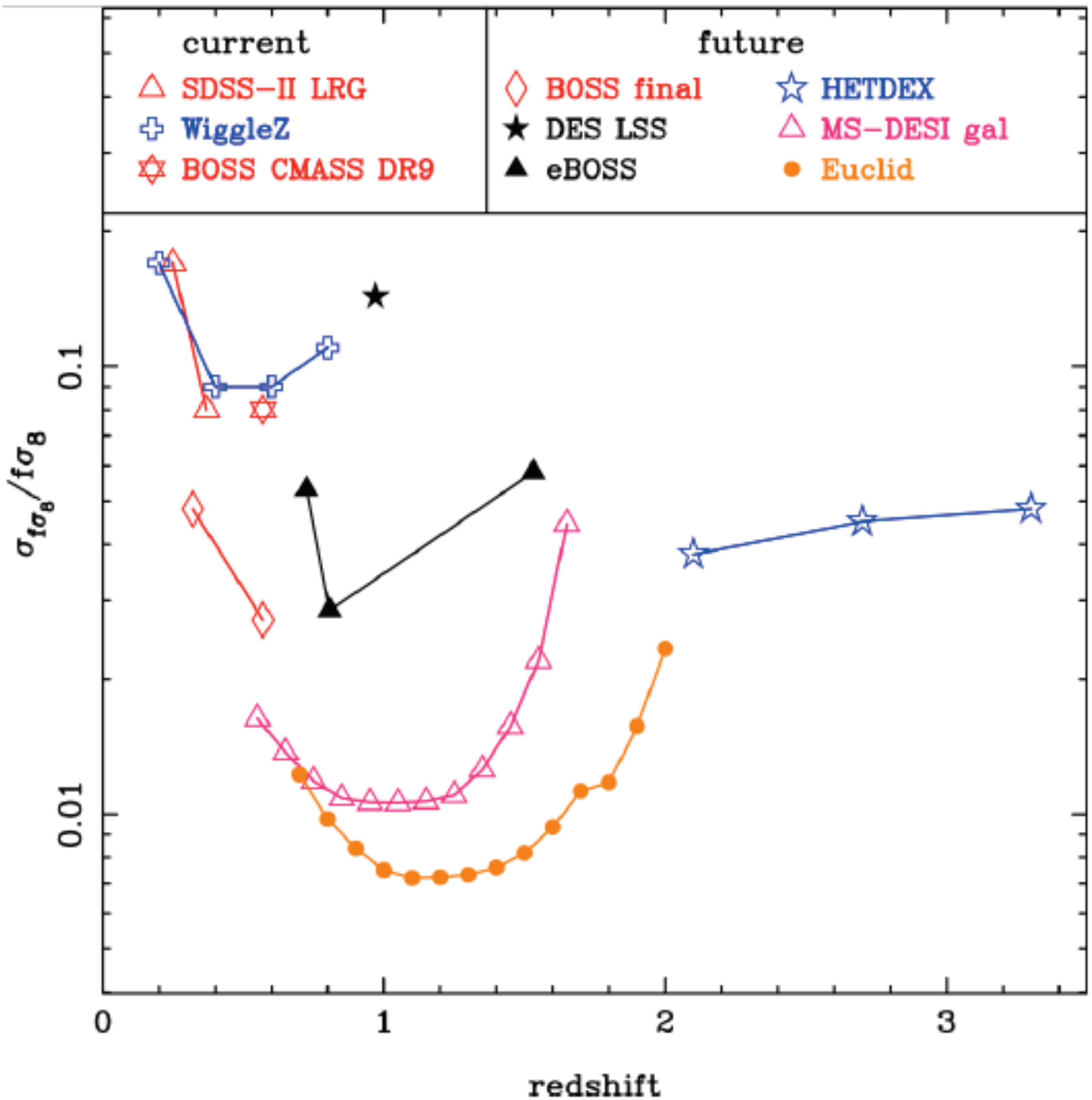
The Square Kilometre Array
The International Radio Telescope for the 21st Century

LSST
Large Synoptic Survey Telescope

The Future is now

Data Type	Now	Soon	Future
Photo-z:LSS (weak lensing)	DES, RCS, KIDS	HSC	LSST, Euclid, SKA
Spectro-z (BAO, RSD, ...)	BOSS	MS-DESI,PFS,HETDEX, Weave	Euclid, SKA
SN Ia	HST, Pan-STARRS, SCP, SDSS, SNLS	DES, J-PAS	JWST,LSST
CMB/ISW	WMAP	Planck	
sub-mm, small scale lensing, SZ	ACT, SPT	ACTPol,SPTPol, Planck, Spider,Vista	CCAT, SKA
X-Ray clusters	ROSAT, XMM, Chandra	XMM, XCS, eRosita	
HI Tomography	GBT	Meerkat, Baobab, Chime, Kat 7	SKA

The Future: Redshift Space Distortions



Percival 2013

Model Dependent Constraints

Theory	parameter	now	future
Brans-Dicke	$1/\omega$	0.006	4.19×10^{-4}
Einstein-Aether	c_1	few	0.222
	c_3	few	1.736
	α	few	0.244
DGP	$1/(r_c H_0)$	0.075	0.004

Summary

- The large scale structure of the Universe can be used to test gravity (different eras probe different scales).
- There is an immense landscape of gravitational theories (how credible or natural is open for debate).
- We need a unified framework to test gravity ("PPF"modelled on PPN).
- Focus on linear scales at late times (for now).
- Non-linear scales can be incredibly powerful but much more complicated
- Need new methods and observations to access the really large scales (is HI tomography the future?).
- Current measurements are not constraining.
- There are a plethora of new experiments to look forward to.