$h \to \gamma\gamma$, Gauge Invariance, and Minimal Coupling

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References

- C. Grojean, E. Jenkins, A. Manohar, and M. Trott, JHEP 1304 (2013) 016
- E. Jenkins, A. Manohar, and M. Trott, arXiv:1305.0017
- A. Manohar, arXiv:1305.0663
- J. Elias-Miro, J. Espinosa, E. Masso, A. Pomarol, arXiv:1302.5661

Pre-LHC

- Previous results on precision electroweak measurements and flavor physics consistent with the standard model and the GIM mechanism to high accuracy.
- Rare decays such as $B \to X_s \gamma$ which occur at loop level in the standard model agree with theory to within the errors (sub 10%).
- These were supposed to occur at rates at factors of 100 times the standard model rate (until they did not) in models such as SUSY.
- Flavor physics sector and CKM unitarity works well, including second order weak processes such as $K^0 - \overline{K}^0$, $B^0 - \overline{B}^0$ and $B_s^0 - \overline{B}_s^0$ mixing.
- The only untested sector was the electroweak symmetry breaking sector, though it had been tested indirectly through electroweak fits.

LHC Summary

- Standard Model provides a good description of all observations so far at the LHC at 7 and 8 TeV.
- A particle has been seen with a mass $M_h \sim 125$ GeV consistent with the Higgs boson of the standard model
- 0^+ quantum numbers favored
- Production rate times branching ratios consistent with the standard model, but with large error bars.
- No evidence for any new particles, dimensions, etc. up to energies of ~ 1 TeV

Assume that at the scale $M_h \sim 125$ GeV, the standard model including the scalar doublet H is a good description.

All new physics is then parameterized by higher dimensional gauge invariant operators made of standard model fields.

At dimension 5, we have $\Delta L = 2$ operators which give neutrino masses. The scale Λ_5 is very high and does not affect Higgs physics.

For Higgs decays, the dominant effect is from dimension six operators due to new physics at some scale Λ, which is taken to be ~ 1 TeV.

If Λ is much higher than this, then the effects of NP become too small to see.

Dimension Six Operators

Grzadkowski et al. JHEP 1010 (2010) 085

Buchmuller and Wyler, Nucl.Phys. B268 (1986) 621

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{d}_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

59 baryon number conserving operators, not including flavor indices.
Field redefinitions (equations of motion) used to eliminate operators.

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_6 + \ldots$$
$$\mathcal{L}_6 = -\sum_i \frac{c_i}{\Lambda^2} O_i$$

Lots of terms!
For $gg \to h$ and $h \to \gamma\gamma$ decay, which operators can contribute at tree level?
Tree-level means tree-level in $\mathcal{L}_{\mathrm{EFT}}$.

$$\mathcal{O}_G = \frac{g_3^2}{2\,\Lambda^2}\, H^\dagger\, H\, G^A_{\mu\nu} G^{A\,\mu\nu}, \qquad \widetilde{\mathcal{O}}_G = \frac{g_3^2}{2\,\Lambda^2}\, H^\dagger\, H\, G^A_{\mu\nu} \widetilde{G}^{A\,\mu\nu},$$
$$\mathcal{O}_B = \frac{g_1^2}{2\,\Lambda^2}\, H^\dagger\, H\, B_{\mu\nu} B^{\mu\nu}, \qquad \widetilde{\mathcal{O}}_B = \frac{g_1^2}{2\,\Lambda^2}\, H^\dagger\, H\, B_{\mu\nu} \widetilde{B}^{\mu\nu},$$
$$\mathcal{O}_W = \frac{g_2^2}{2\,\Lambda^2}\, H^\dagger\, H\, W^a_{\mu\nu} W^{a\,\mu\nu}, \qquad \widetilde{\mathcal{O}}_W = \frac{g_2^2}{2\,\Lambda^2}\, H^\dagger\, H\, W^a_{\mu\nu} \widetilde{W}^{a\,\mu\nu},$$
$$\mathcal{O}_{WB} = \frac{g_1\, g_2}{2\,\Lambda^2}\, H^\dagger\, \tau^a\, H\, W^a_{\mu\nu} B^{\mu\nu}, \qquad \widetilde{\mathcal{O}}_{WB} = \frac{g_1\, g_2}{2\,\Lambda^2}\, H^\dagger\, \tau^a\, H\, W^a_{\mu\nu} \widetilde{B}^{\mu\nu}.$$

The $\widetilde{O}$ operators are CP and P odd.

Constraint on c_{WB} from the S parameter.

$$c_{WB} = -\frac{1}{8\pi}\frac{\Lambda^2}{v^2}S.$$

The hgg amplitudes get contributions from c_G and $\widetilde{c}_G$.

For $h\gamma\gamma$,

$$c_{\gamma\gamma} = c_W + c_B - c_{WB}, \qquad \widetilde{c}_{\gamma\gamma} = \widetilde{c}_W + \widetilde{c}_B - \widetilde{c}_{WB}$$

For $h\gamma Z$,

$$\begin{aligned} c_{\gamma Z} &= c_W \cot\theta_W - c_B \tan\theta_W - c_{WB}\cot 2\theta_W, \\ \widetilde{c}_{\gamma\gamma} &= \widetilde{c}_W \cot\theta_W - \widetilde{c}_B \tan\theta_W - \widetilde{c}_{WB}\cot 2\theta_W \end{aligned}$$

$$\frac{\Gamma(h \to \gamma\gamma)}{\Gamma^{\rm SM}(h \to \gamma\gamma)} \simeq \left|1 - \frac{4\pi^2 v^2 c_{\gamma\gamma}}{\Lambda^2 I^\gamma}\right|^2 + \left|\frac{4\pi^2 v^2 \tilde{c}_{\gamma\gamma}}{\Lambda^2 I^\gamma}\right|^2,$$

and $I^\gamma \approx -1.64$.

The $\tilde{c}$ terms do not interfere with the standard model amplitude.

Similar expressions for $gg \to h$ and $h \to \gamma Z$.

Looked at the anomalous dimensions of the dimension six operators listed. This is a submatrix of the full 59×59 matrix.

The values of $c_i(M_h)$ determine the Higgs decay rates

Considered these operators in an earlier work, and an explicit model that produced these operators.

AM, M.B. Wise, PLB636 (206) 107, PRD74 (2006) 035009

Anomalous dimensions can be computed in the unbroken theory.

λ coupling and y_t enter.

In a gauge theory, the operators

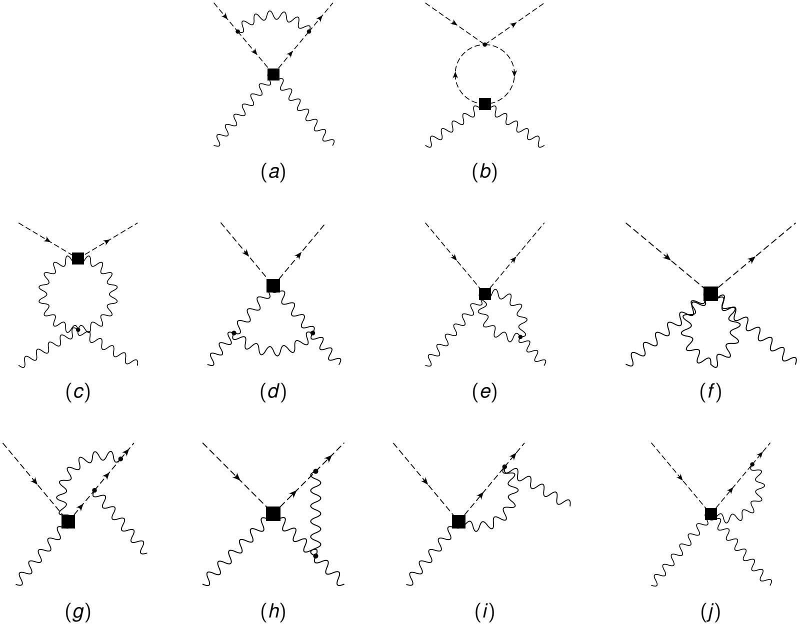
$$\mathcal{O}_+ = \sum \frac{\beta(g)}{2g} F^A_{\mu\nu} F^{A\,\mu\nu}, \qquad \mathcal{O}_- = g^2 F^A_{\mu\nu} \widetilde{F}^{A\,\mu\nu},$$

are not multiplicatively renormalized to all orders in perturbation theory.

At one-loop,

$$g^2 F^A_{\mu\nu} F^{A\,\mu\nu}, \qquad g^2 F^A_{\mu\nu} \widetilde{F}^{A\,\mu\nu},$$

are not renormalized.


(a)
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(h)
(i)
(j)

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} c_G = \gamma_G \, c_G,$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{bmatrix} c_B \\ c_W \\ c_{WB} \end{bmatrix} = \gamma_{WB} \begin{bmatrix} c_B \\ c_W \\ c_{WB} \end{bmatrix},$$

where the anomalous dimensions are

$$\gamma_G = \frac{1}{16\pi^2} \left[-\frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 + 12\lambda + 2Y \right],$$

$$\gamma_{WB} = \frac{1}{16\pi^2} \begin{bmatrix} \frac{1}{2} g_1^2 - \frac{9}{2} g_2^2 + 12\lambda + 2Y & 0 & 3g_2^2 \\ 0 & -\frac{3}{2} g_1^2 - \frac{5}{2} g_2^2 + 12\lambda + 2Y & g_1^2 \\ 2g_1^2 & 2g_2^2 & -\frac{1}{2} g_1^2 + \frac{9}{2} g_2^2 + 4\lambda + 2Y \end{bmatrix},$$

and

$$Y = \mathrm{Tr}\left[3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e \right] \approx 3y_t^2.$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\widetilde{c}_G = \gamma_G\,\widetilde{c}_G,$$

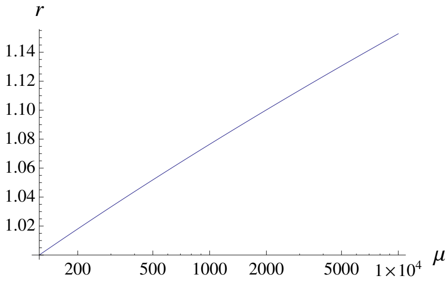
$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\begin{bmatrix}\widetilde{c}_B\\ \widetilde{c}_W\\ \widetilde{c}_{WB}\end{bmatrix} = \gamma_{WB}\begin{bmatrix}\widetilde{c}_B\\ \widetilde{c}_W\\ \widetilde{c}_{WB}\end{bmatrix}.$$

and $\widetilde{\gamma} = \gamma$ at one loop.

Largest contribution is the Yukawa coupling, and can be integrated exactly.

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} r(\mu) = \frac{3y_t^2(\mu)}{8\pi^2} r(\mu) \,.$$

Only ratios of $r(\mu)$ enter, so the overall scale of r is irrelevant.
A plot of $r(\mu)$ normalized so that $r(\mu = 125\,\mathrm{GeV}) = 1$



The correction is about 8% to the amplitude for $\mu = 1$ TeV.

$$c(M_h) = \frac{r(M_h)}{r(\Lambda)} \left[1 - \gamma_{WB}(Y \to 0)\ \log \frac{\Lambda}{M_h} \right] c(\Lambda).$$

This equation is accurate to about 3% for Λ less than 10 TeV.

$$
\begin{aligned}
\frac{r(\Lambda)c_{\gamma\gamma}(M_h)}{r(M_h)} &= \left[1 + \frac{3}{32\pi^2}\left(g_1^2 + 3g_2^2 - 8\lambda\right)\log\frac{\Lambda}{M_h}\right] c_{\gamma\gamma}(\Lambda) \\
&+ \frac{1}{8\pi^2}\left(3g_2^2 - 4\lambda\right)\log\frac{\Lambda}{M_h} c_{WB}(\Lambda), \\
\frac{r(\Lambda)c_{\gamma Z}(M_h)}{r(M_h)} &= \left[1 + \frac{1}{32\pi^2}\left(g_1^2 + 7g_2^2 - 24\lambda\right)\log\frac{\Lambda}{M_h}\right] c_{\gamma Z}(\Lambda) \\
&+ \frac{1}{8\pi^2}\left(g_1 g_2 + 4g_2^2 \cot 2\theta_W - 4\lambda \cot 2\theta_W\right)\log\frac{\Lambda}{M_h} c_{WB}(\Lambda) \\
&- \frac{\underline{1} g_1^2 - \underline{2} g_2^2}{16\pi^2}\left(c_{\gamma\gamma}(\Lambda)\sin 2\theta_W + c_{\gamma Z}(\Lambda)\cos 2\theta_W\right)\log\frac{\Lambda}{M_h}.
\end{aligned}
$$

S-Parameter

$$S = -\frac{8\,\pi\,v^2}{\Lambda^2} c_{WB}(M_h)\,.$$

$$\begin{aligned} c_{WB}(M_h) &= \frac{r(M_h)}{r(\Lambda)}\, c_{WB}(\Lambda)\, \left[1 + \frac{g_1^2 - 9\,g_2^2 - 8\lambda}{32\,\pi^2}\,\log\frac{\Lambda}{M_h}\right] \\ &- \frac{r(M_h)}{r(\Lambda)}\,\frac{1}{8\,\pi^2}\,\left[g_2^2\, c_W(\Lambda) + g_1^2\, c_B(\Lambda)\right]\,\log\frac{\Lambda}{M_h}, \end{aligned}$$

K. Hagiwara, S. Ishihara, R. Szalapski, and D. Zeppenfeld, PRD48 (1993) 2182

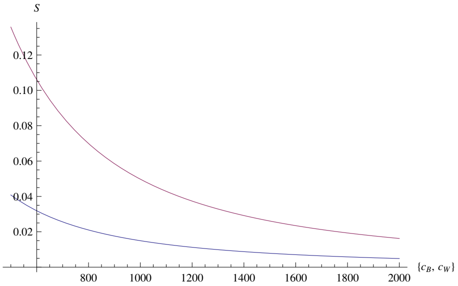
S. Alam, S. Dawson, and R. Szalapski, PRD57 (1998) 1577

$$S = -\frac{8\,\pi\,v^2}{\Lambda^2}\left(c_{WB}(\Lambda) - \frac{1}{8\,\pi^2}\,\left[g_2^2\, c_W(\Lambda) + g_1^2\, c_B(\Lambda)\right]\,\log\frac{\Lambda}{M_h}\right),$$

From finite parts of graphs in broken theory.

$$\mu_{\gamma\gamma} = 1 - 0.02\, S \log \frac{\Lambda}{M_h} + 2.7 \left(\frac{1\,\text{TeV}}{\Lambda}\right)^2 \left\{1 + 0.0035 \log \frac{\Lambda}{M_h}\right\} c_{\gamma\gamma}(\Lambda)$$

$$\simeq 1 - 0.02\, S \log \frac{\Lambda}{M_h} + 0.02 \left(\frac{1\,\text{TeV}}{\Lambda}\right)^2 \left(16\pi^2 c_{\gamma\gamma}(\Lambda)\right)$$



ATLAS:

$$\mu_{\gamma\gamma} = 1.80 \pm 0.30(\text{stat})\,^{+0.21}_{-0.15}(\text{syst})\,^{+0.20}_{-0.14}(\text{theory}),$$

for $M_h = 126.6 \pm 0.3(\text{stat}) \pm 0.7(\text{syst})\,\text{GeV}$
CMS:

$$\mu_{\gamma\gamma} = 1.56 \pm 0.43\,,$$

for $M_h = 125\,\text{GeV}$.
Naive combination of these results gives

$$\mu_{\gamma\gamma} \simeq 1.7 \pm 0.3$$

If due to $c_{\gamma\gamma}$:

$$\frac{v^2}{\Lambda^2}\, c_{\gamma\gamma}(M_h) \simeq -0.1,\ 0.01.$$

The second solution is preferred. The first solution is when $c_{\gamma\gamma}$ switches the sign of the standard model $h \to \gamma\gamma$ amplitude.

Elias-Miro, Espinosa, Masso, Pomarol:

- The specific choice of operator basis is crucial for the analysis, and there is a special basis which makes the analysis very simple,
- There is a powerful and general classification of EFT coefficients based on a “tree” and “loop” classification, even when the underlying theory is strongly interacting and non-perturbative. [This tree-loop classification is not the usual EFT use of tree and loop depending on which EFT diagrams are being computed.]
- The principle of minimal coupling defines an unambiguous classification scheme in which higher-dimensional operators which violate minimal coupling are suppressed by loop factors of $g^2/(16\pi^2)$, where g is a coupling constant.

Minimal coupling is the replacement of an ordinary derivative ∂_μ by the covariant derivative $D_\mu = \partial_\mu + igA_\mu$ to construct a theory with gauge interactions from a theory without gauge interactions.

EFT: Standard Results

Have some scale Λ, write down all possible local, gauge and Lorentz invariant operators: [can have topological terms]

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\leq 4} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \ldots$$

Can have multiple scales. E.g. in the standard model, Λ_ν for $\Delta L \neq 0$ interactions, and Λ_G for $\Delta B \neq 0$ interactions.

For Higgs physics, some scale $\Lambda \sim 1$ TeV.

- EFT can be used to do calculations.
- Expansion in some small parameter.
- EFT does not have to be perturbative, e.g. HQET has an expansion in $\alpha_s(m_Q)$ and in $\Lambda_{\mathrm{QCD}}/m_Q$.
- Tree and Loop refer to diagrams actually being computed in the EFT.
- Coefficients/Operators c_i, O_i in $\mathcal{L}_{\mathrm{EFT}}$ are not assigned a tree or loop number.

- Remember that we are using the EFT because we do not know the theory at Λ.
- If we know the theory, then all the coefficients c_i have a fixed value
- Some information from symmetries: lepton number and baryon number $\Rightarrow c_i = 0$ for $\not{L}$ and $\not{B}$ operators.
- If the theory at Λ is strongly coupled, we may know the theory but not how to compute c_i in $\mathcal{L}_{\mathrm{EFT}}$
- If we know the theory, and it is weakly coupled, then we can compute c_i.
- EFT is a model independent way of analyzing the data without assuming prior knowledge of the theory at Λ.

Wild Speculation

Not going to bother with speculations about the UV theory, without any calculations to back them up.

e.g. could speculate that there is a UV theory that solves the hierarchy problem, quantum gravity and gives a 126 GeV scalar with standard model quantum numbers. Does this solve the problems of particle physics?

or speculate that there is a UV theory that is a walking technicolor model with a small S parameter and a light scalar at 126 GeV. There might be, but it is up to the people making the claims to justify them, not up to everyone else to exactly solve all possible QFTs and show that it is impossible.

or speculate that strongly coupled theories produce a composite vector boson with a mass much smaller than the Λ_χ scale of the theory, and nothing else.

What is Minimal Coupling?

Start with a theory with a global symmetry

Gauge the symmetry using

$$\partial_\mu \to D_\mu = \partial_\mu + i\, e\, q\, A_\mu \qquad \partial_\mu + i\, e\, g\, A_\mu^A\, T^A$$

Usually called the minimal coupling prescription.

Non-minimally coupled terms:

$$\overline{\psi}\, \sigma^{\mu\nu} g F_{\mu\nu}\, \psi$$

Inverse procedure: gauge theory $\to$ global theory well-defined by

$$A \to 0 \qquad\qquad g \to 0$$

Tree vs. Loop

Can we classify operators in the EFT Lagrangian in terms of the number of loops (n-loop term, $n = 0$ being tree-level).

Let's start with the case where the theory at Λ is strongly coupled. In QCD

$$\mu \frac{\mathrm{d}g}{\mathrm{d}\mu} = -\frac{b_0 g^3}{16\pi^2}, \qquad \left(\frac{\Lambda_{\mathrm{QCD}}}{\mu}\right)^{b_0} = e^{-8\pi^2/\left[\hbar\, g^2(\mu)\right]}$$

Λ_{QCD} is non-analytic in $\hbar$ — neither tree nor loop nor anything. Similarly for $f_\pi, \Lambda_\chi \propto \Lambda_{\mathrm{QCD}}$.

χPT is an expansion in ∂/Λ_χ. No way to classify terms as n-loop in terms of the underlying theory.

True for any non-perturbative strongly coupled theory with a scale — such as those used in composite Higgs models.

- In weakly coupled theories, you can look at diagrams in the full theory that generate higher dimension operators.
- Field redefinitions in the EFT can move coefficients around, and change loop counting.
- Classification depends on the underlying theory

Tree-level in the standard model:

$$\mathcal{L}_{\Delta S=1} = -\frac{4G_F}{\sqrt{2}} V_{ud} V_{us}^{*} \, \overline{u}\gamma^{\mu} P_L d \, \overline{s}\gamma_{\mu} P_L u \,,$$

This has current-current form which seems to play an important role in people's thinking.
A very similar current-current operator

$$\overline{s}\gamma^{\mu} P_L d \, \overline{s}\gamma_{\mu} P_L d \,.$$

Is generated at loop level depending because of the GIM mechanism. GIM violation would produce this operator at tree level.

Current-current operators are not really products of currents even though they may look that way.

Composite operators are not products of their pieces.

$$[\bar{s}\gamma^\mu P_L u\, \bar{u}\gamma_\mu P_L d] \neq [\bar{s}\gamma^\mu P_L u]\,[\bar{u}\gamma_\mu P_L d]$$

because of renormalization.

In QCD, the $\Delta I = 1/2$ rule: $A_{1/2,3/2}$ in $K \to \pi\pi$ decay.

$$\text{QCD} : A_{1/2} \approx 20\, A_{3/2} \qquad \text{Product of Currents} : A_{1/2} = \sqrt{2}\, A_{3/2}$$

ϕ has $q = 1$, Φ has $q = 0$:

$$O_1 = \phi^\dagger \phi D_\mu \phi^\dagger D_\mu \phi, \quad O_2 = e^2 \phi^\dagger \phi F_{\mu\nu} F^{\mu\nu}, \quad O_3 = e^2 \Phi^\dagger \Phi F_{\mu\nu} F^{\mu\nu},$$

$$\widetilde{O}_1 = \phi^\dagger \phi\, \partial_\mu \phi^\dagger \partial_\mu \phi, \qquad \widetilde{O}_2 = 0, \qquad \widetilde{O}_3 = 0\,.$$

So minimal coupling gives $\widetilde{O}_1 \to O_1$, but not for O_2, O_3.

$$\widetilde{O}_2 = -\phi^\dagger \left[\partial_\mu, \partial_\nu\right] \left[\partial^\mu, \partial^\nu\right] \phi = 0$$

Now,

$$\left[\partial^\mu, \partial^\nu\right] \phi = 0 \qquad \left[D^\mu, D^\nu\right] \phi = i\, e\, q\, F^{\mu\nu} \phi$$

cannot do this for Φ since $q_\Phi = 0$ The charge in D is the charge of the field on which D acts.

Now what do you say?

- minimal coupling:
  c_1 is "tree" and $c_{2,3}$ are "loop" and suppressed by $1/(16\pi^2)$?
- next-to-minimal coupling:
  $c_{1,2}$ are "tree" and c_3 is "loop" and suppressed by $1/(16\pi^2)$?

minimal coupling: terms with $F_{\mu\nu}$ are suppressed

next-to-minimal coupling: terms with $F_{\mu\nu}$ and neutral fields are suppressed.

Non-Abelian Case

$$O_4 = \varphi^\dagger G_{\mu\nu} G^{\mu\nu} \varphi, \qquad\qquad O_5 = \varphi^\dagger \varphi \, \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu},$$

where $G_{\mu\nu} \equiv G^a_{\mu\nu} T^a$.

Neither is generated by minimal coupling.

next-to-minimal coupling:

$$\widetilde{O}_4 = \left([\partial_\mu, \partial_\nu] \, \varphi \right)^\dagger \left([\partial_\mu, \partial_\nu] \, \varphi \right) \ .$$

can generate O_4.

But $D_\mu X$ does not change the representation of X. Can't generate the invariant O_5. So in a non-Abelian theory with colored particles, cannot generate all the $G_{\mu\nu}$ terms.

Look at a general renormalizable gauge theory. Then the gauge interactions of fermions and bosons is given by

$$\overline{\psi}\, i\partial\!\!\!/\, \psi \to \overline{\psi}\, iD\!\!\!\!/\, \psi \qquad\qquad \partial_\mu \phi^\dagger \partial_\mu \phi \to D_\mu \phi^\dagger D_\mu \phi$$

This is because there is only a very limited set of operators with dimension ≤ 4.

Not true for the full Lagrangian.

$$\mathcal{L} = -\frac{1}{4} G^A_{\mu\nu} G^{A\,\mu\nu} + \frac{g^2\theta}{32\pi^2} G^A_{\mu\nu} \widetilde{G}^{A\,\mu\nu}$$

$G\widetilde{G}$ not “loop suppressed” since the effects of θ are order unity, i.e coefficient of θ is the winding number which is an integer.

Why is there no anomalous magnetic term in QED?

$$\frac{1}{\Lambda}\overline{\psi}\,\sigma^{\mu\nu}\,eF_{\mu\nu}\,\psi$$

The reason is not minimal coupling, but renormalizability.

S. Weinberg, Brandeis Summer School Lectures (1970).

If we want the theory to be valid up to an arbitrarily high scale, $\Lambda \to \infty$, and the term can be dropped.

Quantum mechanical Hamiltonian for a non-relativistic particle,

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m}\mathbf{1},$$

where $\mathbf{1}$ is a unit matrix of dimension $(2s+1)$ for a particle of spin s.

The Hamiltonian for the interaction with the electromagnetic field is

$$\mathcal{H}_{\text{em}} = \frac{(\mathbf{p} - qe\mathbf{A})^2}{2m}\mathbf{1} + eqA^0\mathbf{1}$$

Alternate form for spin-1/2:

$$\mathcal{H} = \frac{(\boldsymbol{\sigma}\cdot\mathbf{p})^2}{2m} = \frac{\mathbf{p}^2}{2m}\mathbf{1}.$$

identical to first version.

$$\mathcal{H}_{\text{em}} = \frac{(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})^2}{2m} + eqA^0\mathbf{1} = \frac{(\mathbf{p} - qe\mathbf{A})^2}{2m}\mathbf{1} + eqA^0\mathbf{1} - \frac{eq}{2m}\boldsymbol{\sigma}\cdot\mathbf{B},$$

Ambiguity once again related to $[\partial_\mu, \partial_\nu] = 0$ but $[D_\mu, D_\nu] \neq 0$

H. Weyl, Phys. Rev. 77 (1950) 699

Look at an electron in gravity plus EM.

Can use the first or second order formalism, i.e. treat e^a_μ and ${\omega^a}_{b\mu}$ as independent, and vary them separately, or treat ω as a function of e, and e is varied.

The two methods differ by a $F_{\mu\nu}$ term.

Correct Hamiltonian is neither, but has $a = 1 + \alpha/(2\pi) + \ldots$ in front of the magnetic moment term.

Maybe a is just a “loop” correction, and you have to use the second form of $\mathcal{H}$.

What about the proton? $\mu = 2.793 = 1 + 1.793$. Either method has a big magnetic moment term that is not loop suppressed.

The neutron is neutral and has $\mu = -1.91$. For the neutron, $D = \partial$, so the entire magnetic moment is a non-minimally coupled term.

Dashen, EJ, AM

$$\frac{\mu_n}{\mu_p} = -\frac{2}{3} + \mathcal{O}\left(\frac{1}{N_c^2}\right),$$

Minimal coupling result would be 0.

Hydrogen Atom

Atom in an electric field

$$\mathcal{H} = -\frac{1}{2}\alpha_E \mathcal{E}^2,$$

with $\alpha_E = 9a_0^3/2$.

The H atom is neutral, but atomic transitions take place, and power counting scale is $\Lambda = 1/a_0$.

No loops in QM, so no “loop factors”

EFT: pNRQCD Brambilla, Pineda, Soto, Vairo

Chiral Perturbation Theory

A. Pich, Chiral perturbation theory, Rept. Prog. Phys. 58 (1995) 563Ð610

$$
\begin{aligned}
\mathcal{L}_4 = {} & L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle \\
& + L_3 \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U \rangle + L_4 \langle D_\mu U^\dagger D^\mu U \rangle \langle U^\dagger \chi + \chi^\dagger U \rangle \\
& + L_5 \langle D_\mu U^\dagger D^\mu U \left(U^\dagger \chi + \chi^\dagger U \right) \rangle + L_6 \langle U^\dagger \chi + \chi^\dagger U \rangle^2 \\
& + L_7 \langle U^\dagger \chi - \chi^\dagger U \rangle^2 + L_8 \langle \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi \rangle \\
& - i L_9 \langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U \rangle + L_{10} \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu} \rangle \\
& + H_1 \langle F_{R\mu\nu} F_R^{\mu\nu} + F_{L\mu\nu} F_L^{\mu\nu} \rangle + H_2 \langle \chi^\dagger \chi \rangle \, .
\end{aligned}
$$

L_{10} bigger than most of the other coefficients

$$\sum_n f^2\Lambda_\chi^2 \frac{\mathcal{L}_n}{\Lambda_\chi^n} = f^2\Lambda_\chi^2 \left[\frac{\mathcal{L}_2}{\Lambda_\chi^2} + \frac{\mathcal{L}_4}{\Lambda_\chi^4} + \ldots\right] = \frac{\Lambda_\chi^4}{16\pi^2}\left[\frac{\mathcal{L}_2}{\Lambda_\chi^2} + \frac{\mathcal{L}_4}{\Lambda_\chi^4} + \ldots\right]$$

$$\mu\frac{\mathrm{d}L_i}{\mathrm{d}\mu} = \frac{\gamma_i}{16\pi^2},$$

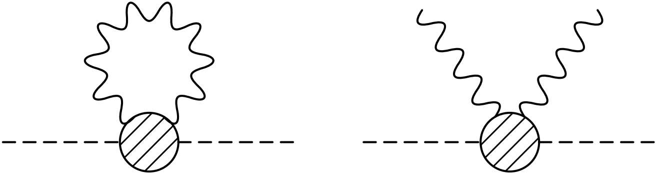
where γ_i are pure numbers e.g. $\gamma_{10} = -1/4$.

No QCD tree vs loop classification

PGBH Models

e.g. composite Higgs, Little Higgs, ...

$$\mathcal{L}_2 = \frac{f^2}{2}(D^\mu\Sigma)^\dagger(D_\mu\Sigma) + \ldots$$



$$\mathcal{L}_M = c\,\mathrm{Tr}\,UQU^\dagger Q,$$

Generate $\pi^+ - \pi^0$ mass difference by using L_{10}.

PGBH Models

In PGBH Models, the Higgs is by construction a pseudo-Goldstone Boson that get's its mass from weakly gauging the flavor symmetries.

Might occur at order g^4 in Little Higgs models.

$$\mathcal{L} = \frac{g^2 c_6}{\Lambda^2} H^\dagger H F_{\mu\nu} F^{\mu\nu} \ .$$

Closing the gauge loop to generate the mass:

$$m_H^2 \sim \frac{g^2 \Lambda^2}{16\pi^2} c_6 \, .$$

$$\mathcal{O}_{BB} = g_1^2\, H^\dagger\, H\, B_{\mu\nu} B^{\mu\nu},$$
$$\mathcal{O}_{WW} = g_2^2\, H^\dagger\, H\, W^a_{\mu\nu} W^{a\,\mu\nu},$$
$$\mathcal{O}_{WB} = g_1\, g_2\, H^\dagger\, \tau^a\, H\, W^a_{\mu\nu} B^{\mu\nu},$$

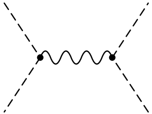
O_{BB} and

$$\mathcal{P}_{HW} = -i\, g_2\, (D^\mu H)^\dagger\, \tau_a\, (D^\nu H)\, W^a_{\mu\nu}, \qquad \mathcal{P}_{HB} = -i\, g_1\, (D^\mu H)^\dagger\, (D^\nu H)\, B_{\mu\nu},$$
$$\mathcal{P}_W = -\frac{i\, g_2}{2}\, (H^\dagger\, \tau_a\, \overleftrightarrow{D}^\mu H)\, (D^\nu W^a_{\mu\nu}), \qquad \mathcal{P}_B = -\frac{i\, g_1}{2}\, (H^\dagger\, \overleftrightarrow{D}^\mu H)\, (D^\nu B_{\mu\nu}),$$

for a total of five operators.

$$\mathcal{P}_B = \mathcal{P}_{HB} + \frac{1}{4}\mathcal{O}_{BB} + \frac{1}{4}\mathcal{O}_{WB}, \qquad \mathcal{P}_W = \mathcal{P}_{HW} + \frac{1}{4}\mathcal{O}_{WW} + \frac{1}{4}\mathcal{O}_{WB}.$$

Large N dynamics has an infinite tower of meson states with particles of arbitarily high spin.



Ecker, Gasser, Pich, de Rafael, NPB321 (1989) 311.

- Generates the chiral Lagrangian with all the higher dimension operators.
- Infinite number of meson states to get the log Q^2 behavior
- Arbitrarily high spin to get all QCD operators such as $\overline{\psi}\gamma^{\mu_0} D^{\mu_1} \ldots D^{\mu_n}\psi$
- A strongly interacting theory is not just a theory with an additional light ρ meson

You Cannot Assume Minimal Coupling

Can you assume the UV theory is such that the low energy theory is minimally coupled?

Consider a much simpler example, with far greater freedom to construct the theory: quantum mechanics.

Bound states with an arbitrary potential $V(\mathbf{x})$. Can we find $V(\mathbf{x})$ so that the bound state dynamics is minimally coupled?

$\boxed{\text{NO}}$

$$1 = \sum_f \frac{2m\,(E_f - E_i)}{\hbar^2} \left|\langle f|z|i\rangle\right|^2$$

oscillator-strength sum rule: cannot make all transitions "loop" suppressed.

Very basic point:

You cannot assume minimal coupling

It is like assuming that you can arbitrarily adjust the dynamics of a theory

In QCD, you cannot just adjust the interactions of the π^0 without changing anything else.

Otherwise, I can just assume that I have a strong coupling theory that reduces to the standard model, and has no hierarchy problem.