Understanding INFLATION

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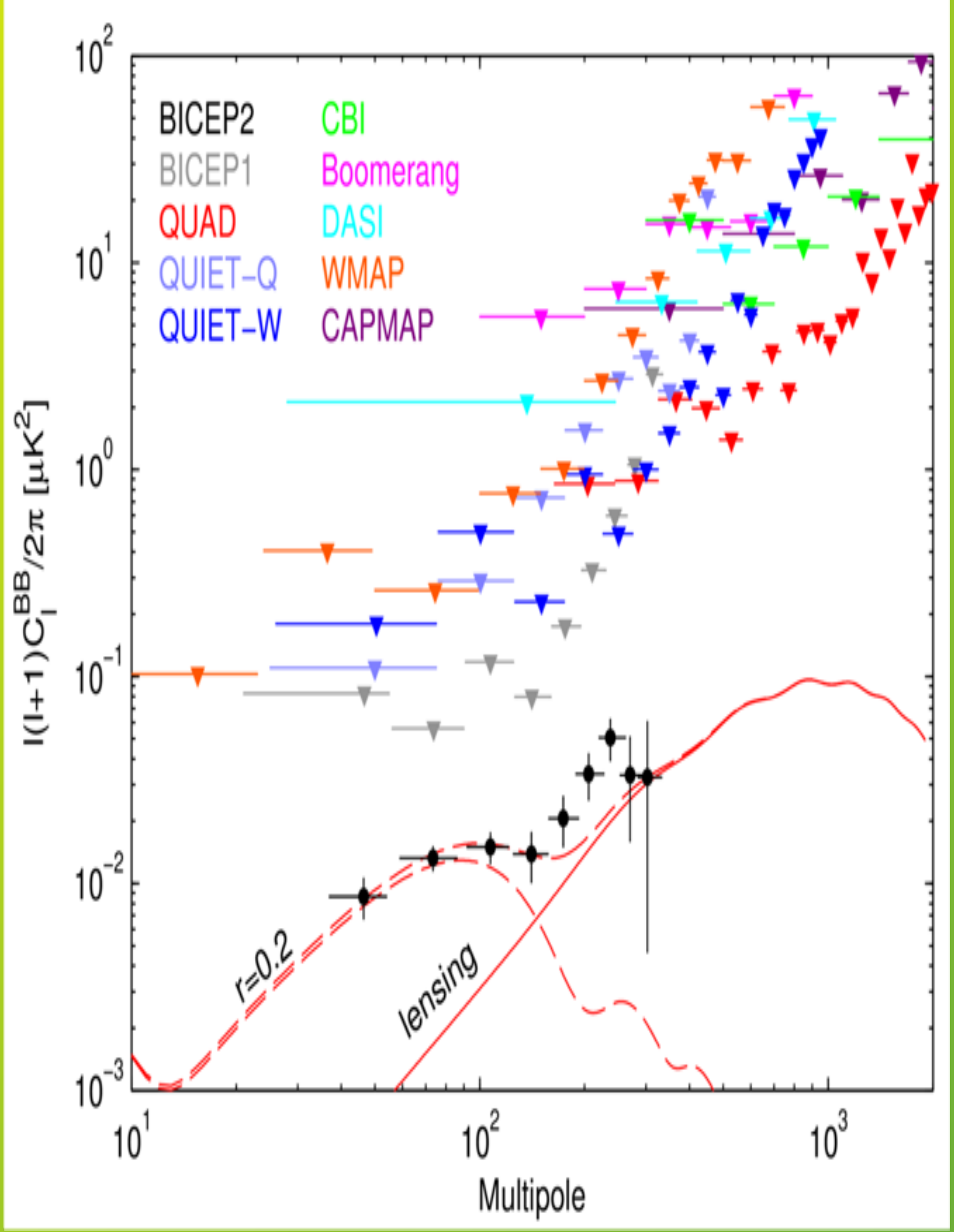
GENEVA U.

0) Motivation

1) Implementation
2) Predictions

3) Observables



0) Motivation/s (The need of Inflation)

UNIVERSE

1) **Gravity Th:** GENERAL RELATIVITY (**GR**)

$$G_{\mu\nu}[g_{**}] = \frac{1}{m_p^2} T_{\mu\nu}[\psi]$$

2) **P. Symmetry:** HOMOGENEITY & ISOTROPY (**H & I**)

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - K r^2} + r^2 d\Omega^2\right]$$

(FRW)

Scale Factor — $a^2(t)$

Curvature — K

0) Motivation/s (The need of Inflation)

UNIVERSE → 1) GR + 2) H & I ⇒

Friedmann Equations

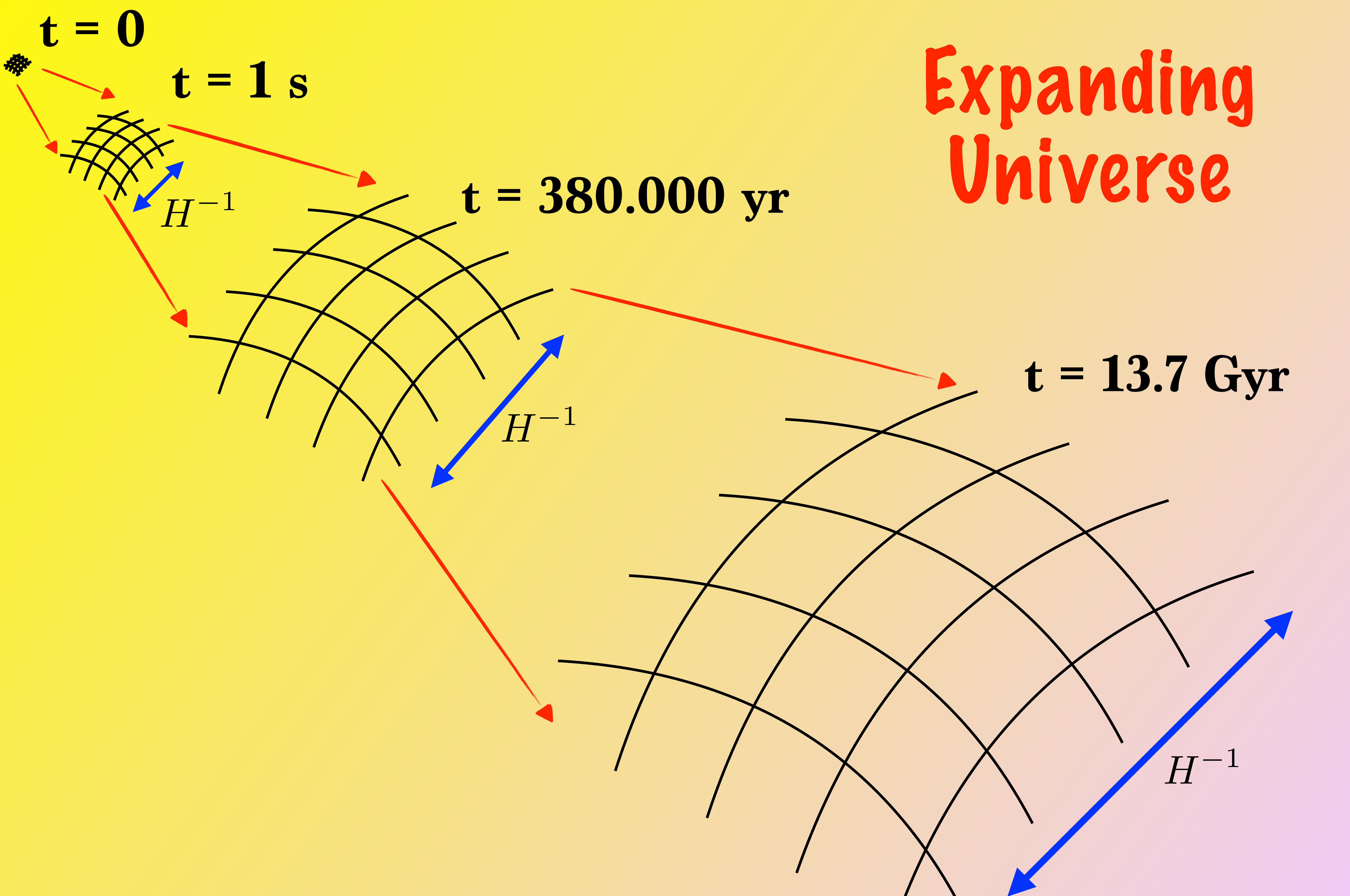
$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{\rho}{6m_p^2}(1+3w)$$

$$H^2 \equiv \left(\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2}$$

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{a}\frac{da}{dt}(1+w)$$

$$\left(w \equiv \frac{p}{\rho}\right)$$

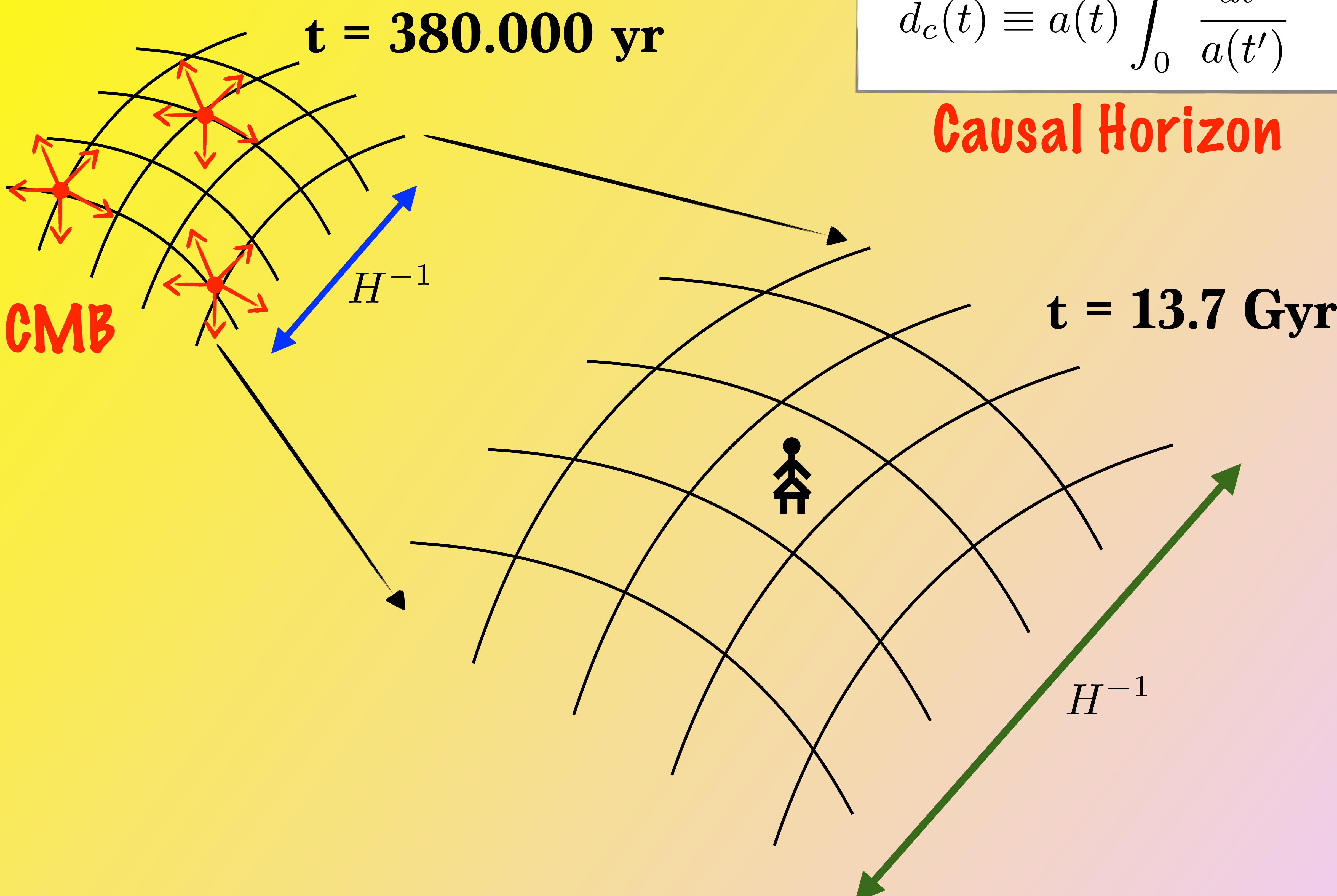
0) Motivation/s (The need of Inflation)



0) Motivation/s (The need of Inflation)

$$d_c(t) \equiv a(t) \int_0^t \frac{dt'}{a(t')}$$

Causal Horizon



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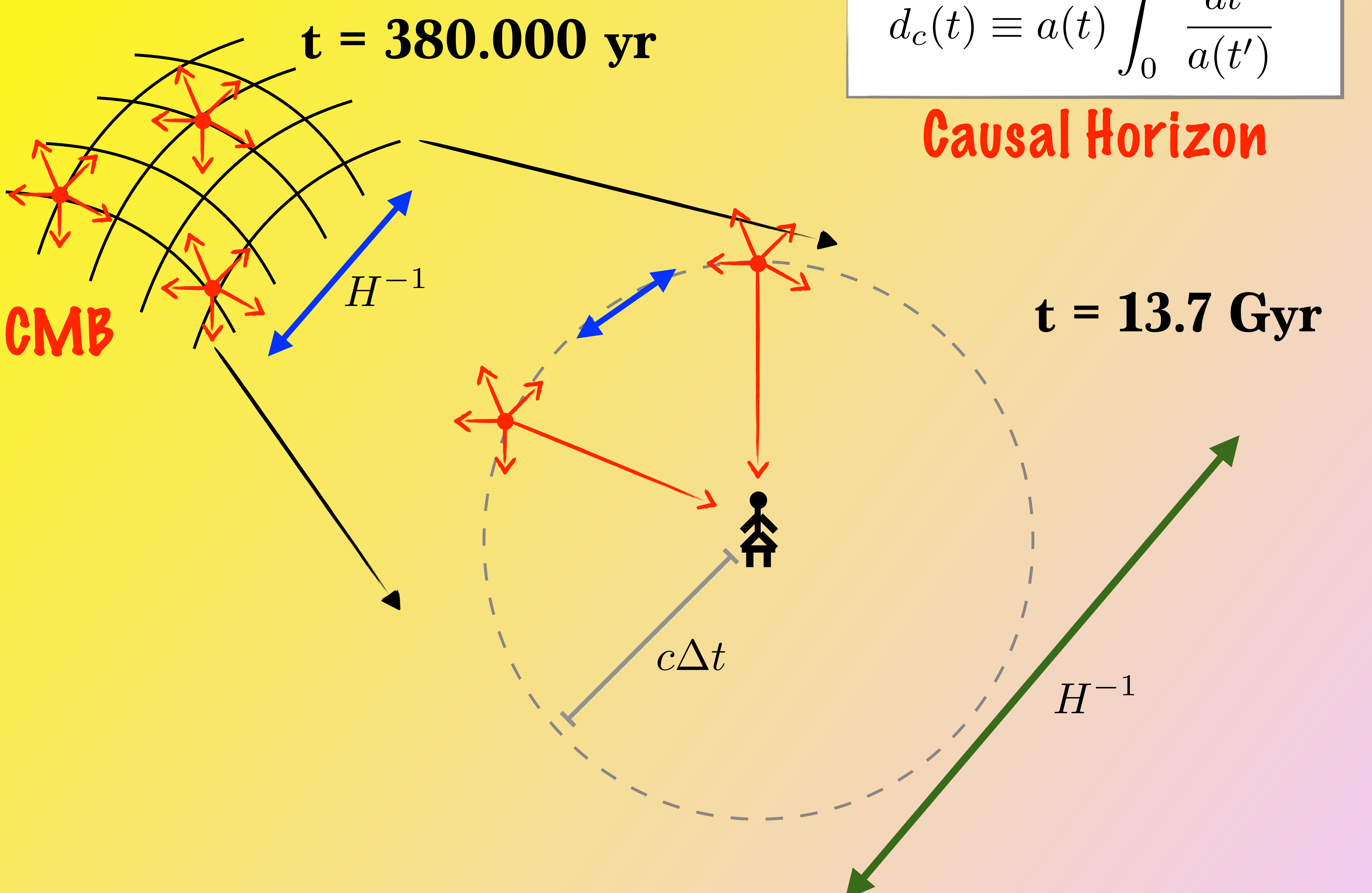
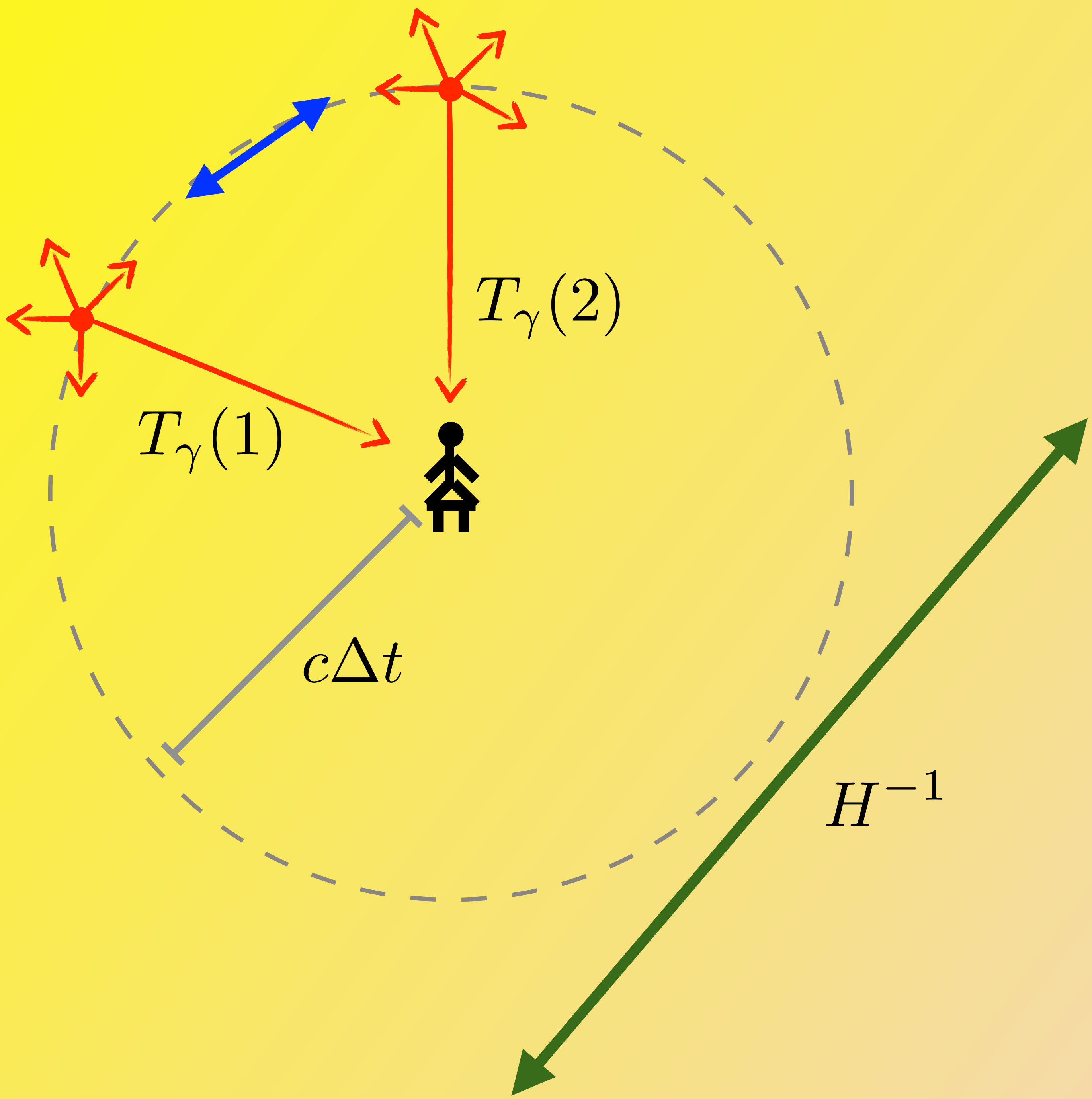


0) Motivation/s (The need of Inflation)

t = 13.7 Gyr



IF $T_\gamma(1) = T_\gamma(2)$

CAUSALITY VIOLATION !!

hBB:

H&I @ Scales >> 1/H

iLL-defined!

0) Motivation/s (The need of Inflation)

1) Horizon Problem → Causality Violation !!!

hBB ↗

0) Motivation/s (The need of Inflation)

hBB → 1) **Horizon Problem** → **Causality Violation !!!**

hBB → 2) **Curvature Problem** (Fine-tuning)

(If curvature $\neq 0$, it grows unstable!)

0) Motivation/s (The need of Inflation)

hBB

- **1) Horizon Problem** → **Causality Violation !!!**
- **2) Curvature Problem (Fine-tuning)**
  (If curvature $\neq 0$, it grows unstable!)

Need extra 'Ingredient' ! → **INFLATION !**

1) Inflation (Definition + Implementation)

INF → * **Definition:**

$$\frac{d^2a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0$$

Comoving Hubble Radious

$$\mathcal{H}^{-1} \equiv \frac{1}{aH} \sim \begin{cases} a^2\ , & \text{hBB} \\ a^{-1}\ , & \text{Inf.} \end{cases}$$

(increasing)

(decreasing)

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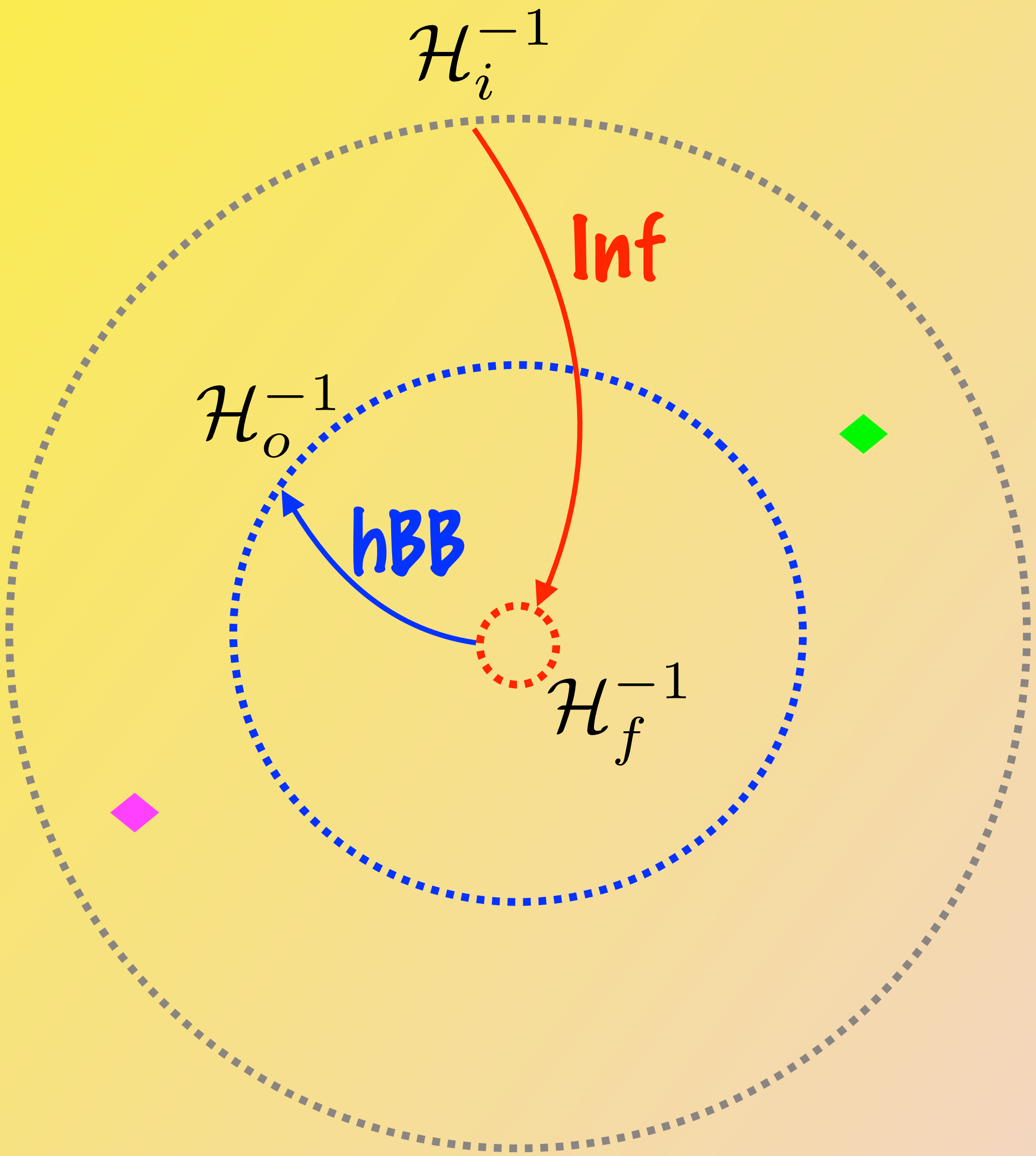

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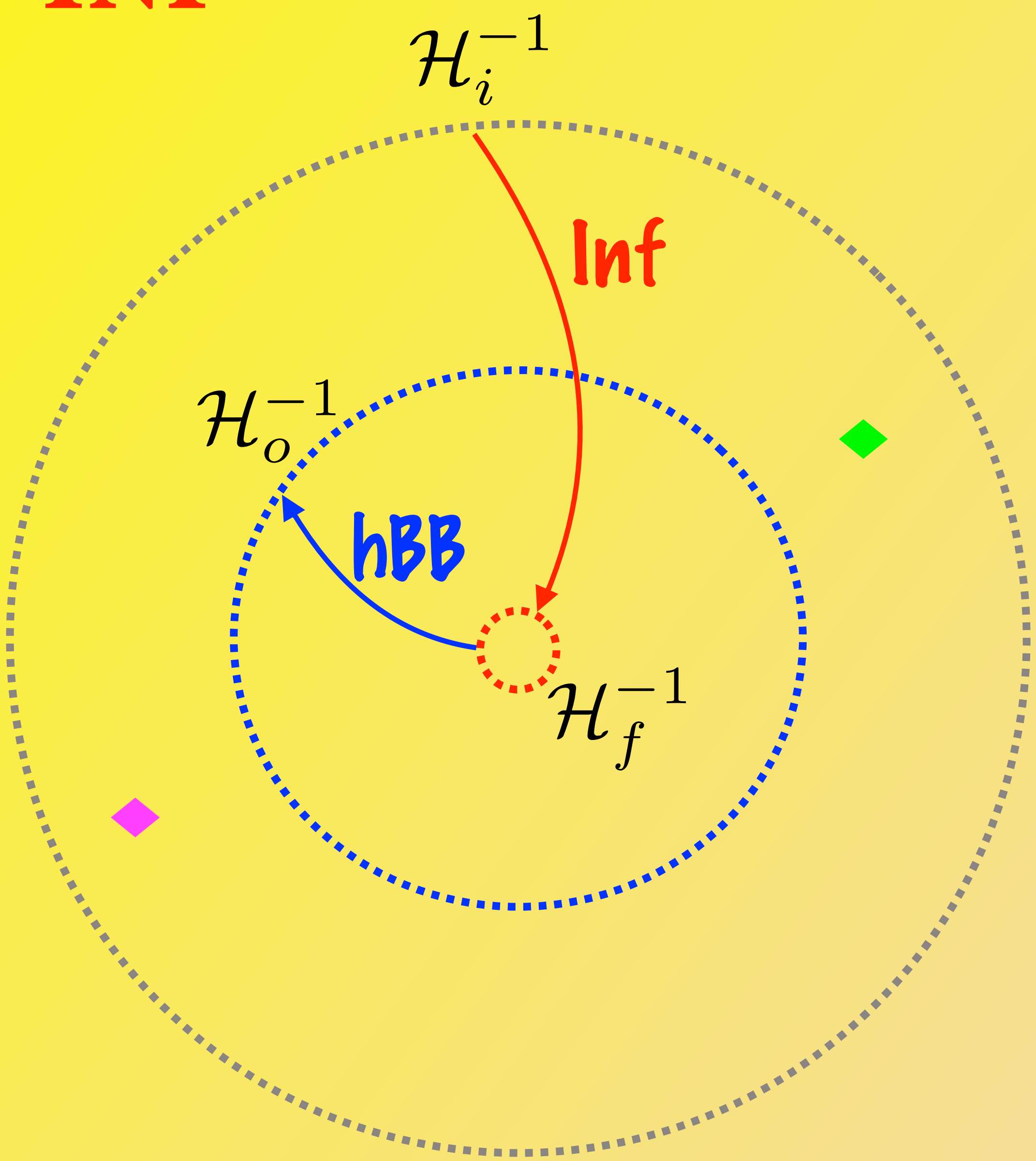

$$\frac{a_f}{a_i} \equiv e^N \quad (\text{\# e-folds})$$

$$\begin{aligned} \mathcal{H}_i^{-1} &= \frac{a_f}{a_i}\,\mathcal{H}_f^{-1} \\ &= e^N\,\mathcal{H}_f^{-1} \geq \mathcal{H}_o^{-1} \end{aligned}$$

$$\begin{aligned} N &\geq \log(\mathcal{H}_f/\mathcal{H}_o) = \log(E_f/E_o) \\ &\gtrsim 60 + \log(E_f[GeV]/10^{16}) \end{aligned}$$

1) Inflation (Definition + Implementation)

INF

* **Definition:** $\frac{d^2 a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0$

* **Consequences:** If N $\gtrsim$ 60
  - Horizon Problem Solved !!!
  - Bonus: Null Curvature !

$$\left(\left| \frac{K}{\mathcal{H}^2} \right| \sim |K/H_i^2| e^{-2N} = |K/H_i^2| e^{-120} \ll 1 \right)$$

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* **Implementation:** $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)$ (ϕ Inflaton)

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IF $V(\phi) \gg \frac{1}{2}\dot{\phi}^2, \frac{1}{2}(\nabla\phi)^2$

$$w \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - \frac{1}{6a^2}(\nabla\phi)^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi)} \simeq \frac{-V(\phi)}{V(\phi)} \simeq -1$$

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$$
\begin{aligned}
&\text{i)} & \frac{d\rho_\phi}{dt} &\simeq 0 \\
&\text{ii)} & H^2 &\simeq \frac{V(\phi)}{3m_p^2} \\
&\text{iii)} & \frac{1}{a}\frac{d^2a}{dt^2} &\simeq +\frac{V(\phi)}{3m_p^2}
\end{aligned}
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\end{aligned}$$

→ $a(t) \simeq a_i e^{\int_t H(\phi)dt'}$

(Quasi) de Sitter

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$$w \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - \cancel{\frac{1}{6a^2}(\nabla\phi)^2} - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + \cancel{\frac{1}{2a^2}(\nabla\phi)^2} + V(\phi)} \simeq -1 + \frac{2}{3}\epsilon$$

$$\boxed{\epsilon \equiv \frac{3}{2}\frac{\dot{\phi}^2}{V(\phi)} \ll 1}$$

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$$\epsilon \equiv \frac{3}{2}\frac{\dot{\phi}^2}{V(\phi)} \ll 1$$

$$\frac{\ddot{a}}{a} = \frac{1}{3m_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \simeq H^2(1-\epsilon)$$

↔

$$\epsilon = -\frac{\dot{H}}{H^2}$$

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⇕

Can $\epsilon \ll 1$ be sustained for $\Delta N = 60$? No, unless $V(\phi)$ is "flat" !

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⇕

Can $\epsilon \ll 1$ be sustained for $\Delta N = 60$? No, unless $V(\phi)$ is "flat" !

$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$ → ϕ accelerates! $\Rightarrow \dot{\phi} \uparrow\uparrow \Rightarrow \epsilon \uparrow\uparrow$

Needed: $|\ddot{\phi}| \ll 3H\dot{\phi}, V'(\phi)$ → $\eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$

1) Inflation (Definition + Implementation)

* **Implementation:** $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)$ (ϕ Inflaton)

Slow Roll (SR) Regime:

$$\epsilon = \frac{\dot{\phi}^2}{2m_p^2 H^2} \ll 1 \; ; \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

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$$\epsilon = \frac{\dot\phi^2}{2m_p^2 H^2} \ll 1 \; ; \quad \eta = -\frac{\ddot\phi}{H\dot\phi} \ll 1$$

$$\epsilon_V \equiv \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta_V \equiv m_p^2\left(\frac{V''}{V}\right)$$

$$(\epsilon \simeq \epsilon_V \, , \quad \eta \simeq \eta_V - \epsilon_V)$$

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$(\epsilon \simeq \epsilon_V \,, \quad \eta \simeq \eta_V - \epsilon_V)$

→ If $\epsilon_V\,, \eta_V \ll 1 \;\Rightarrow\; \epsilon, \eta \ll 1$

SR $\Longrightarrow$ quasi dS for $\Delta N = 60$

$$N(\phi) \simeq \int_{\phi_f}^{\phi} \frac{d\phi'}{\sqrt{2\epsilon(\phi',\dot\phi')}}$$

1) Inflation (Definition + Implementation)

* **Implementation:** $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)$ (ϕ Inflaton)

Case of Study:

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2$$

→ $\epsilon_V(\phi) = \eta_V(\phi) = 2(m_p/\phi)^2$

→ $N(\phi) = (\phi/2m_p)^2 - 1/2$

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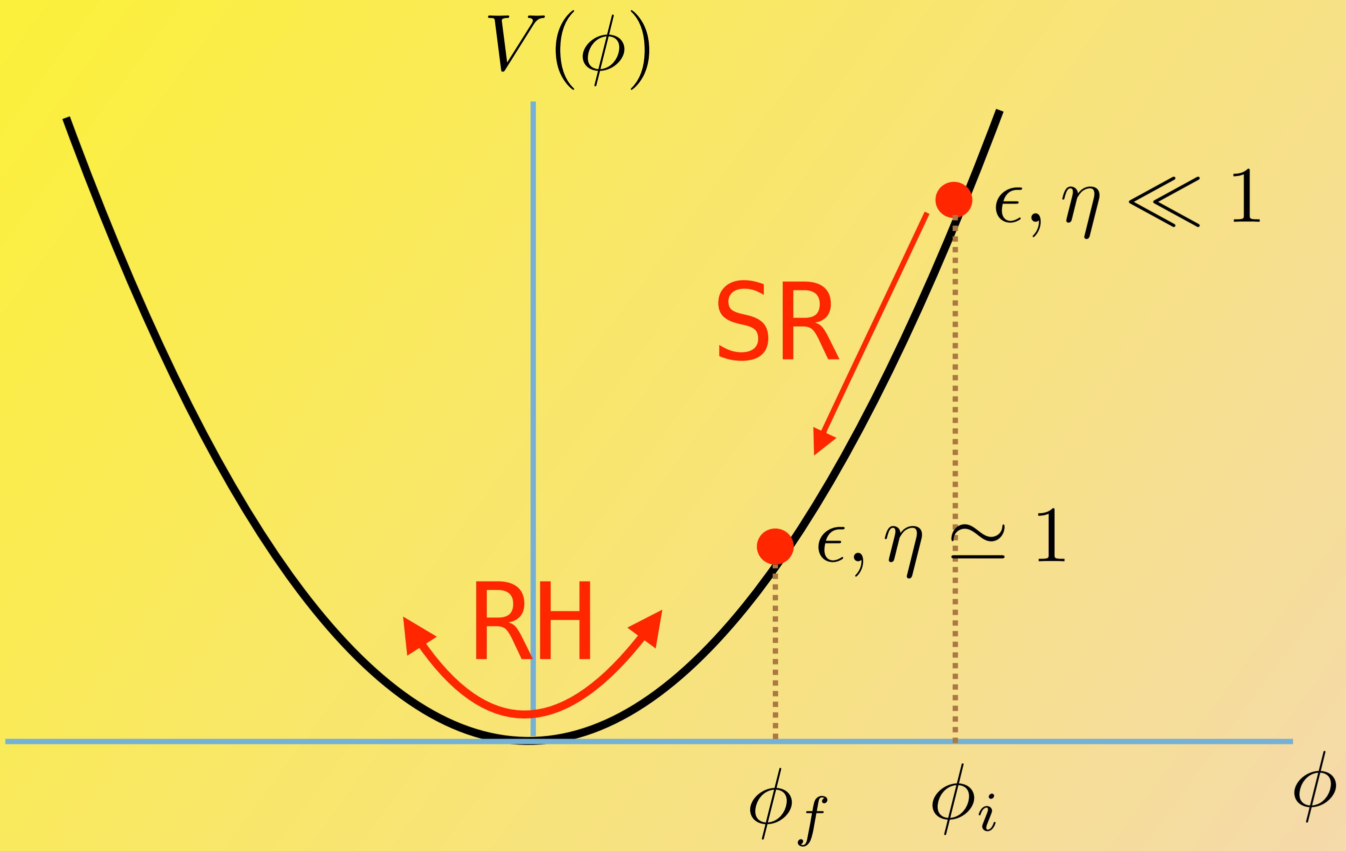

2) Inflation: Basic Predictions

$a \sim e^{\int H dt'} \gtrsim e^{60}$ (qdS)

INF → **SR:** $\epsilon, \eta \ll 1 \rightarrow \epsilon, \eta \simeq 1$

(Start) **(End)**

Flat Universe !

No Hor. Problem !

2) Inflation: Basic Predictions

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(Start) (End)

→ $a \sim e^{\int H dt'} \gtrsim e^{60}$ (qdS)

→ Flat Universe !

→ No Hor. Problem !

* Is that ALL ?? NO!

(Background)		(Fluctuations)
$\phi(t)$	→	$\phi(t) + \delta\phi(\vec{x}, t)$
$g_{\mu\nu}(t)$	→	$g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x}, t)$

INF

↓

Primordial fluctuations!!

2) Inflation: Basic Predictions

Inflation: A generator of Primordial Fluctuations

$\phi(t)$ → $\phi(t) + \delta\phi(\vec{x}, t)$

$g_{\mu\nu}(t)$ → $g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x}, t)$

(Background) **(Fluctuations)**

but WHY fluctuations ?

because of...

Quamtum Mechanics !

2) Inflation: Basic Predictions

Inflation: A generator of Primordial Fluctuations

$\phi(t) \rightarrow \phi(t) + \delta\phi(\vec{x}, t)$

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(Background) **(Fluctuations)**

but WHY fluctuations ?

because of...

Quamtum Mechanics !

QM:

$$\hat{\phi}(\vec{x}, t) \;\rightarrow\; \langle \hat{\phi}(\vec{x}, t) \rangle = \phi(t) \quad \Rightarrow \quad \hat{\phi}(\vec{x}, t) = \phi(t) + \delta\hat{\phi}(\vec{x}, t)$$

VeV

Vacuum Quam. Fluct

$$\langle \delta\hat{\phi}(\vec{x}, t) \rangle = 0$$

but...

$$\left\langle \, [\delta\hat{\phi}(\vec{x}, t)]^2 \, \right\rangle \neq 0$$

Vacuum Quam. Fluct. !!!

2) Inflation: Basic Predictions

Inflation: A generator of Primordial Fluctuations

$$\hat{\phi}(\vec{x},t) = \phi(t) + \hat{\delta\phi}(\vec{x},t) \quad \rightarrow \quad \langle \hat{\delta\phi}^2(\vec{x},t) \rangle \neq 0$$

but ... ~~Minkowski~~ → Curved Space: (quasi)dS

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but ... ~~Minkowski~~ → Curved Space: (quasi)dS

$$S = \frac{m_p^2}{2}\int d^4x\sqrt{-g}\{R - (\partial\phi)^2 - 2V(\phi)\}$$

→ $\phi(t) + \delta\phi(\vec{x},t)$

→ $g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$

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$$\phi(t) + \delta\phi(\vec{x},t)$$

$$g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$$

$$\begin{aligned} ds^2 &= g^{\rm tot}_{\mu\nu}dx^\mu dx^\nu = (g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t))dx^\mu dx^\nu \\ &= -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j \end{aligned}$$

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$$B_i = \partial_i B - S_i$$

$$E_{ij} = 2\partial_{ij}E + 2\partial_{(i}F_{j)} + h_{ij}$$

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Expanding U. → Vector Perturbations $B_i \propto 1/a(t)$

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$$\partial_i h_{ij} = h_{ii} = 0$$

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dof

- scalars → $\delta\phi + \{\Phi, B, \Psi, E\}$
- tensors → h_{ij}

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dof

scalars: $\delta\phi + \{\Phi, B, \Psi, E\}$ —Diff.→ $\delta\phi' + \{\Phi', B', \Psi', E'\}$ ~~Gauge Inv.~~

tensors: h_{ij} —Diff.→ h_{ij} Gauge Inv. !

Diff.: $x^\mu \;\to\; x^\mu + \xi^\mu$

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dof

scalars → $\delta\phi + \{\Phi, B, \Psi, E\}$ →

tensors → h_{ij}

$$\zeta \equiv -[\Psi + (H/\dot{\rho})\delta\rho_\phi] \overset{\text{Diff.}}{\longrightarrow} \zeta$$

$$\mathcal{R} \equiv [\Psi + (H/\dot{\phi})\delta\phi] \overset{\text{Diff.}}{\longrightarrow} \mathcal{R}$$

$$Q \equiv [\delta\phi + (\dot{\phi}/H)\Psi] \overset{\text{Diff.}}{\longrightarrow} Q$$

All Gauge Inv. !

2) Inflation: Basic Predictions

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dof

scalars: $\delta\phi + \{\Phi, B, \Psi, E\}$

tensors: h_{ij}

$$\zeta \equiv -[\Psi + (H/\dot{\rho})\delta\rho_\phi] \xrightarrow{\text{Diff.}} \zeta$$

$$\mathcal{R} \equiv [\Psi + (H/\dot{\phi})\delta\phi] \xrightarrow{\text{Diff.}} \mathcal{R}$$

$$Q \equiv [\delta\phi + (\dot{\phi}/H)\Psi] \xrightarrow{\text{Diff.}} Q$$

All Gauge Inv. !

Curvature Pert.

Tensor Pert. (GW)

Fixing Gauge: e.g. $E, \delta\phi = 0 \Rightarrow g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]$

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$$S = \frac{m_p^2}{2}\int d^4x\sqrt{-g}\{R - (\partial\phi)^2 - 2V(\phi)\}$$

2) Inflation: Basic Predictions

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

$$g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]$$

$$S = \frac{m_p^2}{2}\int d^4x\sqrt{-g}\{R - (\partial\phi)^2 - 2V(\phi)\}$$

$$S = S_{(0)} + S_{(2)}^{(s)} + S_{(2)}^{(t)}$$

$$S_{(2)}^{(s)} = \frac{1}{2}\int d^4x\ a^3\frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]$$

$$S_{(2)}^{(t)} = \frac{m_p^2}{8}\int dt dx^3 a^3\left[(\dot{h}_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right]$$

Background
Inflationary dynamics

2) Inflation: Basic Predictions

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$S^{(s)}_{(2)} = \frac{1}{2}\int d^4x \; a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right]$$

=

?

2) Inflation: Basic Predictions

Inflation: A generator of Primordial Fluctuations

$d\tau \equiv dt/a(t)$

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$$\left[v \equiv z\mathcal{R}\,, \quad z \equiv a\frac{\dot\phi}{H}\right]$$ (Mukhanov variable)

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(F. T.)

$$v''_{\vec k} + (k^2 - z''/z)v_{\vec k} = 0$$

with

$$\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\,, \quad \nu \equiv \frac{3}{2} + 2\epsilon - \delta$$

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Quantization: $v_{\vec{k}}(t) \to v_k(t)\hat{a}_{\vec{k}} + v^*_k(t)\hat{a}^\dagger_{-\vec{k}} \;, \quad [a_{\vec{k}}, a^\dagger_{\vec{k}'}] = (2\pi)^3\delta(\vec{k} - \vec{k}')$

$$v_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\pi(\nu+1/2)/2}(-\tau)^{1/2} H^{(1)}_\nu(-k\tau)$$

(Bunch-Davies)
Vacuum Fluct.

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Inflation: A generator of Primordial Fluctuations

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$$\equiv P_{\mathcal{R}}(k, \eta)$$

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$$\Delta^2_{\mathcal{R}}(k, \tau) \equiv \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k, \tau)$$

$k\tau \ll 1$ (Super-Horizon)

$$\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2 \dot{\phi}^2} \left(\frac{k}{aH} \right)^{2\eta - 4\epsilon}$$

2) Inflation: Basic Predictions

Inflation: A generator of Primordial Fluctuations

Tensor Fluctuations:

$$S^{(t)}_{(2)} = \frac{m_p^2}{8} \int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2} (\partial_l h_{ij})^2 \right] = ?$$

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$$=$$

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$$h_{ij}(\vec{k}, \tau) = \epsilon^{(s)}_{ij} h^{(s)}_{\vec{k}}$$

$$v^{(s)} \equiv \frac{a}{2} m_p h^{(s)}_{\vec{k}}$$

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→ [Same Procedure as with Scalar Pert.

Quantize → Bunch-Davies → Power Spectrum]

Quantization of Gravity dof !

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Quantization of Gravity dof !

$$\Delta^2_h(k,\tau) \equiv \frac{k^3}{2\pi^2} P_h(k,\tau) \xrightarrow[\text{(Super-Horizon)}]{k\tau \ll 1} \Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2 \left(\frac{k}{aH}\right)^{-2\epsilon}$$

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$$\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{n_s-1}$$

$$n_s - 1 \equiv 2(\eta - 2\epsilon)$$

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$$n_t \equiv -2\epsilon$$

@ Super-Horizon Scales:

$$\mathcal{R}(k), h_{ij}(k) \approx \text{Const.}, \quad k\tau \ll 1$$

3) Inflation: Observables

INFLATION →

$$g_{\mu\nu} = g^{FRW}_{\mu\nu} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$$

H & I

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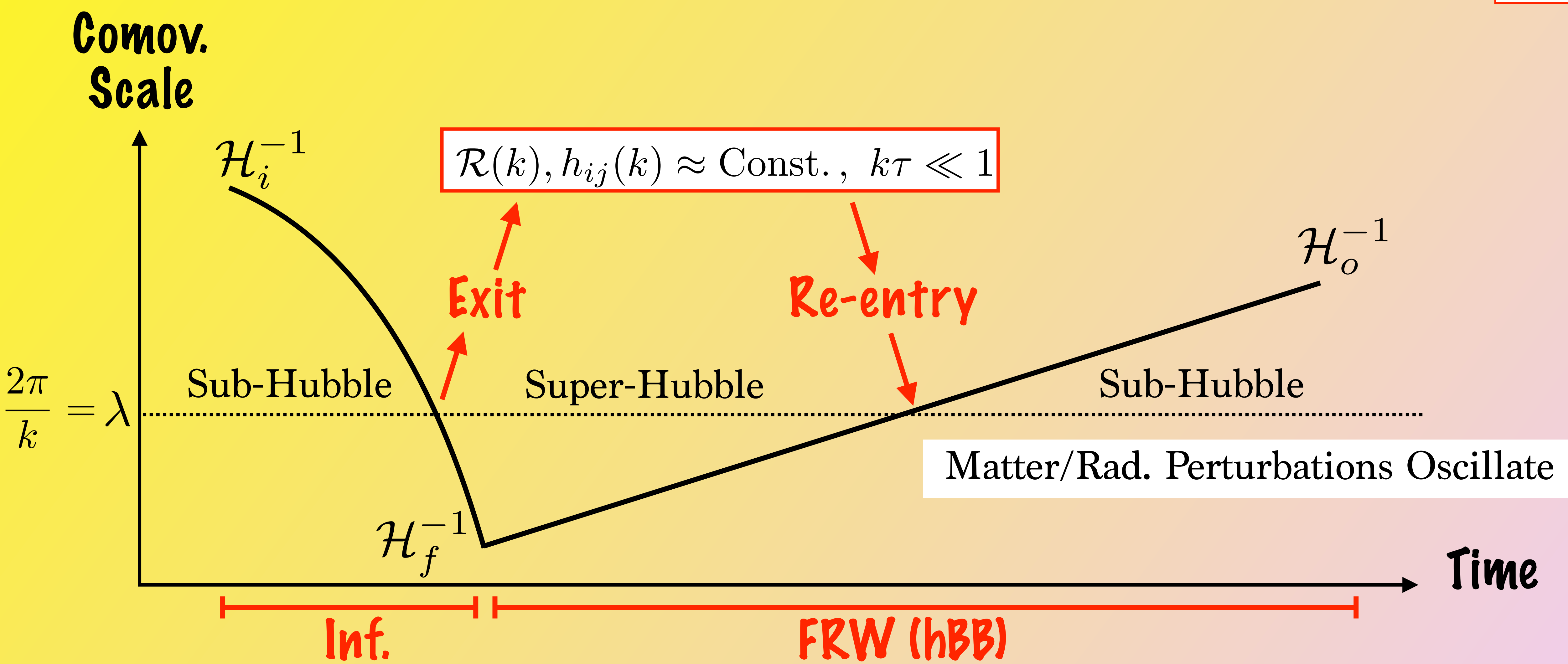
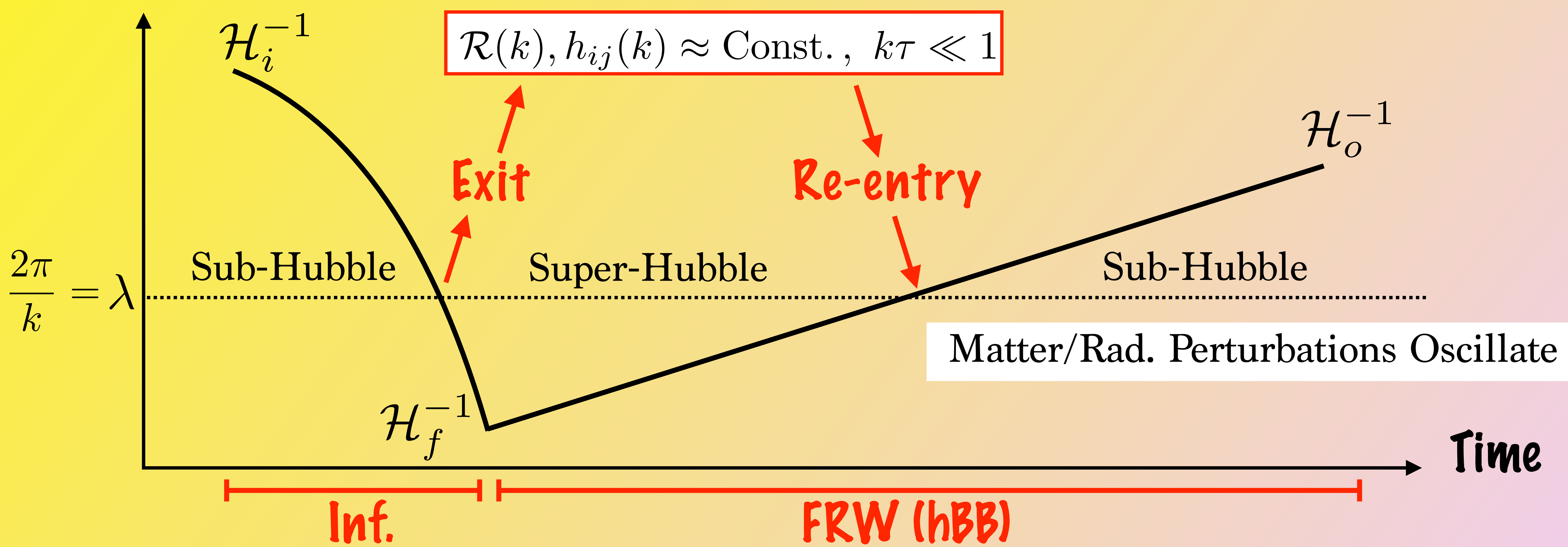


3) Inflation: Observables

Inflation → **Reheating** → **FRW (hBB)**

$a \propto e^{Ht}$ | $\rho_\phi \to \rho(\psi, A_\mu, \ldots)$ | $\rho_\gamma, \rho_b, \rho_{DM}, \ldots$

$g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$ → (**GR**) → $\rho_i + \delta\rho_i$ → (**GR, Int.**) → $\delta\rho_i(t)$ [Evolution]



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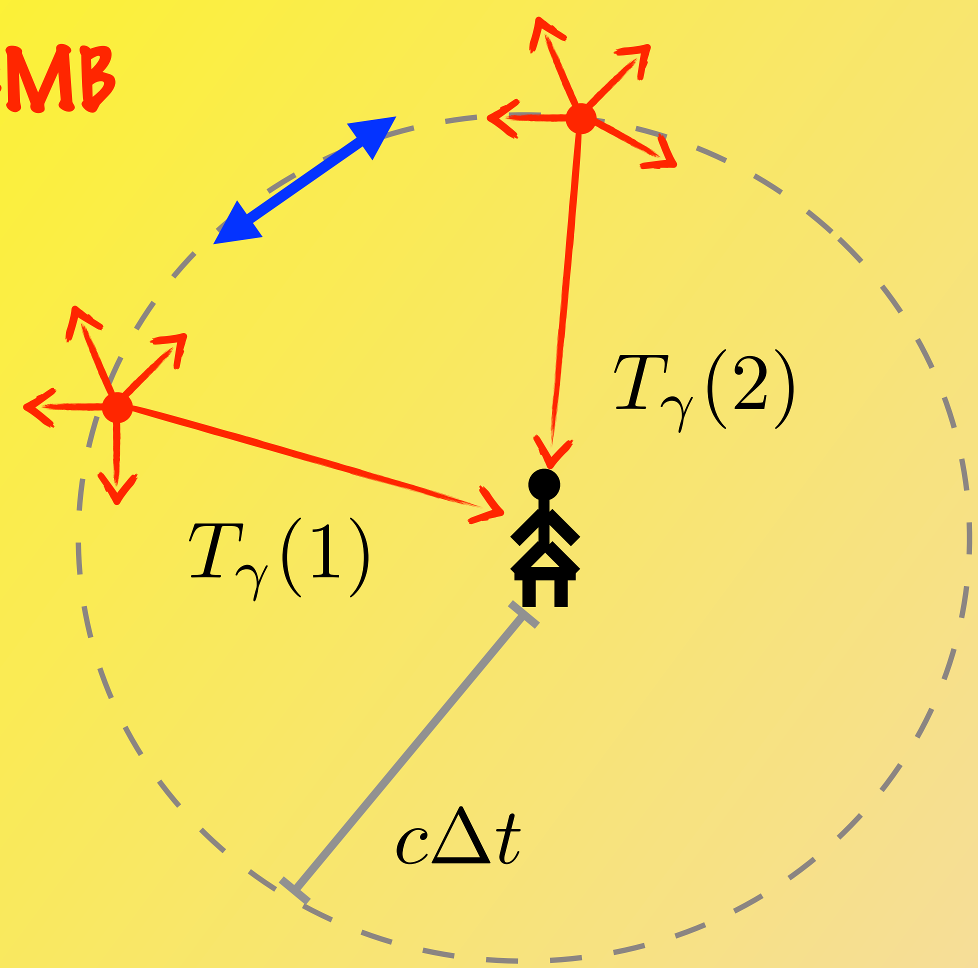
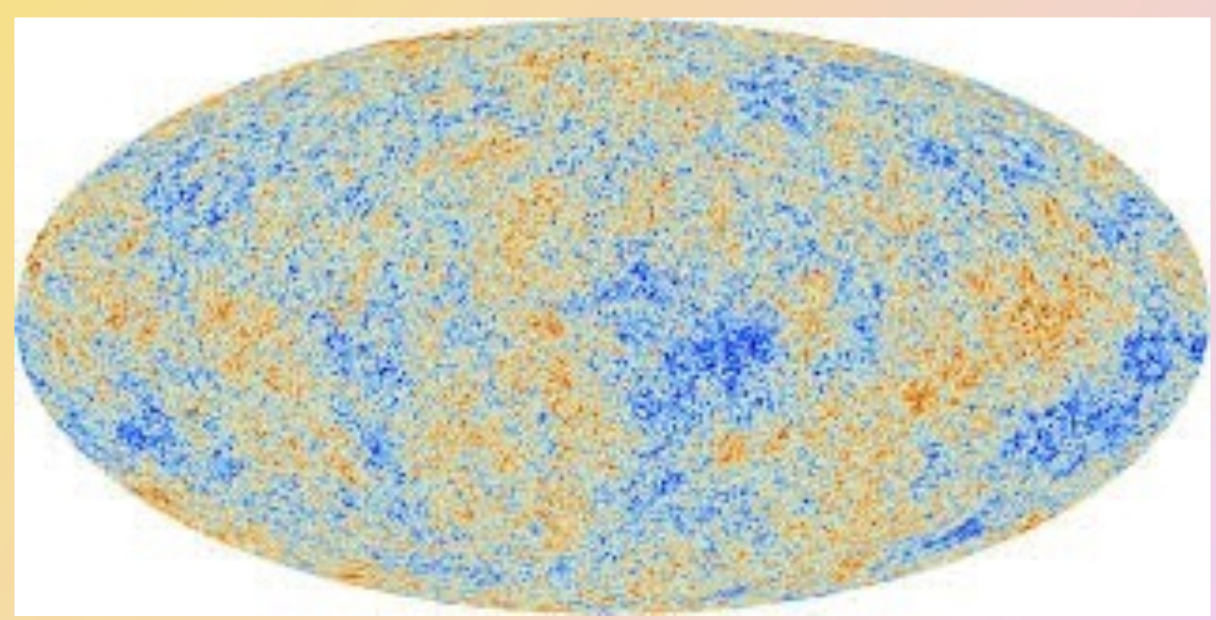
CMB



$$\rho_\gamma + \delta_\gamma \;\Rightarrow\; T_\gamma(\hat{n}) = \bar{T} + \delta T(\hat{n})$$



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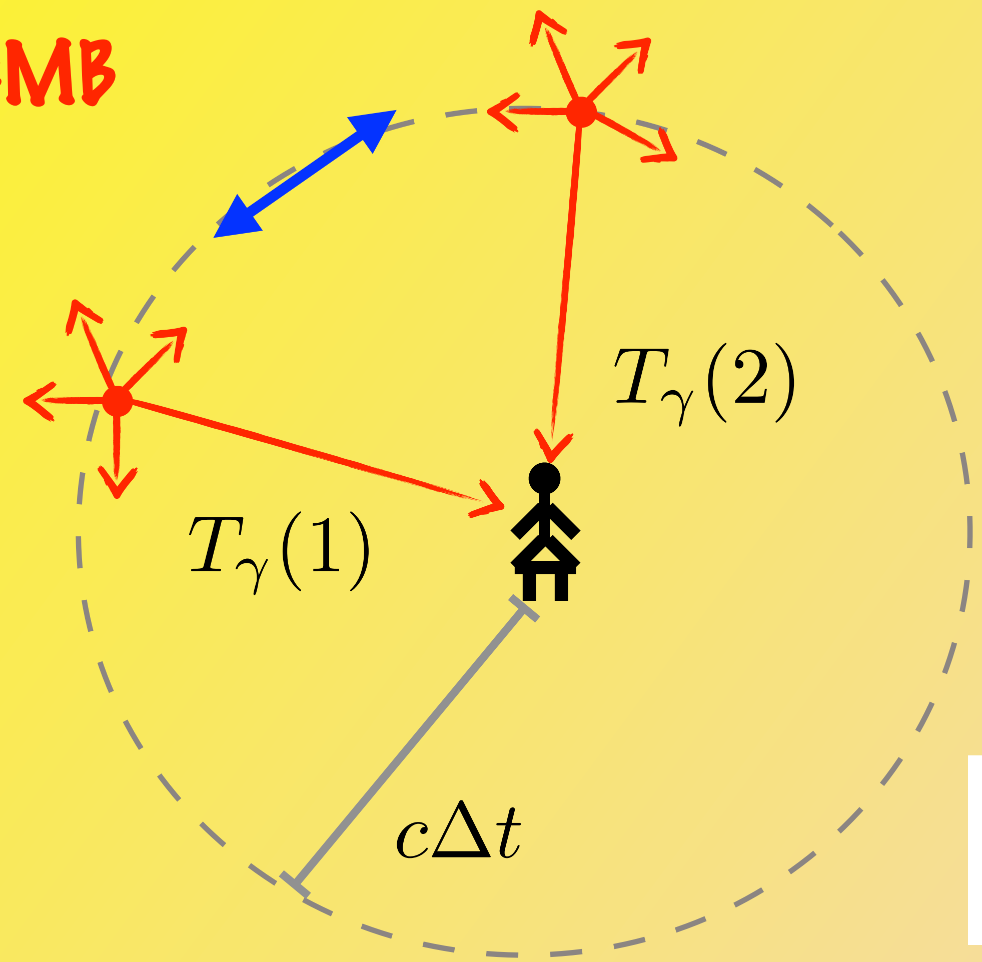

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Temperature Angular Power Spectrum

$$\delta T(\hat{n}) = \sum_{l,m} a_{lm} Y_{lm}(\hat{n}) \;\Rightarrow\; \langle [\delta T]^2 \rangle \;\to\; \langle |a_{lm}|^2 \rangle \equiv C_l$$

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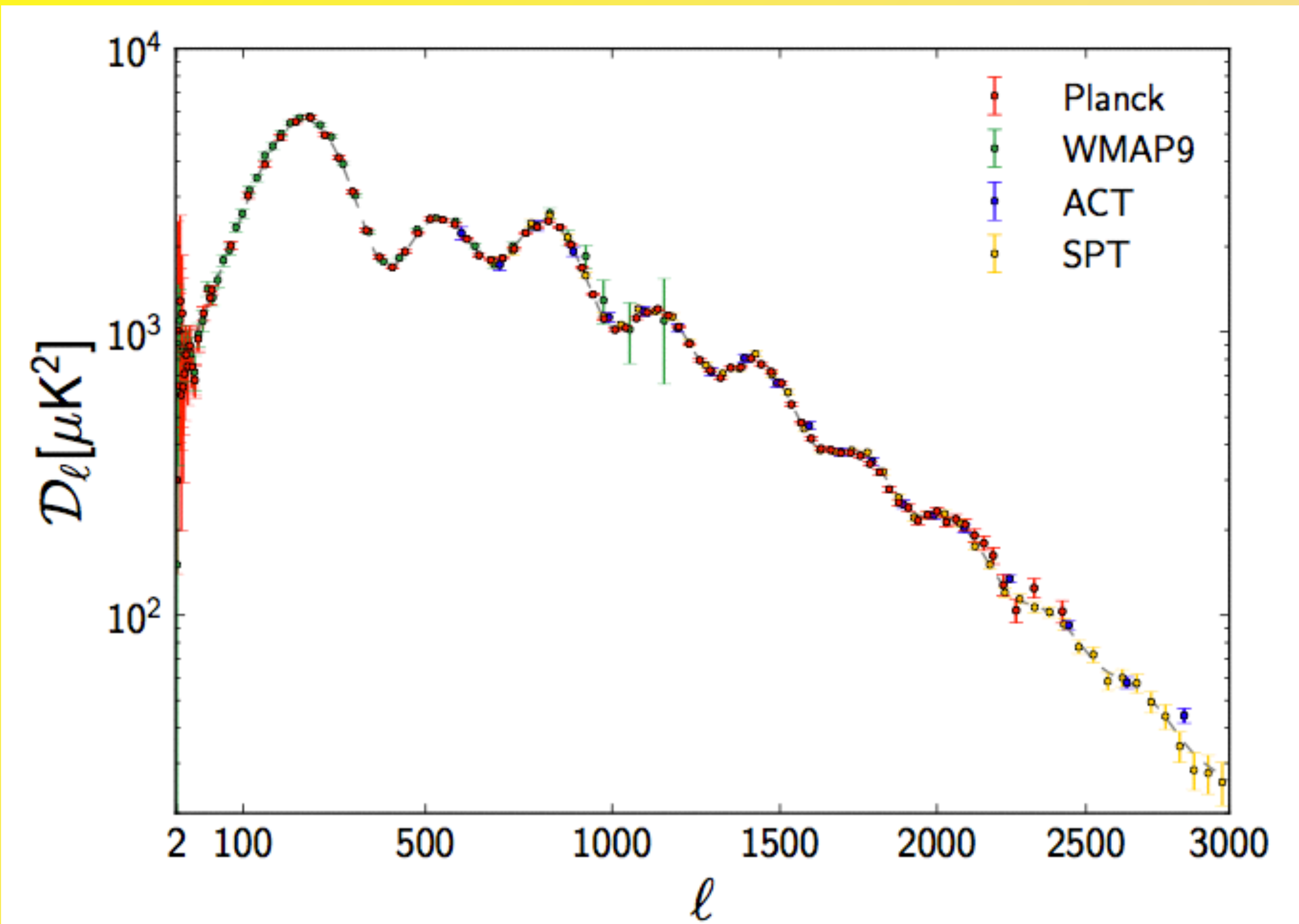


Dashed Line Theoretical Expectation from Inflation

$$|\Omega_k| \ll 1$$

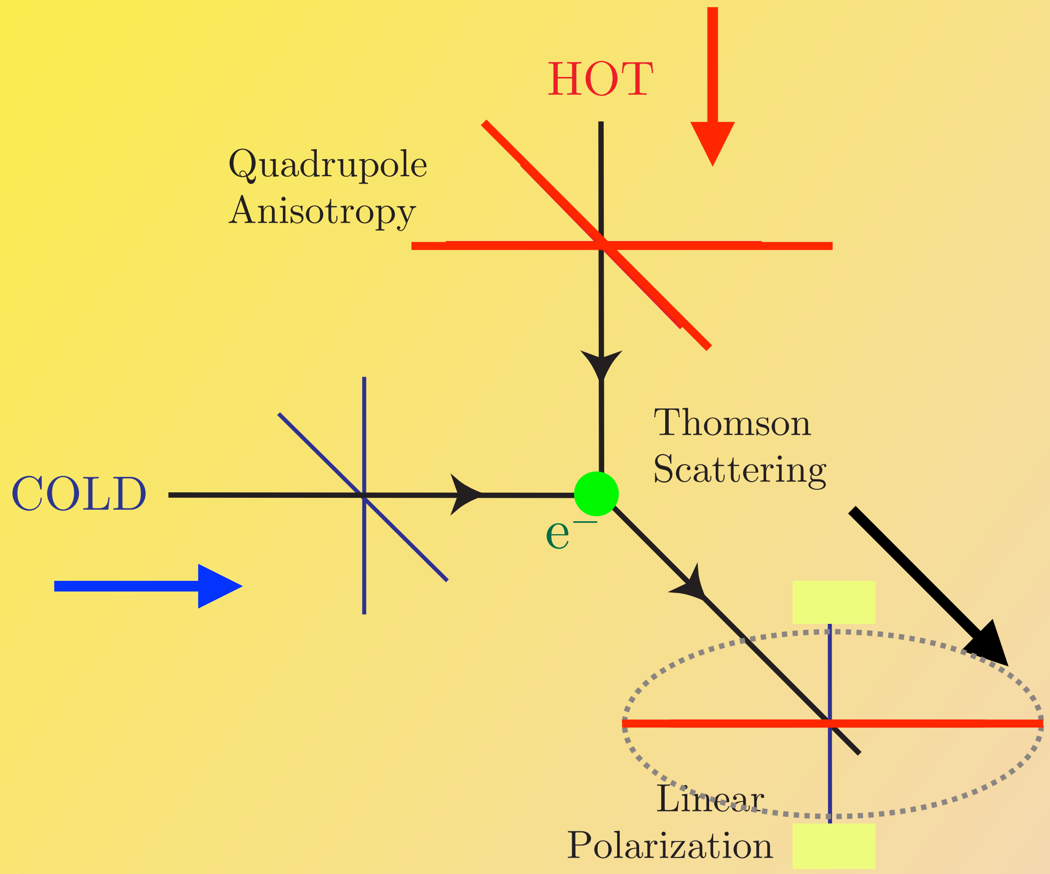
$$n_s \approx 0.96, \Delta_s \simeq 2 \cdot 10^{-9}$$

Adiabatic, Gaussian

3) Inflation: Observables

$\delta\rho_\gamma, \delta\rho_e \Rightarrow$ [Thomson Scattering] $\Rightarrow$ Linear Polarization

Polarization Angular Power Spectrum



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Polarization Angular Power Spectrum

$$\text{Linear Polarization} \rightarrow Q, U \ \text{(Stokes Parameters)}$$

$$(Q \pm iU)(\hat{n}) = \sum_{l,m} a_{lm}^{(\pm 2)} Y_{lm}^{(\pm 2)} = \sum_{l,m} (e_{lm} \pm i b_{lm}) Y_{lm}^{(\pm 2)}(\hat{n})$$

$$\mathcal{E}(\hat{n}) = \sum_{l,m} e_{lm} Y_{lm}(\hat{n}), \ \mathcal{B}(\hat{n}) = \sum_{l,m} b_{lm} Y_{lm}(\hat{n})$$

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Depends on Scalar (also tensor) Perturbations

Depends only on Tensor Perturbations !!!

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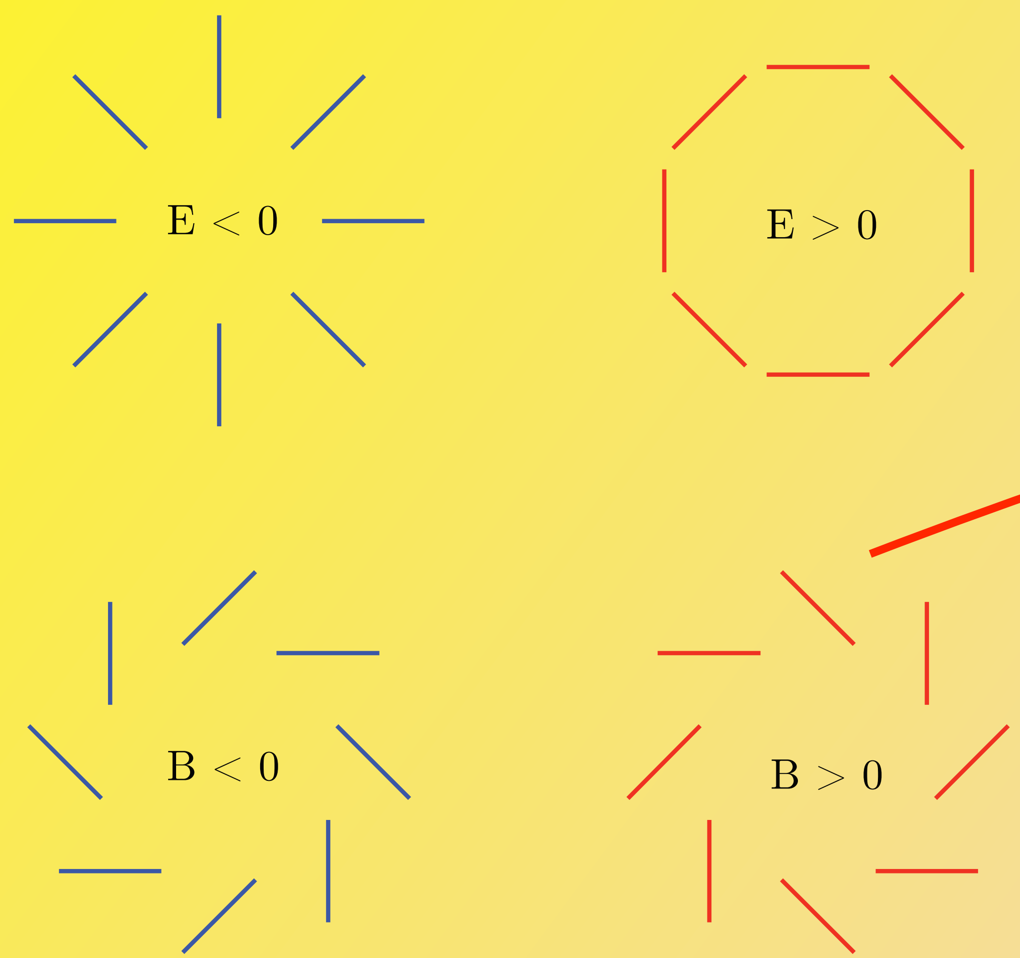

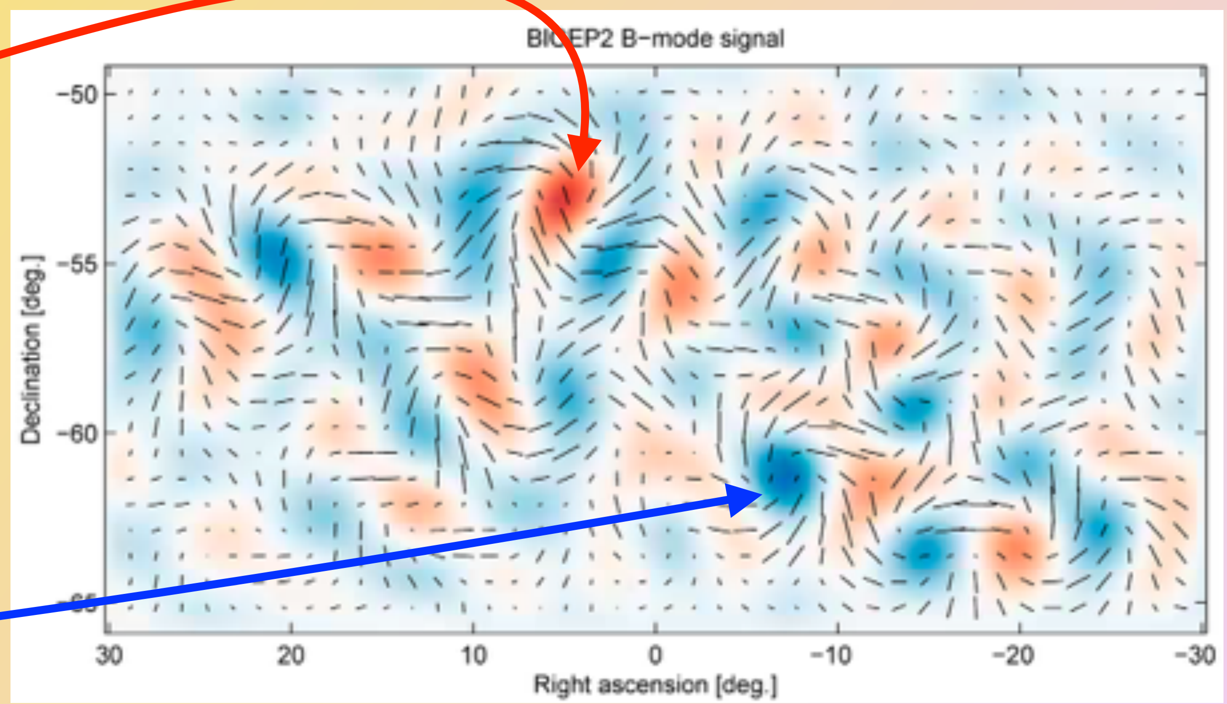

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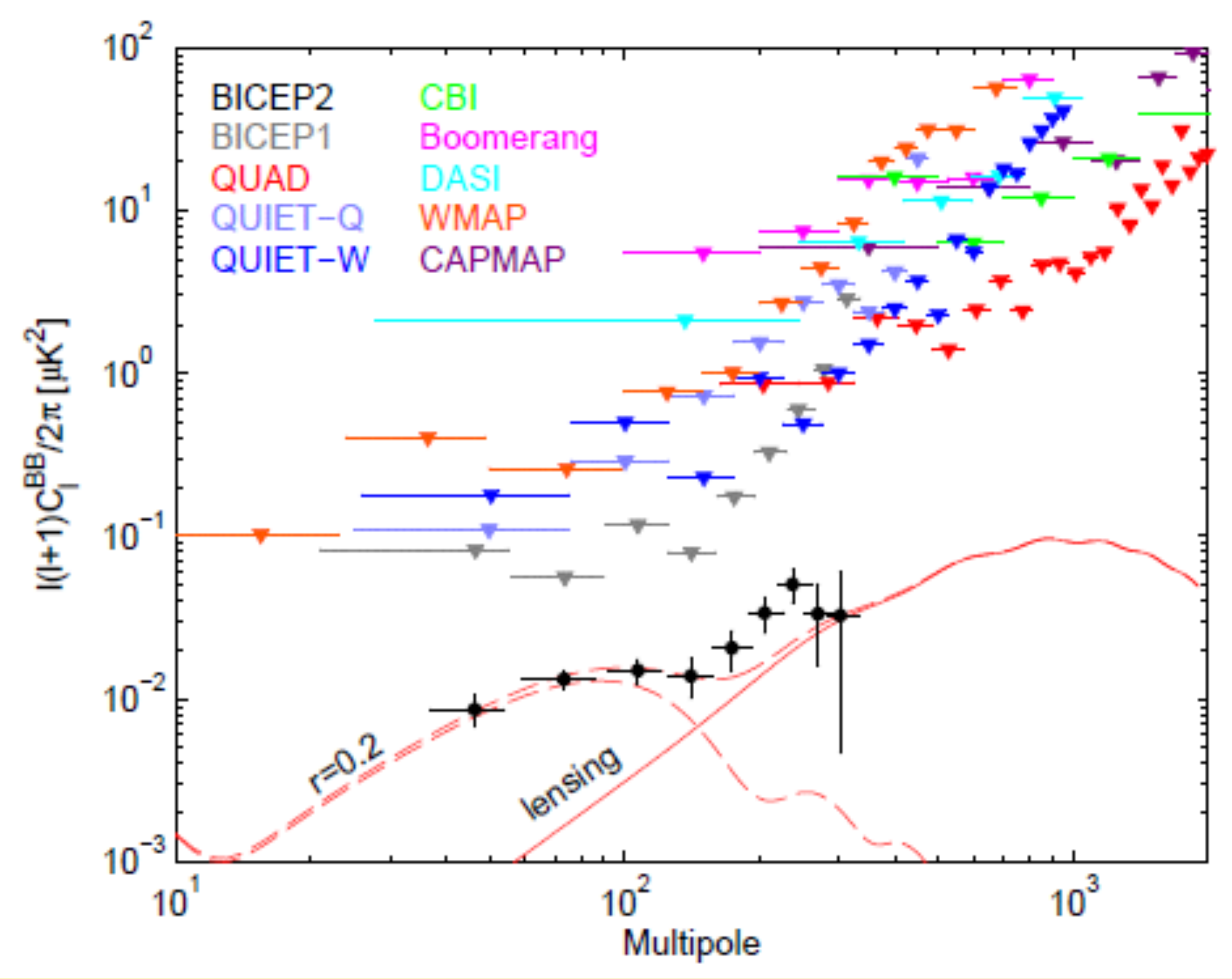

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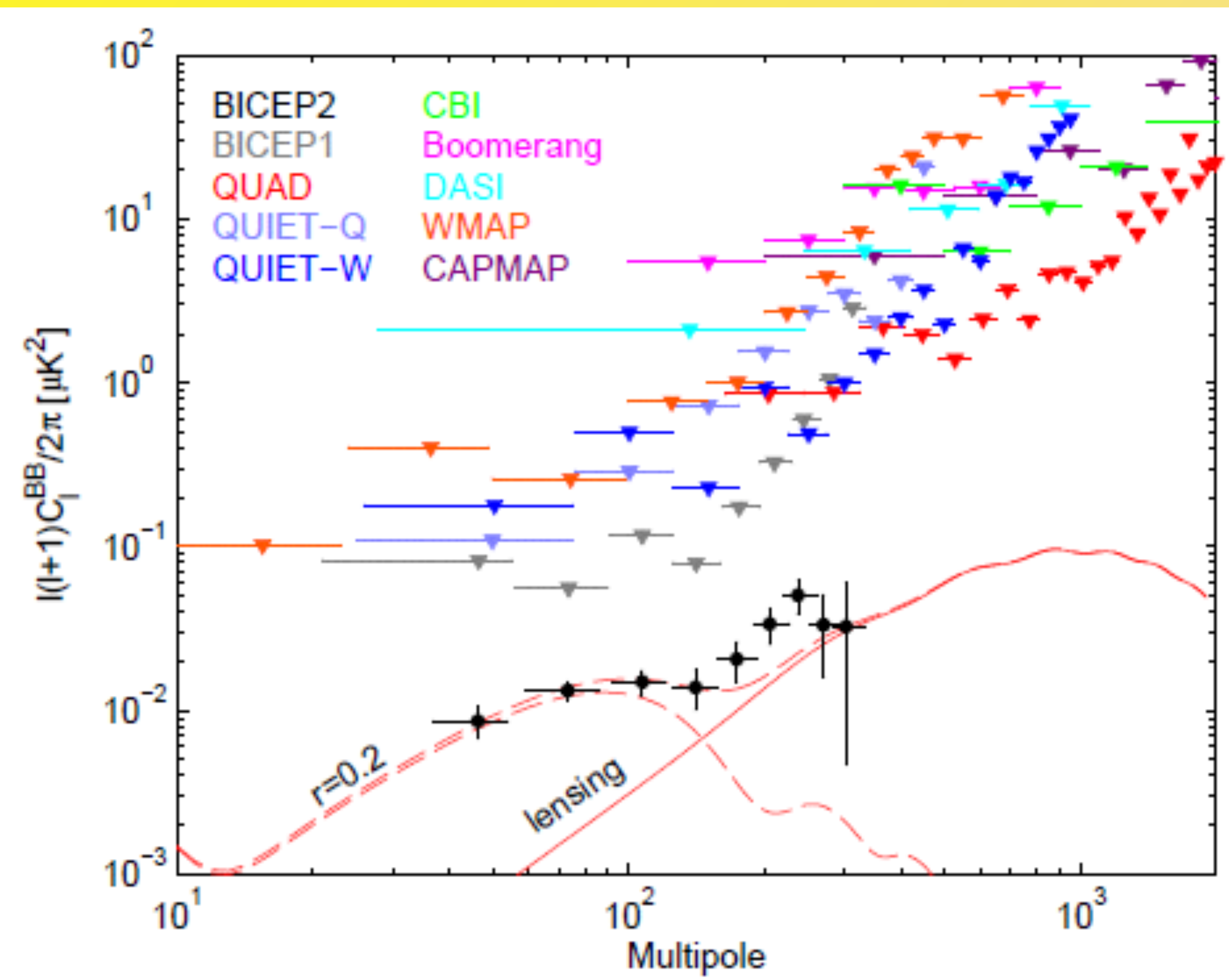

Dashed Line Theoretical Expectation from Inflation

$$r \equiv \Delta_t^2 / \Delta_s^2 \sim \mathcal{O}(0.1) \ \text{[Measured]}$$

$$r \sim \mathcal{O}(0.1) \quad \Rightarrow \quad \Delta_t^2 \sim 10^{-10}$$

$$\Rightarrow \quad E_* \sim 10^{16} \text{GeV}$$

(!)

3) Inflation: Summary

Inflation: Solves Causality Problem. Bonus: Universe Flat

$a \propto e^{Ht}$ → $g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$ → $\Delta T,\ \mathcal{E},\ \mathcal{B}\quad [\text{also } \delta\rho]$

quasi dS (Slow-Roll) | Quantum Origin of Fluctuations | Angular Temperature/ Polarization Anisotropies

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quasi dS (Slow-Roll) | Quantum Origin of Fluctuations | Angular Temperature/ Polarization Anisotropies

Observations:

$\lvert\Omega_k\rvert \ll 1$	$n_s - 1 \sim -0.04$	$\langle \mathcal{R}^3 \rangle \approx 0$	$4\delta_m = 3\delta_\gamma$
Locally Flat	Almost Scale-Inv	Gaussian	Adiabatic

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quasi dS (Slow-Roll) — Quantum Origin of Fluctuations — Angular Temperature/ Polarization Anisotropies

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BICEP2: $H_{\text{inf}} \approx 10^{14} GeV \Rightarrow E_{\text{inf}} \approx 10^{16} GeV$

- Proof of Inflation ?
- GUT physics ?
- Quantum Gravity !
- Super-Planckian Excursion !
- Detection of GWs !

BackSlide: Super Planckian Excursion

BICEP2: $H_{\rm inf} \approx 10^{14} GeV \Rightarrow E_{\rm inf} \approx 10^{16} GeV$ → **Super-Planckian Excursion !**

Super-Planckian Excursion ! → $\Delta\phi \sim \mathcal{O}(10) m_p \times (r/0.1)^{1/2}$

Operators Not Suppressed

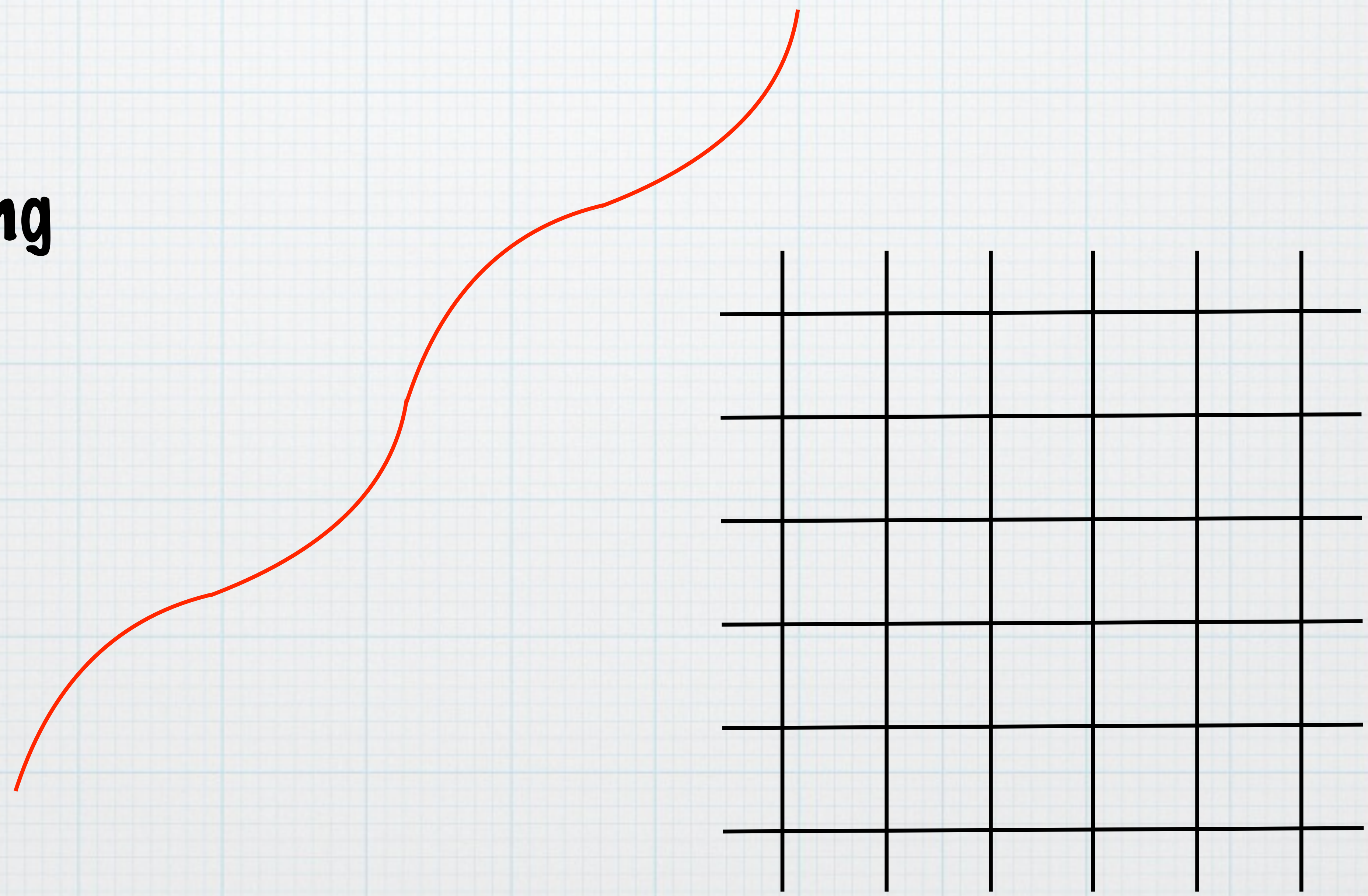
$$\mathcal{O}_6((\phi/m_p)^2), \mathcal{O}_8((\phi/m_p)^4), \ldots \Rightarrow \Delta m_\phi > H$$

Bad !

$$\Rightarrow \begin{cases} \text{No } \delta\phi \text{ Fluctuations!} \\ \eta > 1 \Rightarrow \text{Inflation Stops} \end{cases}$$

(Eta-Problem)

Comoving
Grid



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Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

$$B_i = \partial_i B - S_i$$

$$T_{\mu\nu}(\phi+\delta\phi) = T_{\mu\nu}(t) + \delta T_{\mu\nu}(\vec{x},t)$$

$$E_{ij} = 2\partial_{ij}E + 2\partial_{(i}F_{j)} + h_{ij}$$

$$\partial_i h_{ij} = h_{ii} = 0$$

2) Inflation: Basic Predictions

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

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$$\partial_i h_{ij} = h_{ii} = 0$$

$$\delta T_{\mu\nu}(\vec{x},t) \rightarrow S, V, T$$