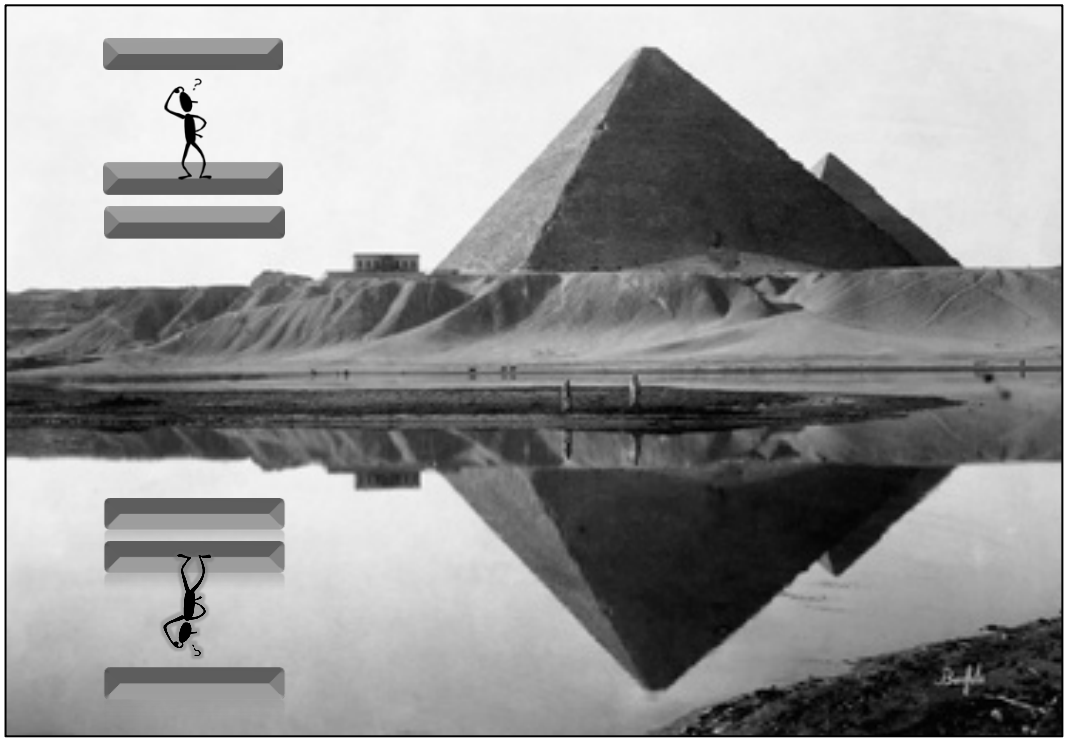
INVISIBLES Journal Club – 26 November 2013

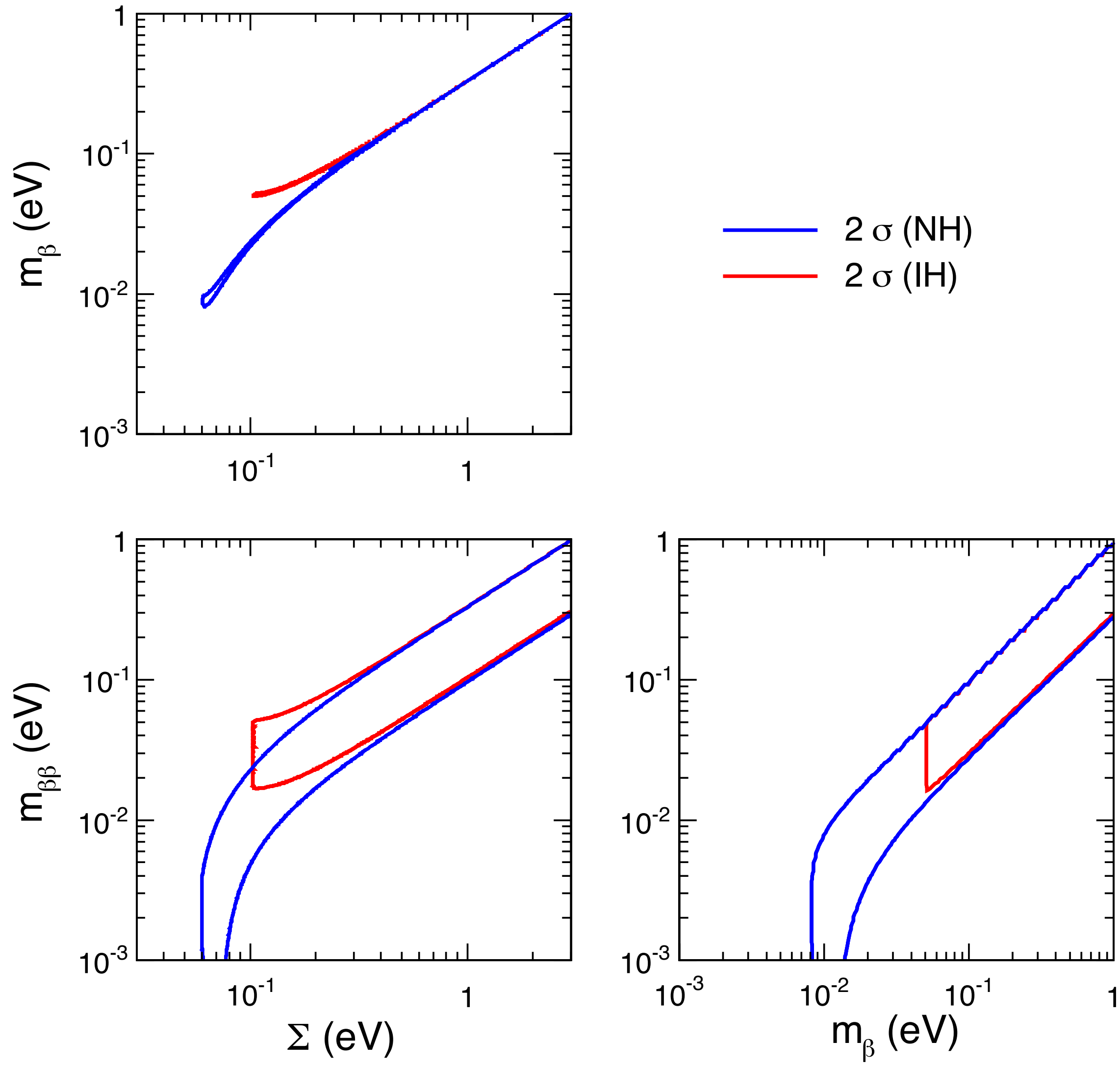
Issues in mass hierarchy discrimination via reactor neutrino oscillations



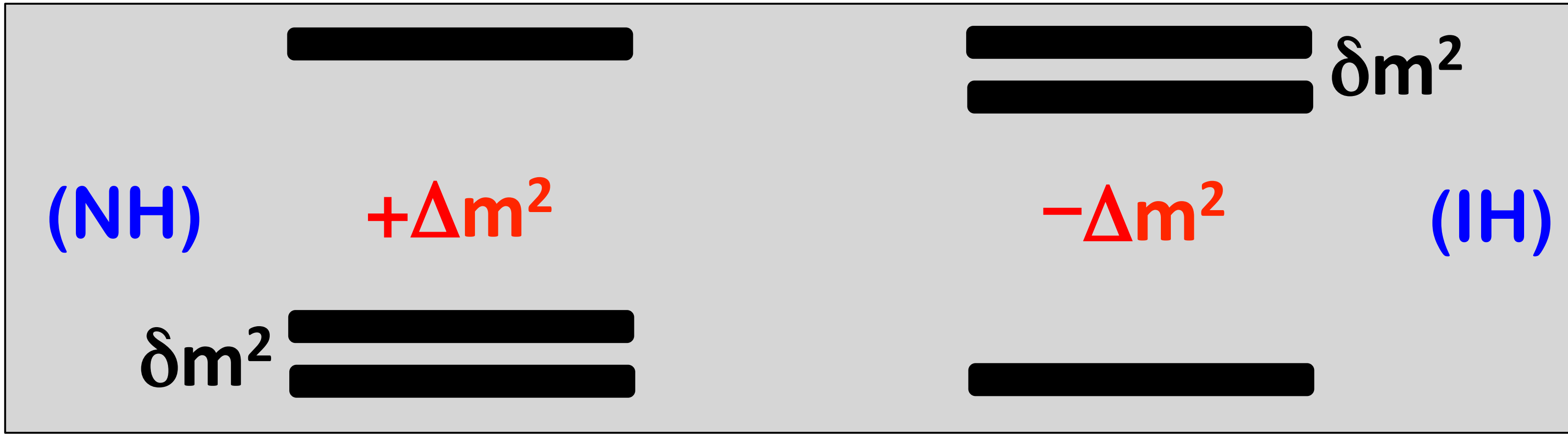
Eligio Lisi
(INFN, Bari, Italy)

Based on: F. Capozzi, E. Lisi, A. Marrone [arXiv:1309.1638] and refs. therein

Observables sensitive to absolute neutrino masses can probe the hierarchy at low mass scales



Neutrino flavor oscillations can also probe the hierarchy…



… if oscillations driven by $\pm\Delta m^2$ interfere with oscillations driven by another "squared mass gap" Q with known sign. Three options:

$Q = \delta m^2$ (focus of this talk)

$Q = 2\sqrt{2}\, G_F N_e E$ (matter effects in Earth or SNe)

$Q = 2\sqrt{2}\, G_F N_\nu E$ (collective effects in SNe)

Early literature :

The full 3ν survival probability of reactor antineutrinos is not invariant under a NH/IH swap, unless $\theta_{12} = \pi/4$

[G.L. Fogli, E. Lisi, A. Palazzo, hep-ph/0105080: 1st NH vs IH analysis of CHOOZ!]

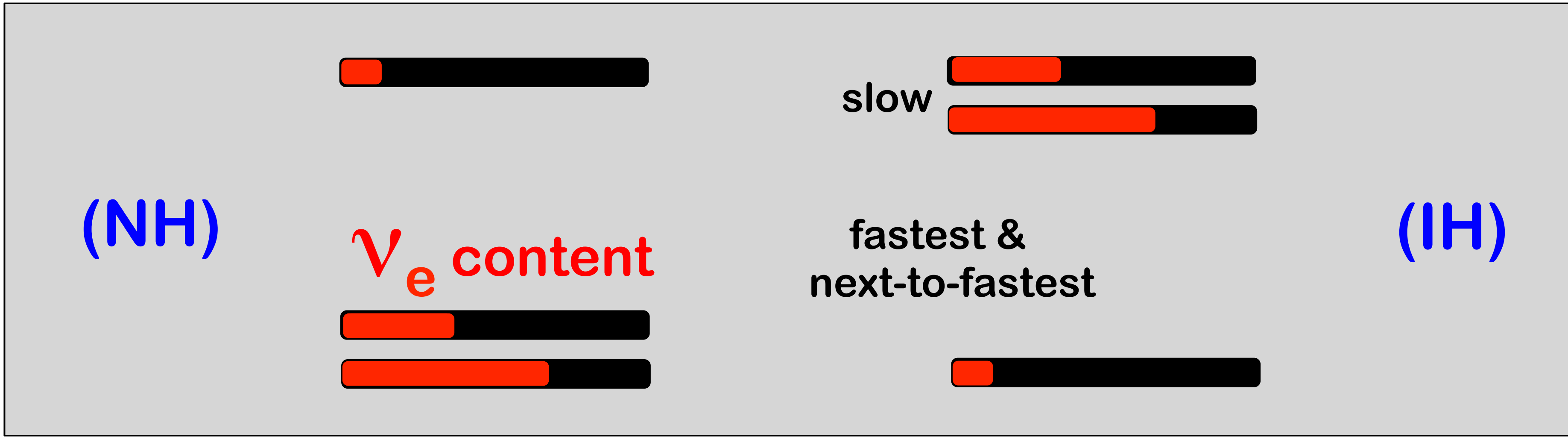
For δm^2 in the LMA region, high-precision reactor experiment at medium baseline can probe the hierarchy

[S.T. Petcov and M. Piai, hep-ph/0112074]

...and can also provide accurate determinations of the "solar" oscillation parameters (δm^2 , θ_{12})

[A. Bandyopadhyay, S. Choubey and S. Goswami, hep-ph/0302243]

Very simple physics: One slow & two fast oscillations



ν_e oscillation amplitude = product of two red bars

Under hierarchy swap:

Amplitude of the slow oscillation does not change, while the (different) amplitudes of the fastest and next-to-fastest oscillations are interchanged

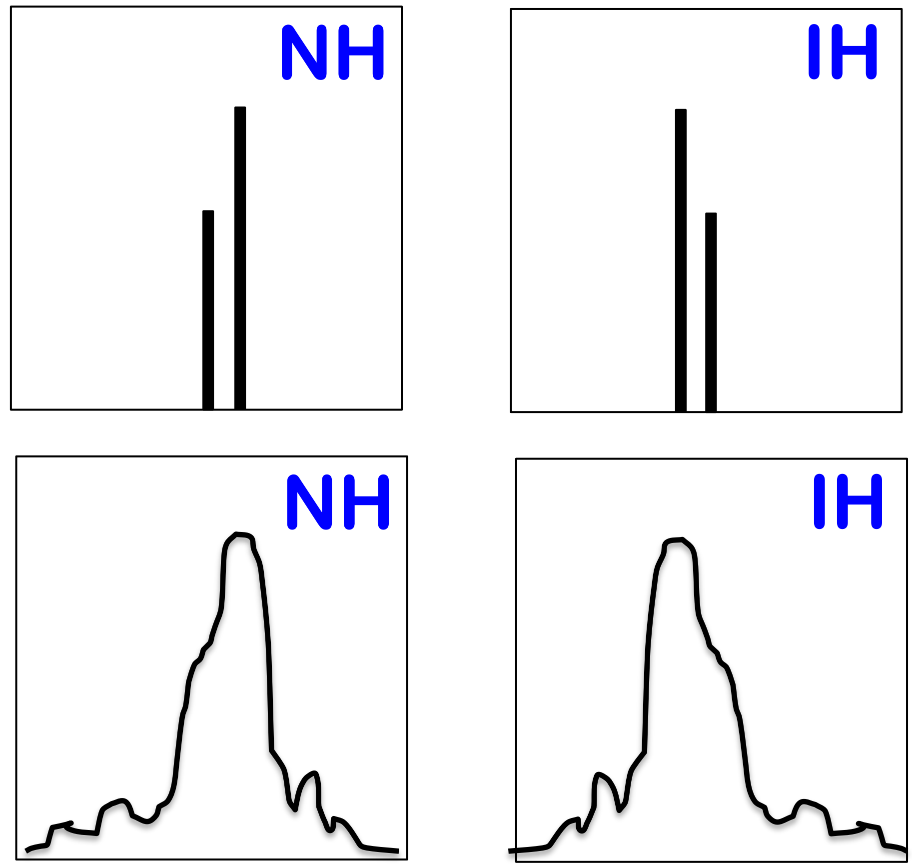
(unless closest red bars were equal, i.e., $\theta_{12}=\pi/4$)

Easiest visualization: Fourier Spectrum

Ideal energy resolution &
infinite oscillation cycles:
two separate peaks
(fastest and next-to-fastest)
with different power

x=frequency
y=amplitude

Finite energy resolution &
finite # of oscillation cycles:
peak + "shoulder" (+noise)



J.G. Learned et al., hep-ex/0612022; L. Zhan et al., arXiv:0807.3203

[But, in my opinion: Fourier Spectrum will not be used with real data: too difficult to include systematics in a transparent way.]

In general: hierarchy discrimin. very difficult & challenging

Need to reach many favorable conditions:

- O(100,000) event statistics
- No destructive interference among various reactors
- Energy resolution 3%/√E or better (~full light collection)
- Energy scale systematics at sub-% level
- Control of reactor flux shape and of its uncertainties

...

Unprecedented, but not proven to be "impossible"!

Actively considered in RENO-50 and JUNO projects

Our contribution to the current discussion (this talk):

- Analytical results on oscillation probability
- Continuous interpolation between NH and IH
- Analytical inclusion of recoil effects in resolution func.
- Interplay between energy scale and flux shape

Osc. probability: improving the vacuum approx.

It is useful to start from a functional form of Pee where **hierarchy-odd terms are all confined in a single phase ϕ** (rather than in amplitudes) in vacuum:

$$P^{3\nu}_{\rm vac} = c^4_{13} P^{2\nu}_{\rm vac} + s^4_{13} + 2 s^2_{13} c^2_{13} \sqrt{P^{2\nu}_{\rm vac}} \cos(2\Delta_{ee} + \alpha\varphi)$$

$$\alpha = \pm 1 \text{ for NH/IH}$$

+1 = advancement (NH), -1 = retardation (IH)

where ϕ is defined via parametric equations:

$$\cos\varphi = \frac{c^2_{12}\cos(2s^2_{12}\delta) + s^2_{12}\cos(2c^2_{12}\delta)}{\sqrt{P^{2\nu}_{\rm vac}}}$$

$$\sin\varphi = \frac{c^2_{12}\sin(2s^2_{12}\delta) - s^2_{12}\sin(2c^2_{12}\delta)}{\sqrt{P^{2\nu}_{\rm vac}}}$$

[Nunokawa et al., hep-ph/0503283, hep-ph/0701151]

$$\boxed{P^{3\nu}_{\rm vac} = c^4_{13} P^{2\nu}_{\rm vac} + s^4_{13} + 2 s^2_{13} c^2_{13} \sqrt{P^{2\nu}_{\rm vac}} \cos(2\Delta_{ee} + \alpha\varphi)}$$

"slow" (solar) L/E osc.

"fast" (atm) L/E osc.

"slow" (hierarchy-dependent, $\alpha=\pm1$) non-L/E osc. phase

L/E phases and related terms:

$$\delta m^2 = \Delta m^2_{21}$$

$$\delta = \delta m^2 L/4E$$

$$\Delta m^2 = |\Delta m^2_{31} + \Delta m^2_{32}|/2$$

$$\Delta m^2_{ee} = \Delta m^2 + \alpha(c^2_{12} - s^2_{12})\delta m^2/2$$

$$\Delta_{ee} = \Delta m^2_{ee} L/4E$$

$$P^{2\nu}_{\text{vac}} = 1 - 4s^2_{12}c^2_{12}\sin^2\delta$$

$$\boxed{P^{3\nu}_{\text{vac}} = c^4_{13}P^{2\nu}_{\text{vac}} + s^4_{13} + 2s^2_{13}c^2_{13}\sqrt{P^{2\nu}_{\text{vac}}}\cos(2\Delta_{ee} + \alpha\varphi)}$$

"slow" (solar) L/E osc.

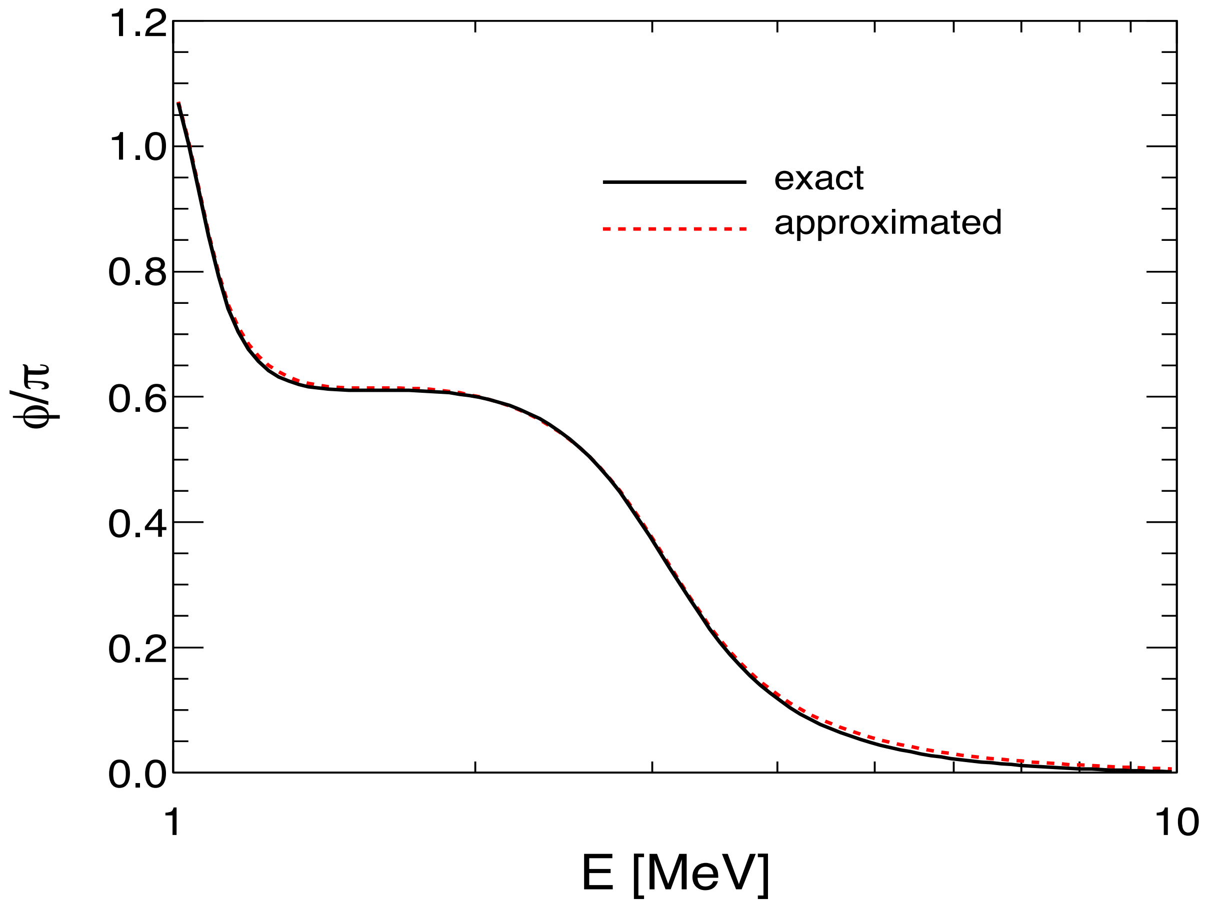
"fast" (atm) L/E osc.

"slow" (hierarchy-dependent, $\alpha=\pm 1$) non-L/E osc. phase

Finding hierarchy = finding evidence for non-L/E phase ϕ with definite sign α: +1 = advancement (NH), -1 = retardation (IH).

Handy approximation for the “non-L/E phase” ϕ:

$$\varphi \simeq 2s_{12}^2\delta\left(1-\frac{\sin\delta}{2\delta\sqrt{P_{\rm vac}^{2\nu}}}\right)$$



Here, for L=52.5 km (JUNO)

This functional form for the survival probability:

$$\boxed{P^{3\nu}_{\rm vac} = c^4_{13} P^{2\nu}_{\rm vac} + s^4_{13} + 2 s^2_{13} c^2_{13} \sqrt{P^{2\nu}_{\rm vac}} \cos(2\Delta_{ee} + \alpha\varphi)}$$

is preserved after some necessary improvements:

- damping effects due to the spread of reactor distances (decrease peak heights by ~28% at low E in JUNO)
- matter effects along L~O(50) km in upper crust (shift solar parameters by ~1σ in JUNO)

vac→mat

w<1

$$\boxed{P^{3\nu}_{\rm mat} \simeq c^4_{13} P^{2\nu}_{\rm mat} + s^4_{13} + 2 s^2_{13} c^2_{13} \sqrt{P^{2\nu}_{\rm mat}}\, w \cos(2\Delta_{ee} + \alpha\varphi)}$$

Pee accuracy at permill level; F. Capozzi, E. Lisi, A. Marrone [arXiv:1309.1638]

Such a functional form allows an independent approach to a statistical issue which has been pointed out recently:

One cannot assume, as usual, $N\sigma = \sqrt{\Delta\chi^2}$(NH-IH) because NH and IH are "disconnected" hypotheses!

[Qian et al., 1210.3651, Ge et al., 1210.8141; Ciuffoli et al., 1305.5150]

Proper statistics lead to $N\sigma \sim 0.5\,\sqrt{\Delta\chi^2}$(NH-IH) (1/2 weaker!) **as a "rule-of-thumb" for hierarchy sensitivity estimates**

[Kettel et al., 1307.7419 & above papers; but: still debate in literature]

We recover this sensitivity estimate by "reconnecting" the discrete hypotheses (NH and IH) as follows in the fit:

$$\alpha = \pm 1\ (\mathrm{NH/IH}) \longrightarrow \alpha = \mathrm{free}$$

$$\boxed{P_{\mathrm{mat}}^{3\nu} \simeq c_{13}^4 P_{\mathrm{mat}}^{2\nu} + s_{13}^4 + 2 s_{13}^2 c_{13}^2 \sqrt{P_{\mathrm{mat}}^{2\nu}}\, w \cos(2\Delta_{ee} + \alpha\varphi)}$$

The experiment is successful if evidence is found for
- either an advancement of phase: $\alpha > 0$, **NH**
- or a retardation of phase: $\alpha < 0$, **IH**
with the correct size expected: $|\alpha| \sim 1$

The experiment fails if
- no evidence is found for extra phase: $\alpha \sim 0$, **"undecided"**
- evidence is found, but with incorrect size: $|\alpha| >> 1$.

$$P_{\rm mat}^{3\nu} \simeq c_{13}^4 P_{\rm mat}^{2\nu} + s_{13}^4 + 2 s_{13}^2 c_{13}^2 \sqrt{P_{\rm mat}^{2\nu}}\, w \cos(2\Delta_{ee} + \alpha\varphi)$$

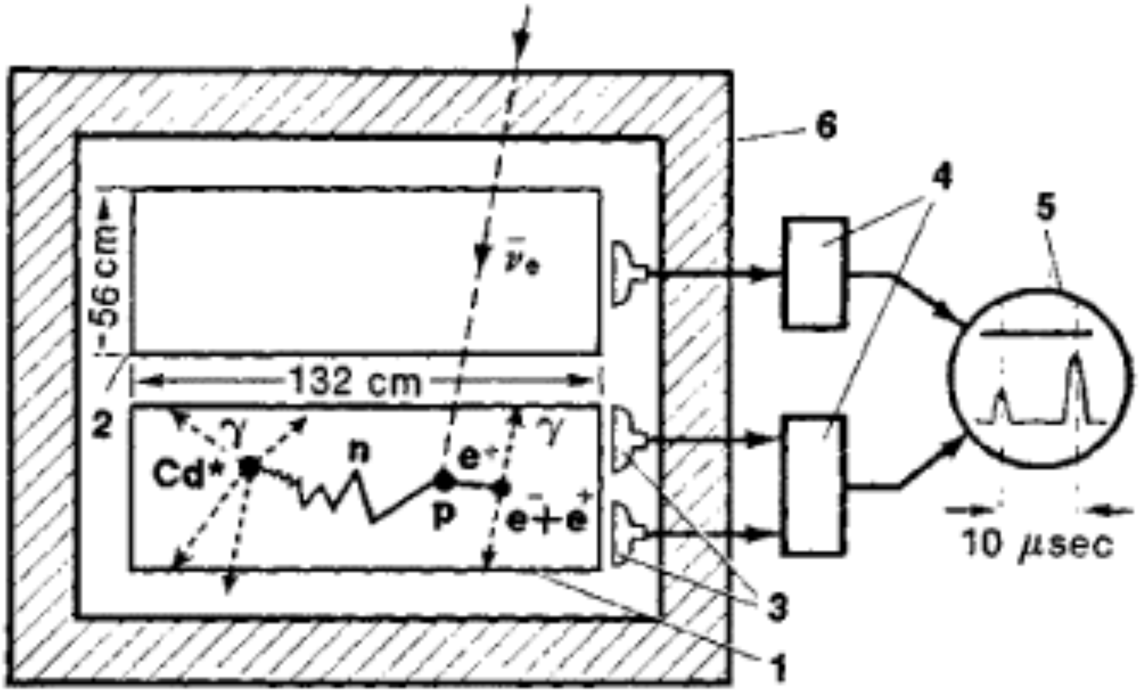
Treating α as a free parameter in a fit to prospective data allows to cover *continuously* this range of possibilities, none of which can be excluded a priori. Sensitivity may be then defined as distance of $|\alpha| = 1$ ("success") from $|\alpha| = 0$ ("failure").*

Experimentally, the possible outcome $\alpha \sim 0$ would mean that two oscillation waves have been observed (slow and fast), but their relative beating (i.e., the hierarchy effect) has not been detected within uncertainties.

****Note: Various definitions of sensitivities are being debated in the literature***

Detection: Improving usual approximations

Detection technique (since Reines & Cowan): Inverse Beta Decay



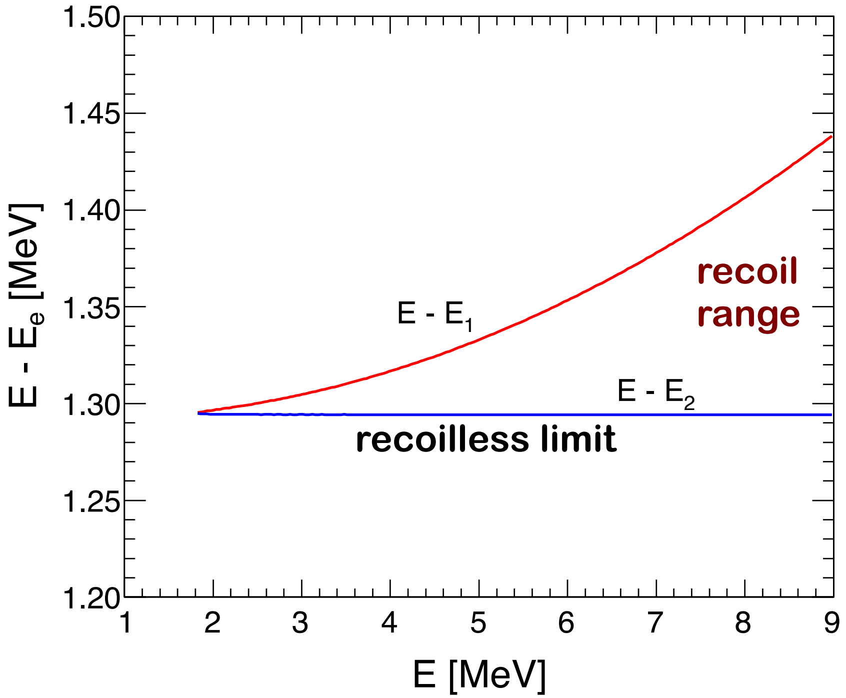
Usual "recoilless" approximation in IBD:

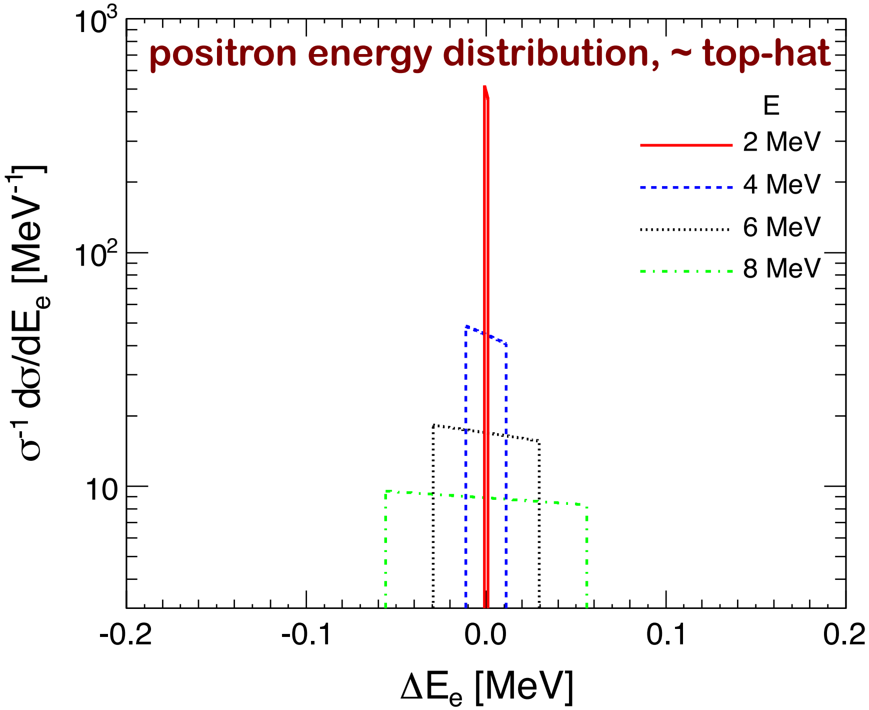
neglect nucleon recoil → one-to-one relation between E(ν) and E(e^+)

No longer adequate for precision reactor experiments.

Necessary improvement: Inclusion of recoil effects

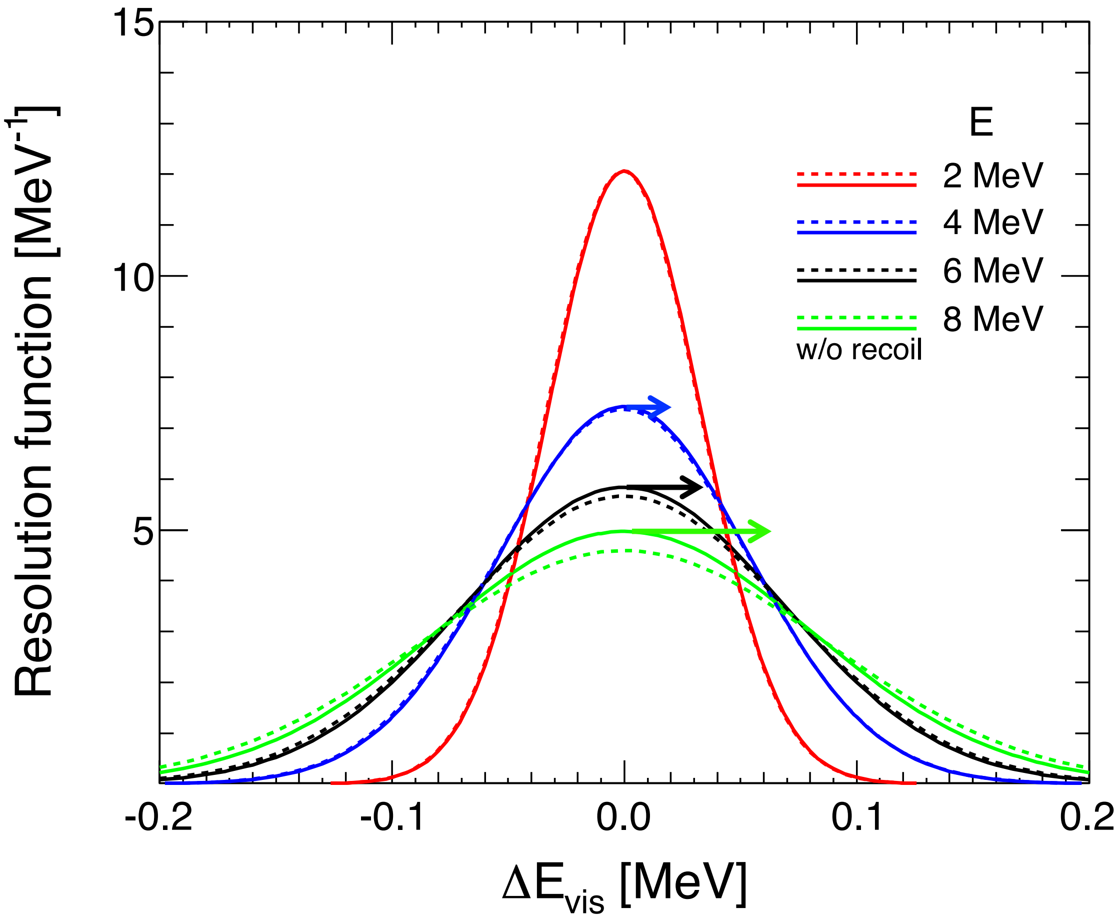
IBD recoilless approximation introduces sub% energy bias + spread.
Two recoil effects of $O(E/m_p)$: shift and spread of positron energy





[we adopted the IBD differential cross section from Strumia & Vissani, astro-ph/0302055]

Both effects can be included analitically via a simple recipe
→ modified (recoil-corrected) energy resolution function:



Gaussian → Gaussian + Erf + Shift

[No need to add further nested integrals]

Statistical analysis of prospective JUNO data
(assume 5 yr run, ~100,000 oscillated reactor events)

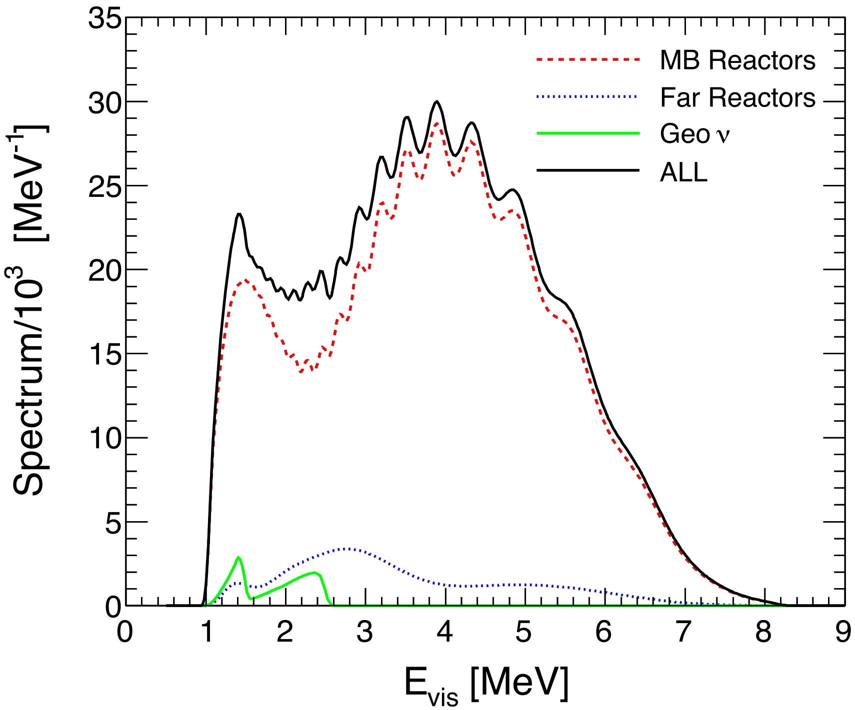
JUNO (Jiangmen Underground Neutrino Observatory) experimental set-up from:
Li, Cao, Wang, Zhan, arXiv:1303.6733

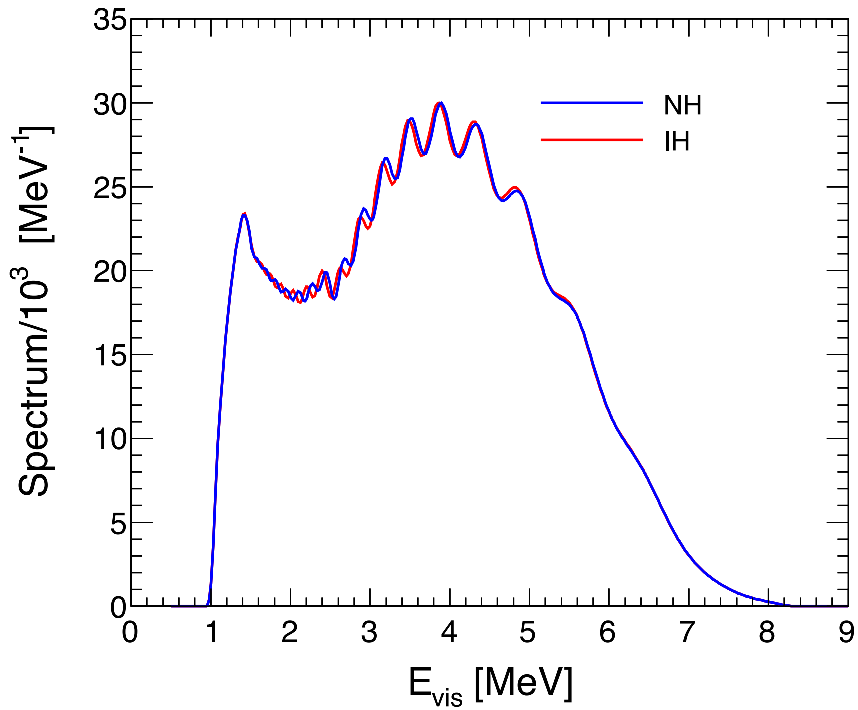
Cores	YJ-C1	YJ-C2	YJ-C3	YJ-C4	YJ-C5	YJ-C6
Power (GW)	2.9	2.9	2.9	2.9	2.9	2.9
Baseline(km)	52.75	52.84	52.42	52.51	52.12	52.21
Cores	TS-C1	TS-C2	TS-C3	TS-C4	DYB	HZ
Power (GW)	4.6	4.6	4.6	4.6	17.4	17.4
Baseline(km)	52.76	52.63	52.32	52.20	215	265

Table 1: Summary of the power and baseline distribution for the Yangjiang (YJ) and Taishan (TS) reactor complexes, as well as the remote reactors of Daya Bay (DYB) and Huizhou (HZ).

Mass = 20 kT (11% protons), resolution = 3%/$\sqrt{E}$, P=35.8 GW.
Geoneutrino and far-reactor contributions are also included.

Typical observable spectra:





separate and total contributions for NH

spectra in NH/IH for the same osc. param. (but: parameters float in fit)

[Note: IBD threshold and geoneutrino “step” energies known a priori]

χ^2 function:

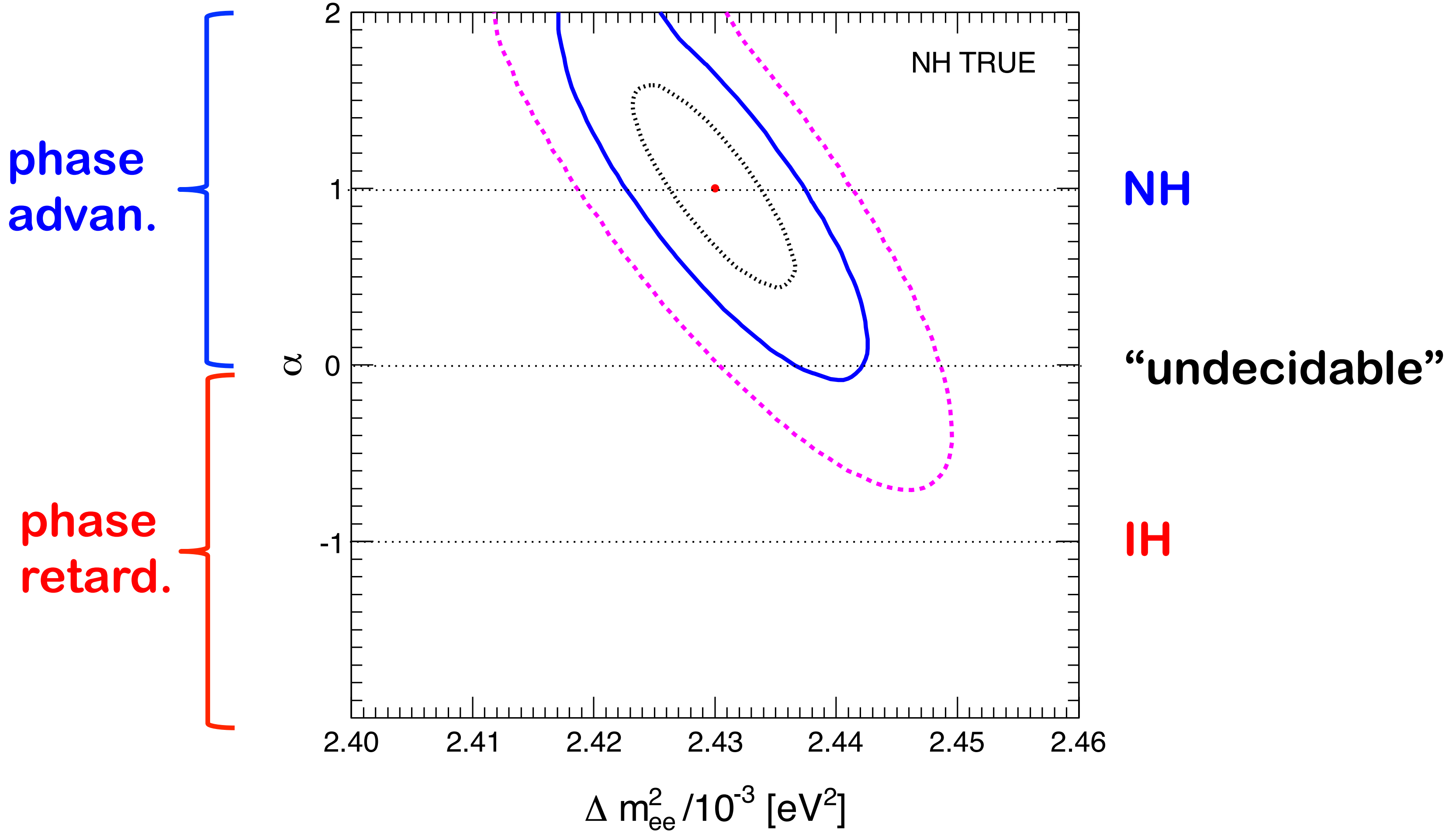
$$\chi^2 = \chi^2_{\rm stat} + \chi^2_{\rm par} + \chi^2_{\rm sys}$$

$$\chi^2 = \chi^2(\underbrace{\delta m^2,\ \Delta m^2_{ee},\ \theta_{12},\ \theta_{13}}_{\substack{\text{oscillation parameters:}\\ \text{floating with penalties}\\ \text{given by current errors}\\ \text{from global fit}}},\ \alpha,\ \underbrace{f_{\rm R},\ f_{\rm U},\ f_{\rm Th}}_{\substack{\text{reactor and U, Th geo-nu}\\ \text{normalizations: floating}\\ \text{with penalties given by}\\ \text{3\%, 20\%, 20\%.}}})$$

hierarchy parameter:
unconstrained a priori

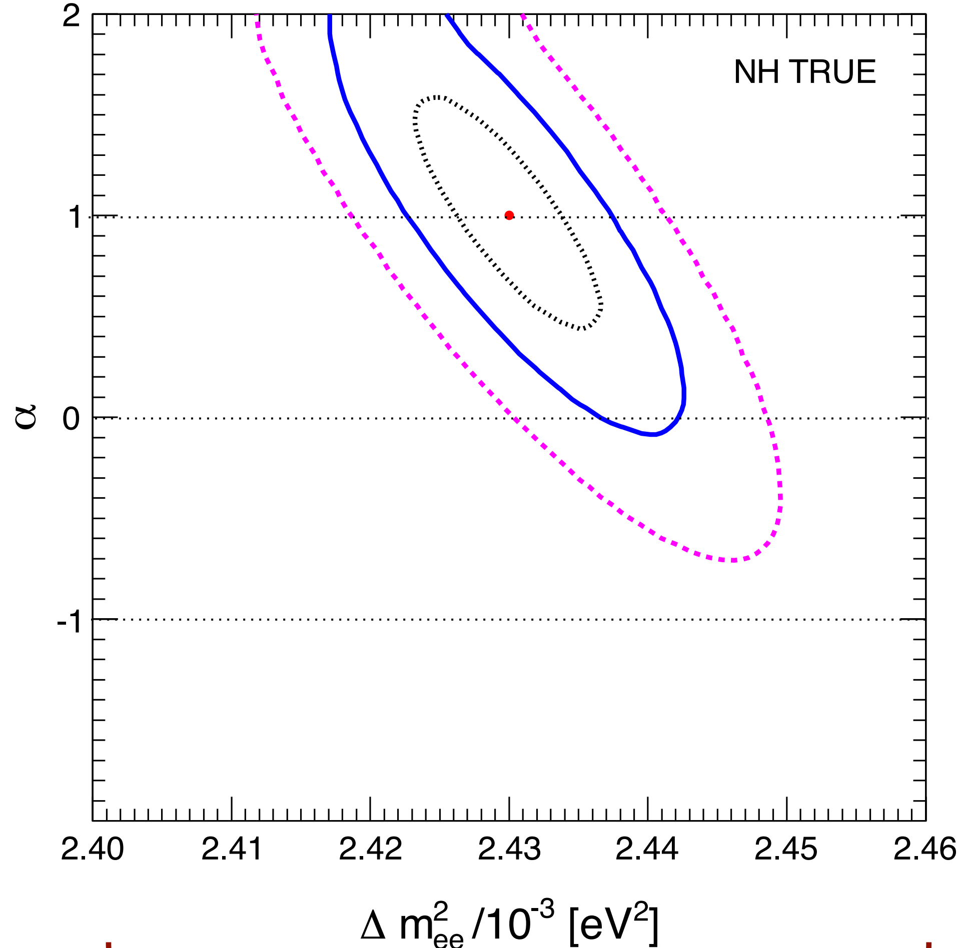
The following results refer to the case of true NH
[the case of true IH is rather symmetrical].
Curves refer to 1σ, 2σ, 3σ (for 1 dof), i.e., $\Delta\chi^2$ = 1, 4, 9

(Δm^2_{ee}, α) fit results



Although the "wrong" hierarchy is ~3.4σ away from the true one, the experiment is already compromised when the "undecidable" case is reached at ~1.7σ = effective sensitivity to hierarchy.

Reminder:



~ size of non-L/E phase

Anti-correl.: one can partly trade one for the other, but not completely: one is L/E, the other is non-L/E

~ size of L/E phase

$$P^{3\nu}_{\rm mat} \simeq c^4_{13} P^{2\nu}_{\rm mat} + s^4_{13} + 2 s^2_{13} c^2_{13} \sqrt{P^{2\nu}_{\rm mat}}\, w\, \cos(2\Delta_{ee} + \alpha\varphi)$$

Other interesting fit results

TABLE I: Statistical analysis of prospective JUNO data: fractional percent errors (1σ) on the free parameters, before and after the fit to prospective JUNO data, assuming either normal or inverted true hierarchy. The hypothetical cases without contributions from far reactors ("all − far") or from geoneutrinos ("all − geo") are also reported. In the latter case, the normalization factors $f_{\mathrm{Th,U}}$ are absent.

Parameter	% error	% error after fit (NH true)			% after fit (IH true)		
	(prior)	all data	all − far	all − geo	all data	all − far	all − geo
α	∞	59.2	59.0	57.0	56.2	55.3	54.0
Δm^2_{ee}	2.0	0.26	0.25	0.26	0.26	0.25	0.25
δm^2	3.2	0.22	0.21	0.16	0.21	0.21	0.16
s^2_{12}	5.5	0.49	0.47	0.39	0.49	0.46	0.42
s^2_{13}	10.3	6.95	6.88	6.95	6.84	6.77	6.84
f_R	3.0	0.66	0.66	0.64	0.65	0.65	0.64
f_{Th}	20.0	15.3	14.6	—	15.5	15.4	—
f_{U}	20.0	13.3	13.3	—	13.3	13.3	—

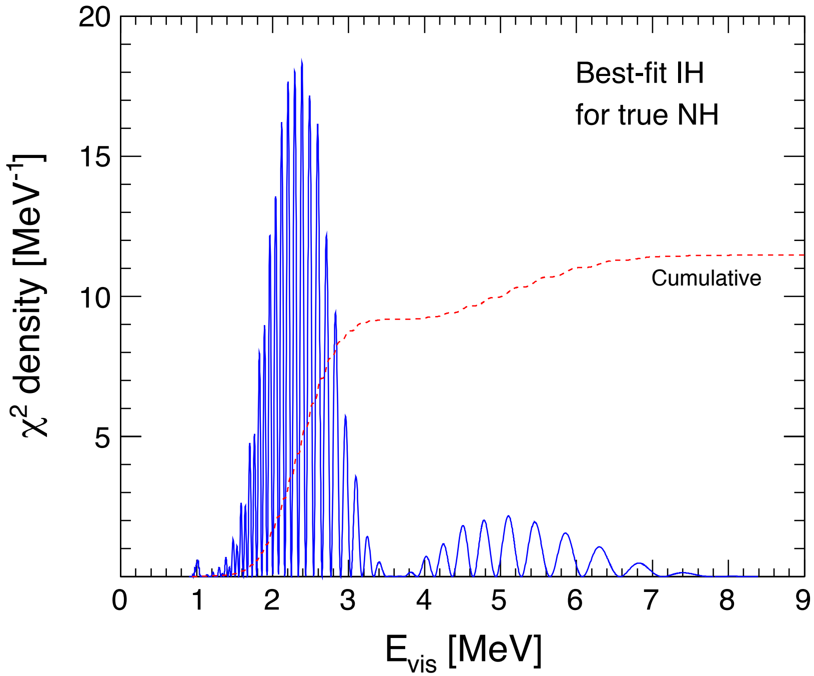
prior errors

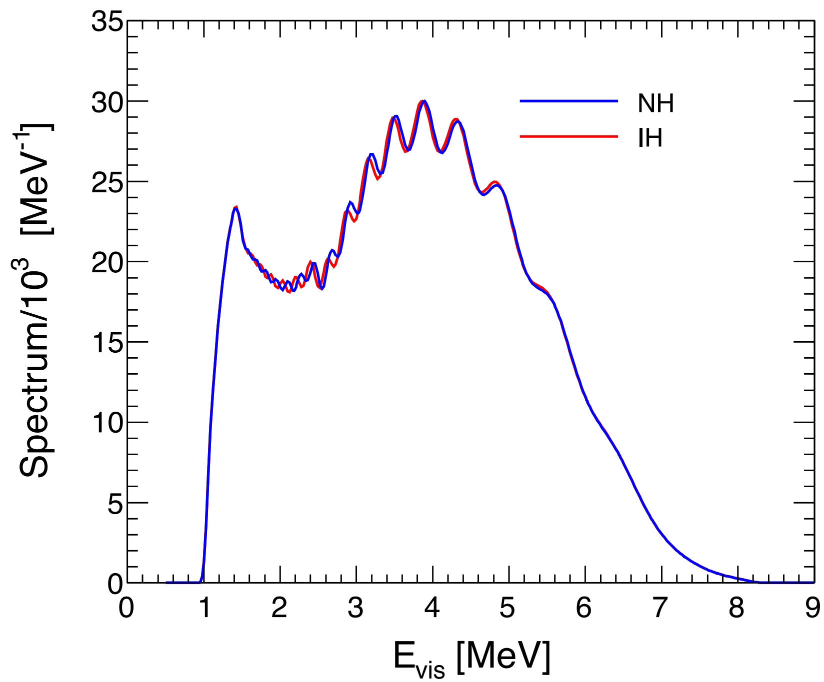
posterior errors

Three oscillation parameters' uncertainties are reduced by about one order of magnitude w.r.t. current errors.

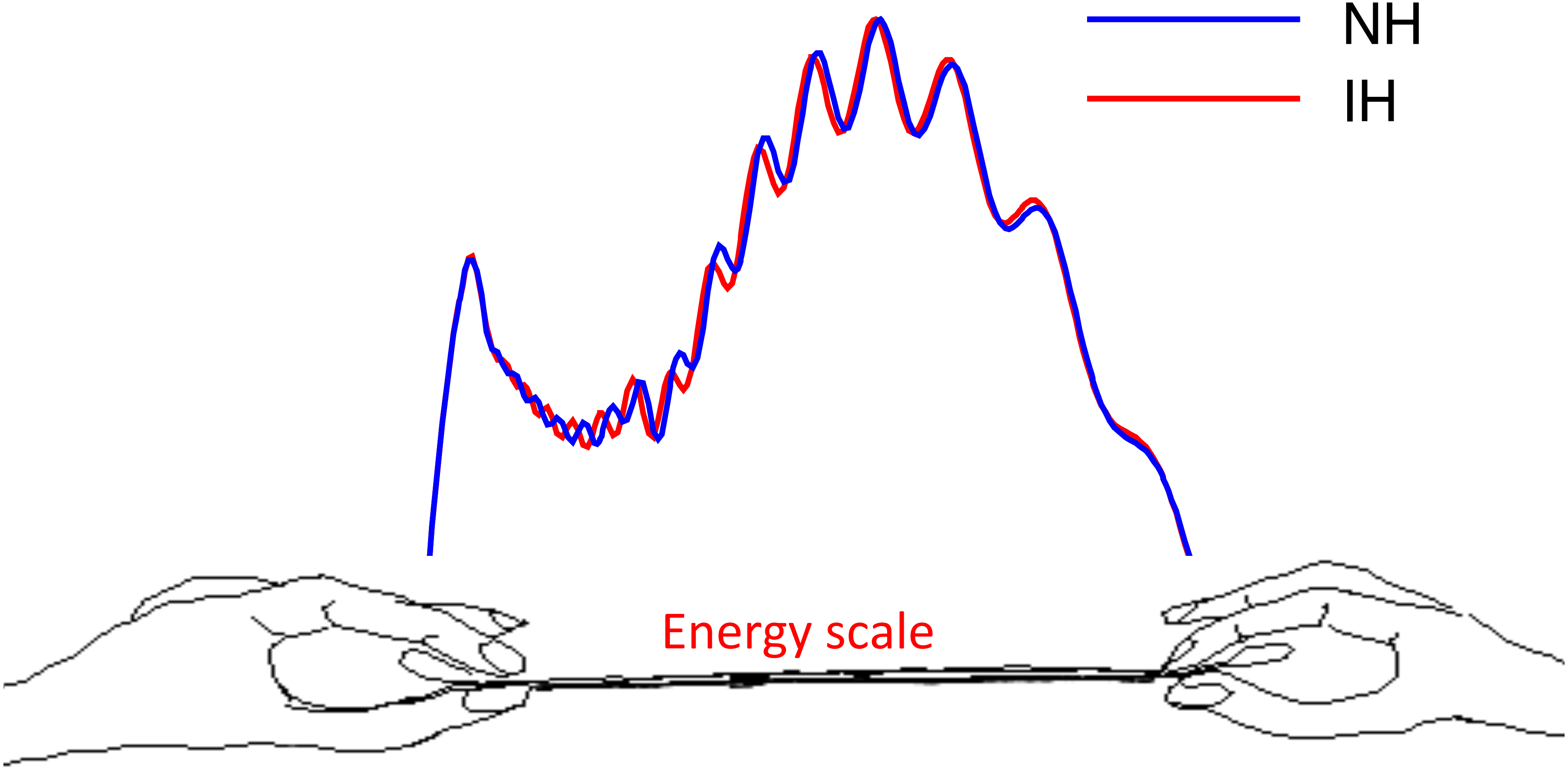
This is enough to justify the experiment (in my opinion) even if the hierarchy sensitivity were limited around the ~2σ level.

Most of the χ^2 contribution comes from low-energy mismatch between oscillation peaks in NH vs IH.





However: What if peaks are "rephased" via a nonlinear transformation of the energy scale, within realistic errors?



In other words: Is there an "elastic" horizontal stretch (+shift) of the red curve, that "realigns" the red and blue oscill. peaks?

Yes – infinite possibilities!

The "energy scale" challenge

There is an infinite family of energy scale transformations, E→ E', which map +α → -α in Pee, and can thus mimic the "wrong hierarchy"

[X. Qian et al., 1208.1551]

+α → -α

$$P_{\mathrm{mat}}^{3\nu} \simeq c_{13}^4 P_{\mathrm{mat}}^{2\nu} + s_{13}^4 + 2 s_{13}^2 c_{13}^2 \sqrt{P_{\mathrm{mat}}^{2\nu}}\, w \cos(2\Delta_{ee} + \alpha\varphi)$$

These transformations can be approximately cast in the form:

[F. Capozzi, E. Lisi, A. Marrone, 1309.1638]

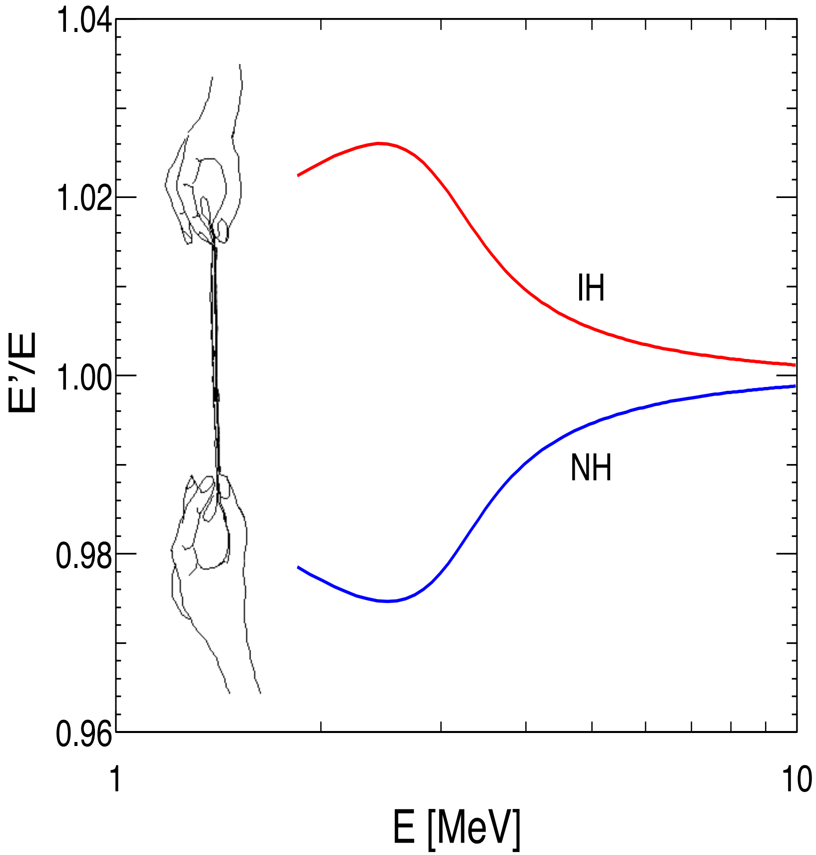
$$\frac{E'}{E} \simeq \frac{\Delta m_{ee}^{2\prime}}{\Delta m_{ee}^2} \mp 2 s_{12}^2 \frac{\delta m^2}{\Delta m_{ee}^2} \left(1 - \frac{\sin \delta(E)}{2\delta(E)\sqrt{P_{\mathrm{mat}}^{2\nu}(E)}} \right)$$

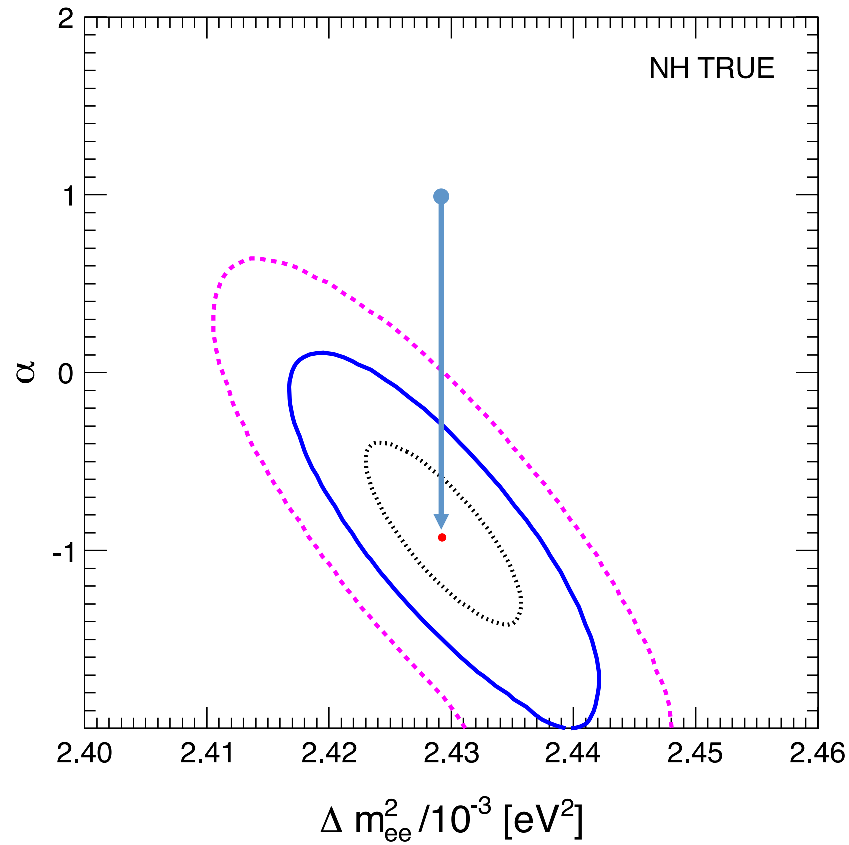
linear term ± nonlinear term (NH/IH)

If the nonlinear term is halved, the transformation maps |α|=1 → α~0 (i.e, into the "undecidable" case)

E.g., choose transformation with linear term=1:

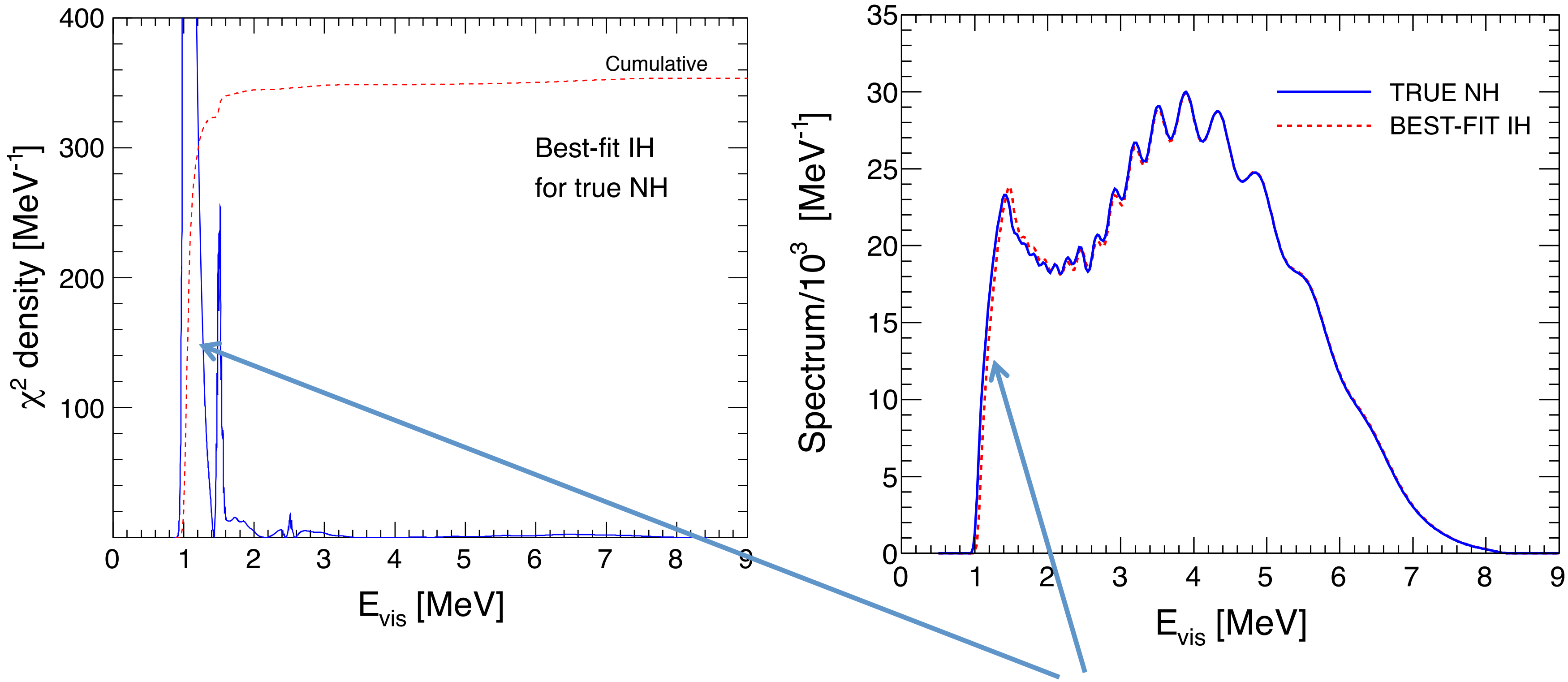
("zero stretching at high energy")





If this transformation is allowed within E-scale errors, then the best fit moves to the wrong hierarchy. However...

… the "best fit" is, in itself, very bad (enormous χ^2):

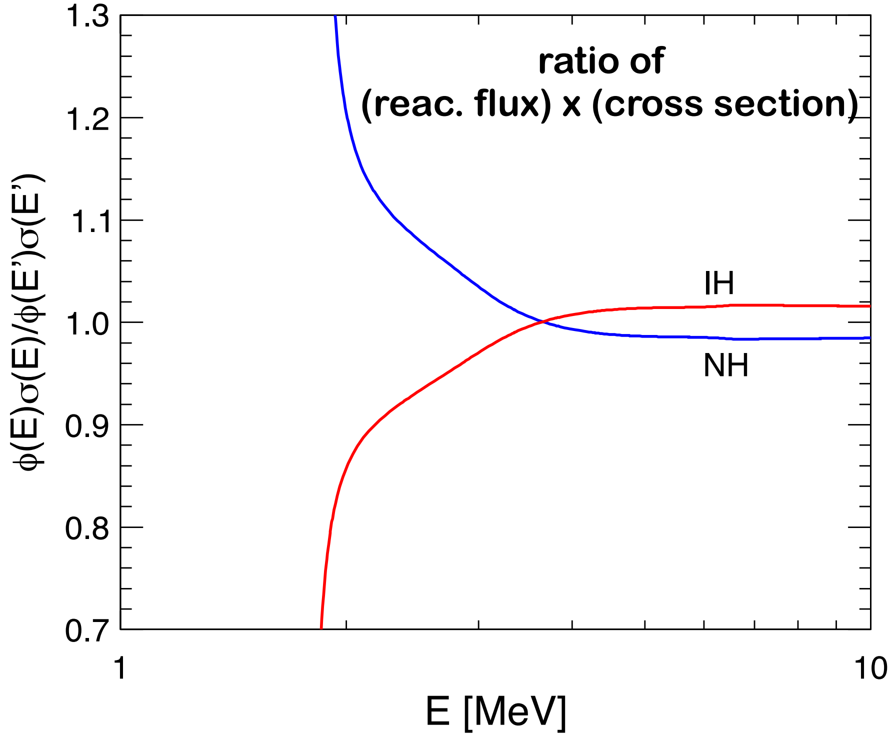


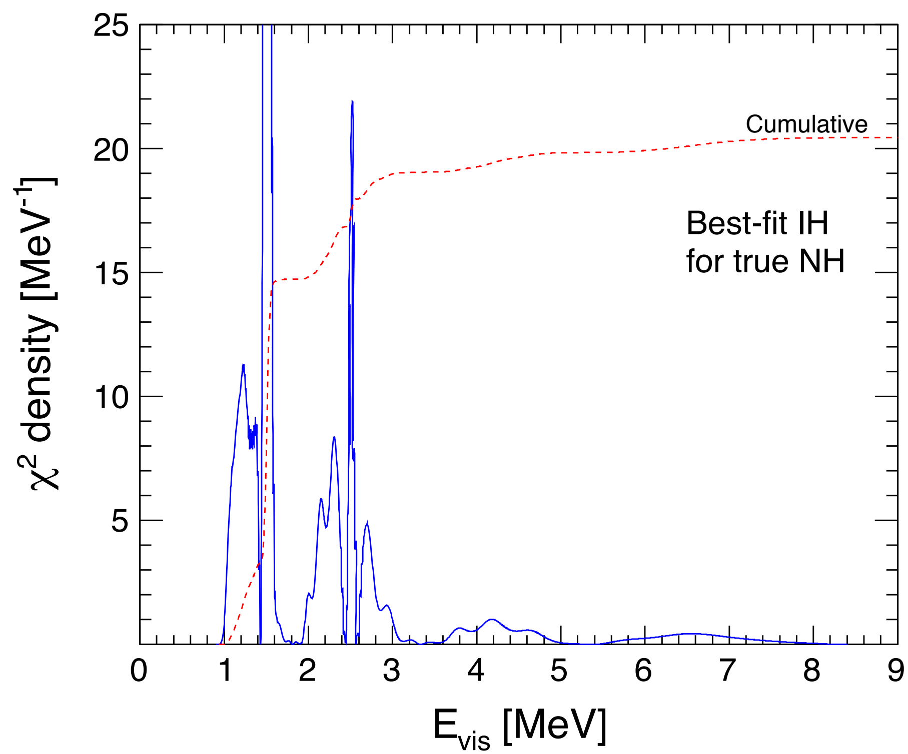
Reason: the transformation realigns osc. peaks, but also shifts the IBD threshold & the step-like features of geo-ν, which are known a priori → "self calibration" at low E.

However, we assumed known reactor spectrum …

but these fractional spectral variations, if allowed within experimental uncertainties...

... can "undo" most of the reactor threshold mismatch, up to a small geo-nu misfit...

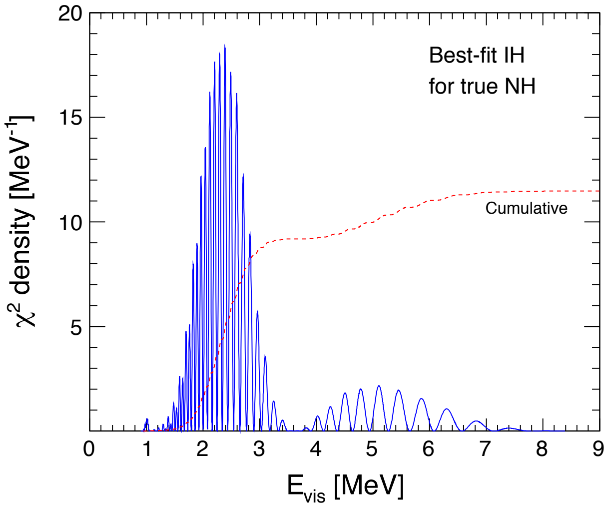


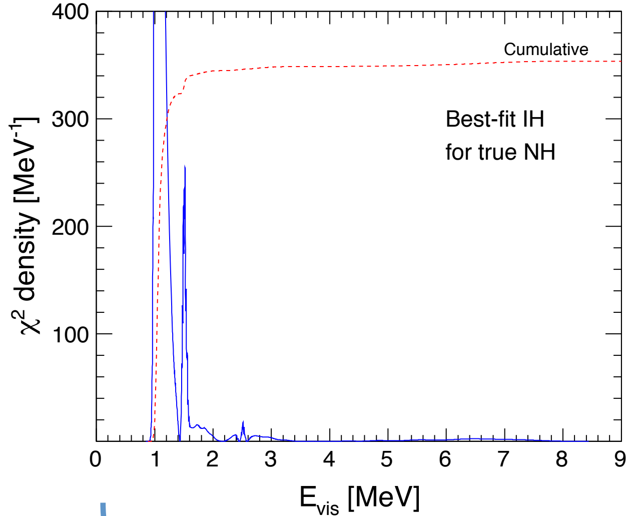


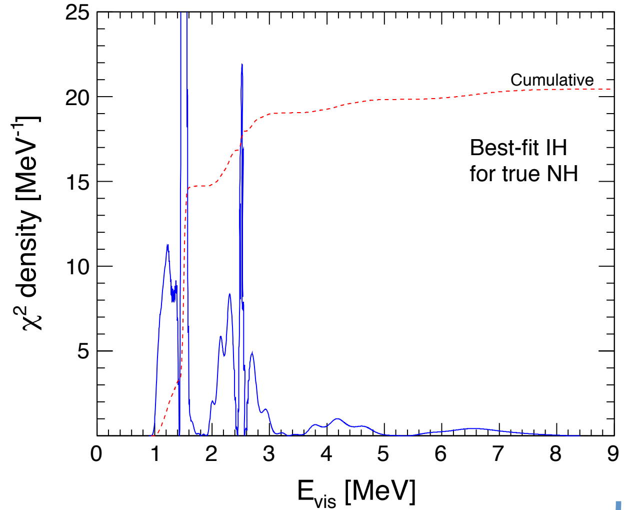
... thus realizing an **almost complete degeneracy between true and wrong hierarchy**, with only a modest χ^2 increase localized around the "steps".

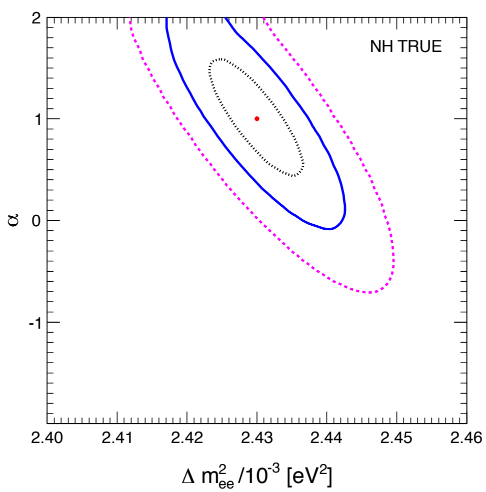
RECAP of possible (mis)fit results:

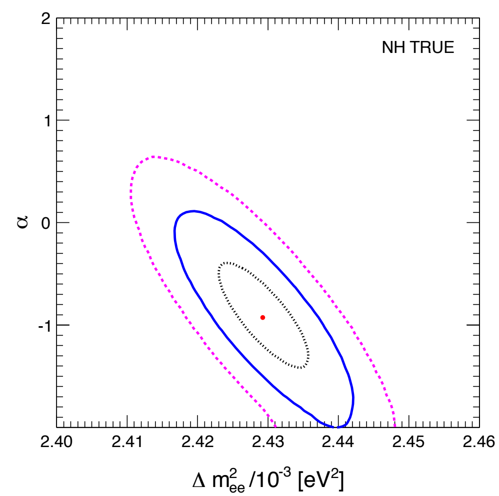
No E-scale or reactor spectrum systematics → **+ ad hoc E-scale variations** → **+ ad hoc reactor spectrum variations**





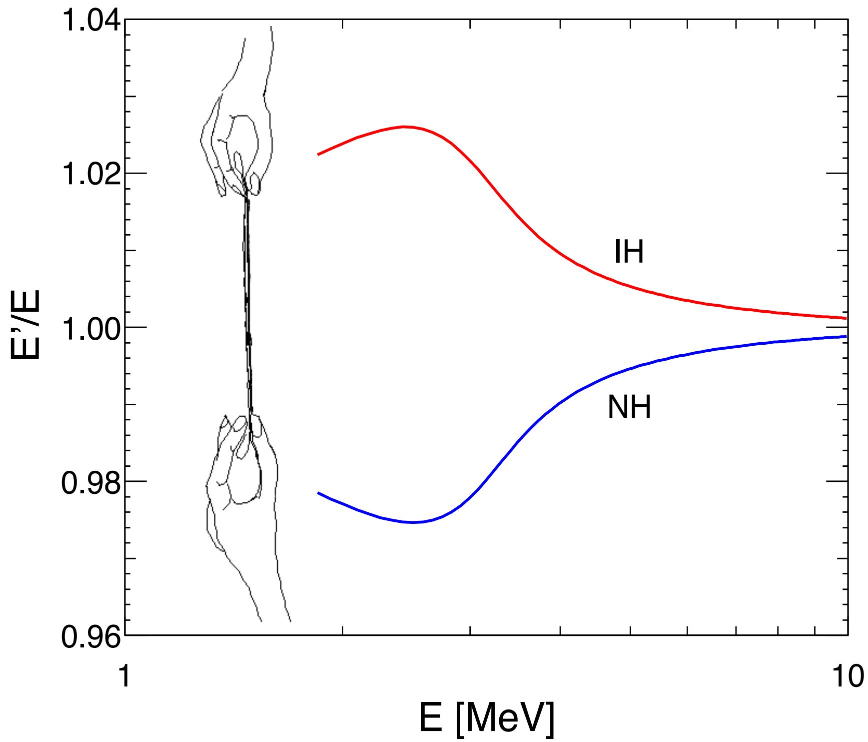


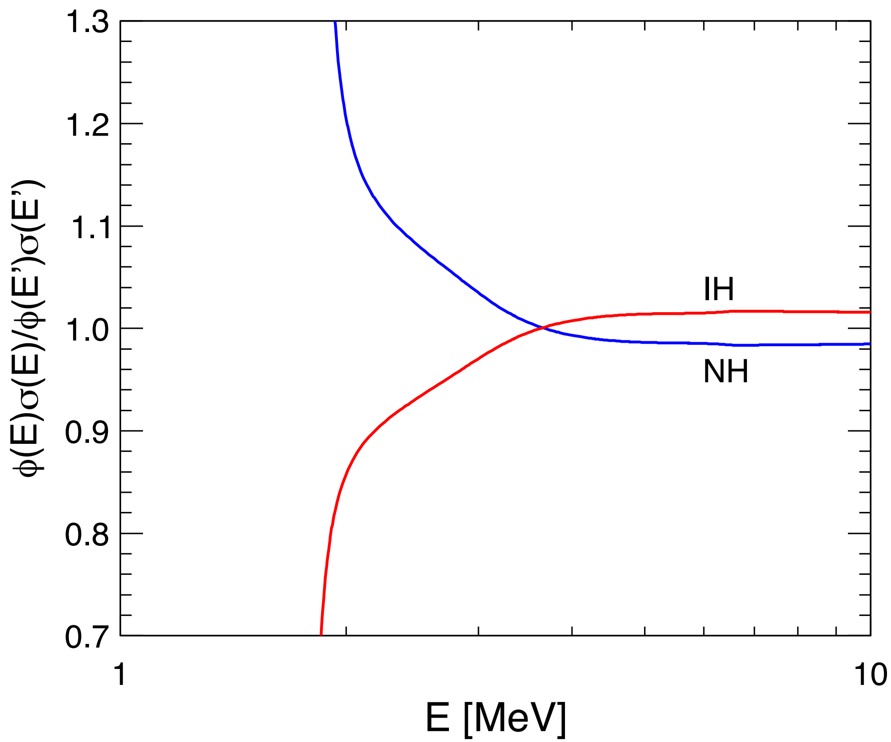




In other words: a peculiar conspiracy of

energy scale variations + spectrum shape variations





may compromise the hierarchy determination.

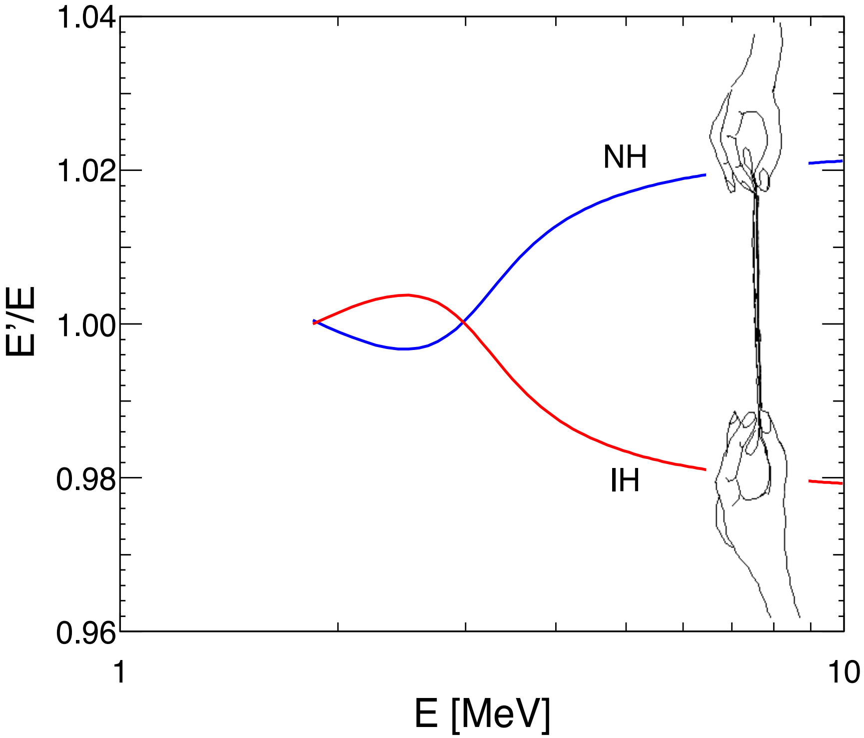
There is an infinity family* of such transformations → the challenge may be transferred from low to high E...

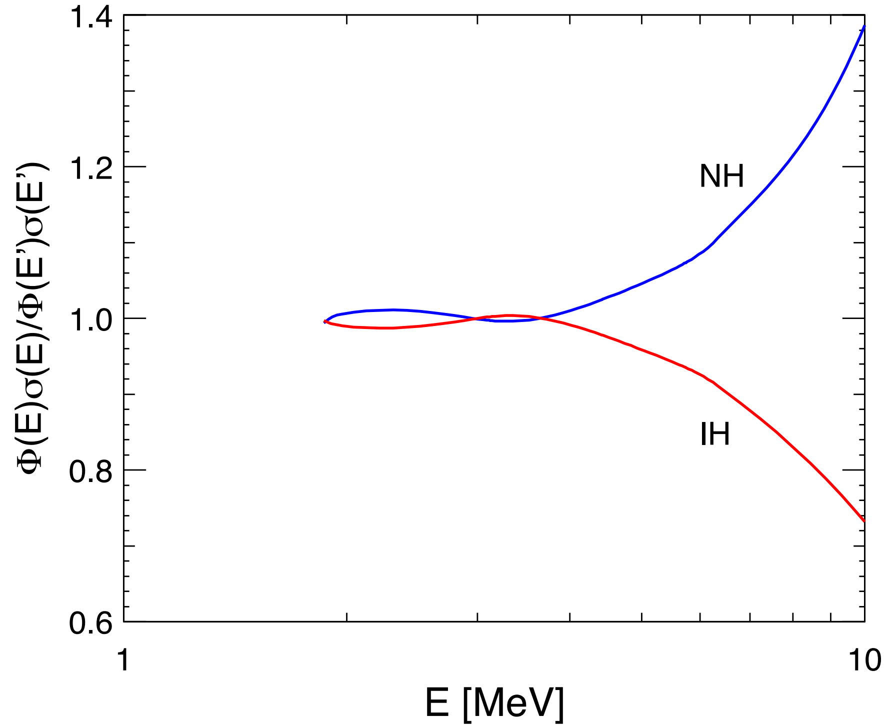
***Not a family of polynomials in E, however!**

Example of an alternative transformation…

("zero stretching at low energy")

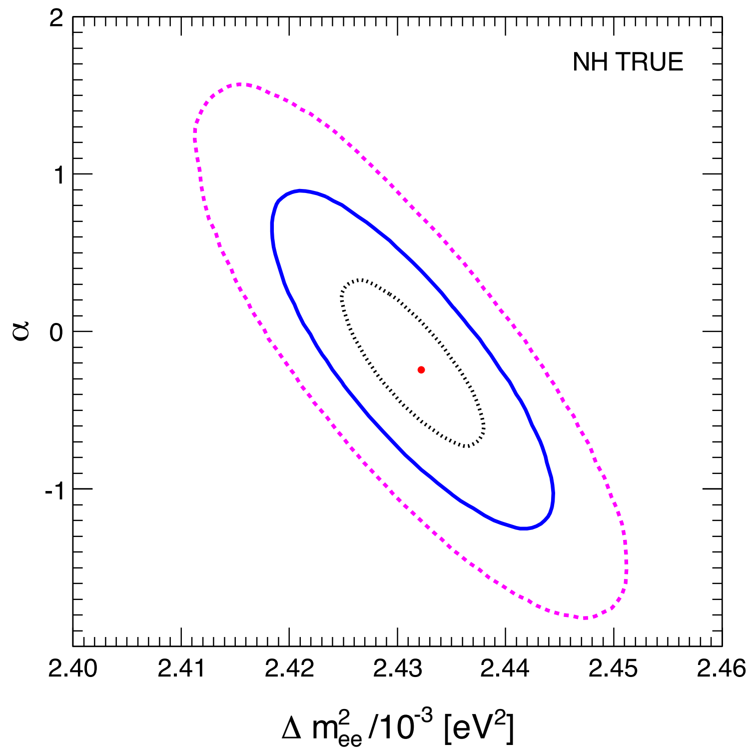
energy scale variations + spectrum shape variations





…which also compromises the hierarchy determination, but without any mismatch at threshold (main effects confined at high energy!). Infinite more possibilities…

... including energy + spectral variations which do not swap the hierarchy but make it nearly “undecidable” (best fit at $\alpha \sim 0$):



Functional uncertainties in the energy scale and in the reactor spectrum need further, dedicated expt+theo studies, in order to reject unlucky combinations which may ruin NH/IH separation

Conclusions

Medium baseline reactor experiments may offer unprecedented opportunities to accurately measure some osc. parameters and to discriminate the mass hierarchy. They ought to be done!

We have (re)considered several issues emerging in the context of hierarchy discrimination: matter, damping & recoil effects, and continuous interpolation between NH and IH in stat. analyses.

For a typical JUNO-like setting, we estimate a sensitivity to the hierarchy around 2σ, and provide an alternative intepretation of the ``rule of thumb'' $N\sigma \sim 0.5 \sqrt{\Delta\chi^2(\text{NH-IH})}$ for discrete hypoth.

Energy scale and reactor flux shape errors represent a serious challenge: specific functional forms may lead to an almost complete degeneracy between NH and IH (up to geo-nu misfits).

Further theo/pheno/expt investigations are needed to understand and to assess the ultimate reactor sensitivity to the mass hierarchy.

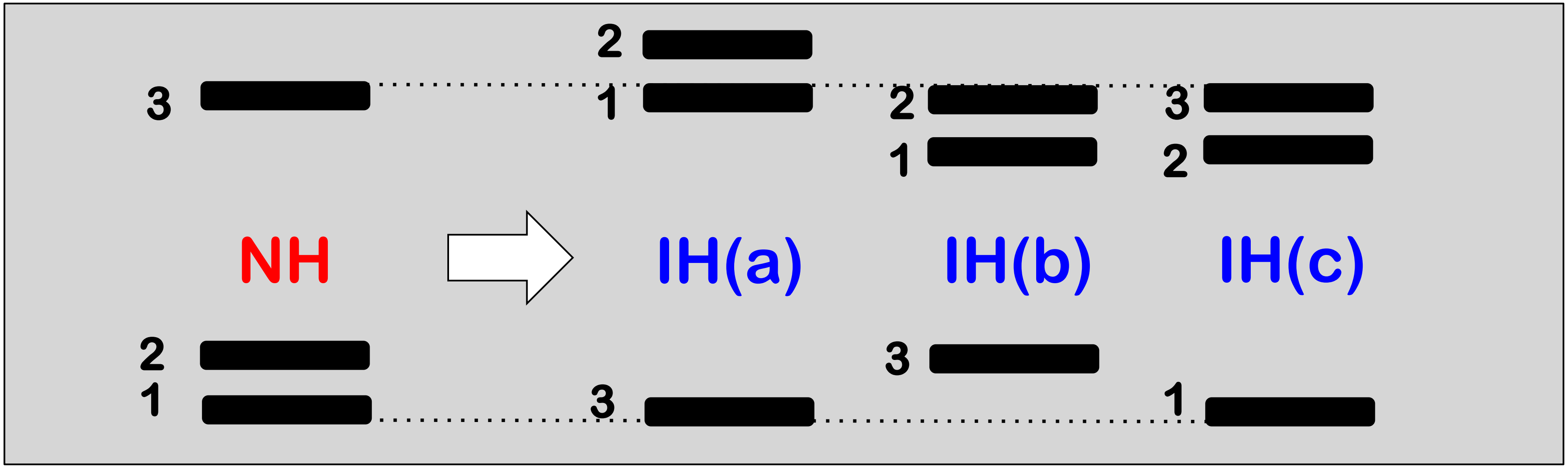
The riddle of ν mass hierarchy is likely to be very difficult…

… but not impossible to solve!

Thank you for your attention

Back-up: osc. probability, a pedagogical issue

What does it mean to “swap” the hierarchy? In various papers:
Fix some atmospheric mass^2 value, change its sign, and the separate hierarchy-odd amplitudes in Pee.
But such odd terms are convention-dependent, e.g., ...



(a) Fix Δm^2_{31}: one frequency increases

(b) Fix Δm^2_{32}: one frequency decreases

(c) Fix $\Delta m^2=(\Delta m^2_{31}+\Delta m^2_{32})/2$: frequencies do not change

→ better to confine odd terms in a phase rather than in amplitudes