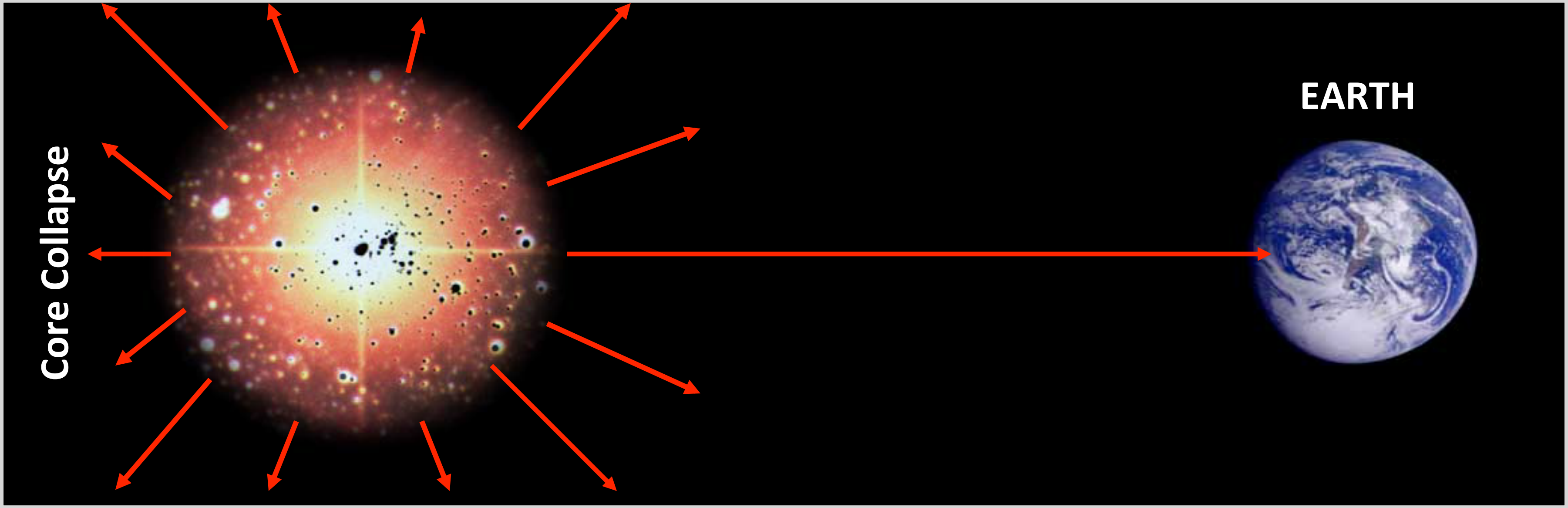
Self-induced Flavor Conversion of Supernova Neutrinos

Sovan Chakraborty
Indian Institute of Technology, Guwahati.

Invisibles Webinar
15th March, 2016.

TYPICAL PROBLEMS IN SUPERNOVA NEUTRINOS



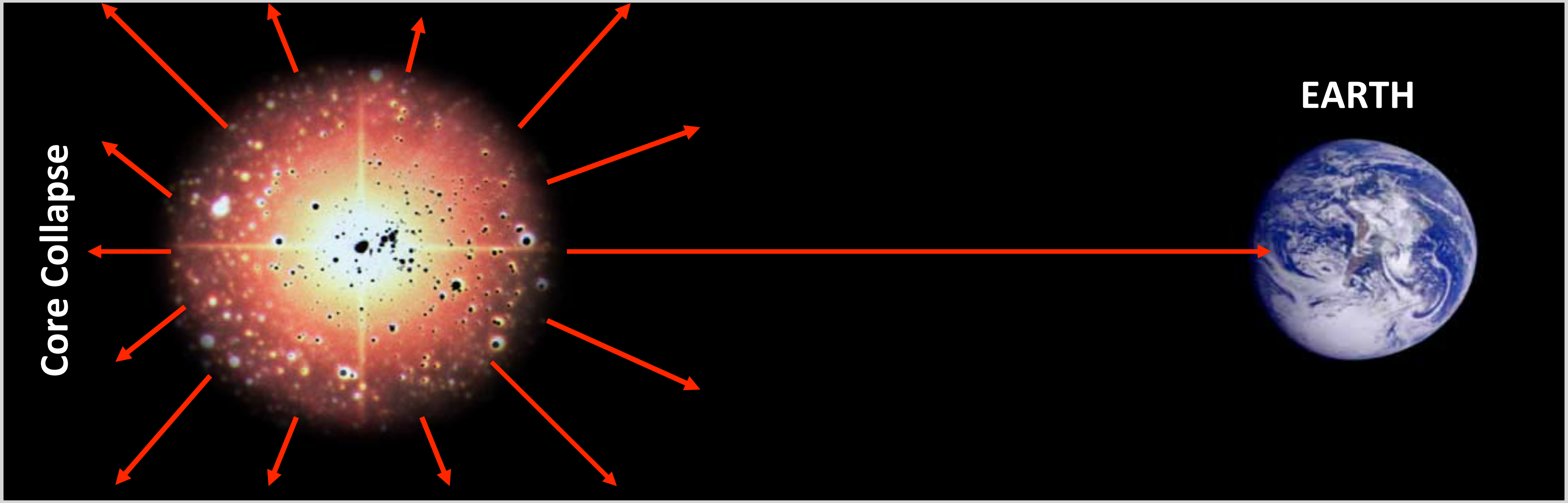
Production
(flavor)

Propagation
(mass,mixing)

Detection
(flavor)

Based on the works
arXiv:1507.07569, arXiv:1602.00698 & arXiv:1602.02766
with
Rasmus Hansen, Ignacio Izaguirre & Georg Raffelt

TYPICAL PROBLEMS IN SUPERNOVA NEUTRINOS

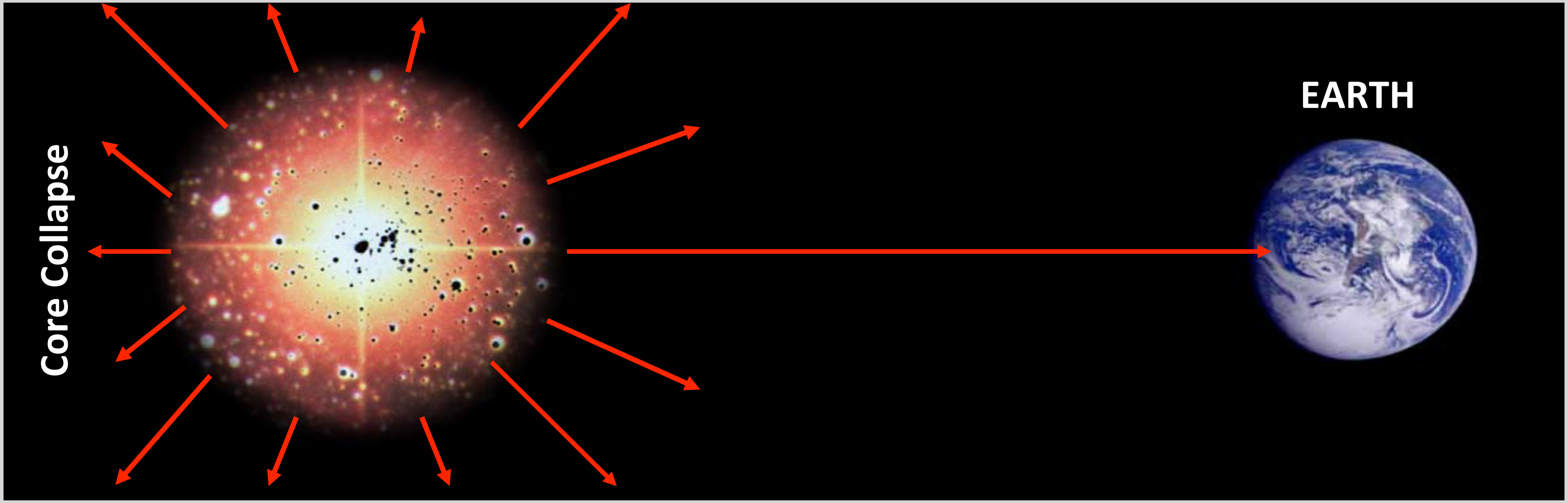


Supernova (SN) as Neutrino Source

SN Neutrino Oscillation: Initial Symmetries

Linear Stability Analysis: Symmetry Breaking

TYPICAL PROBLEMS IN SUPERNOVA NEUTRINOS

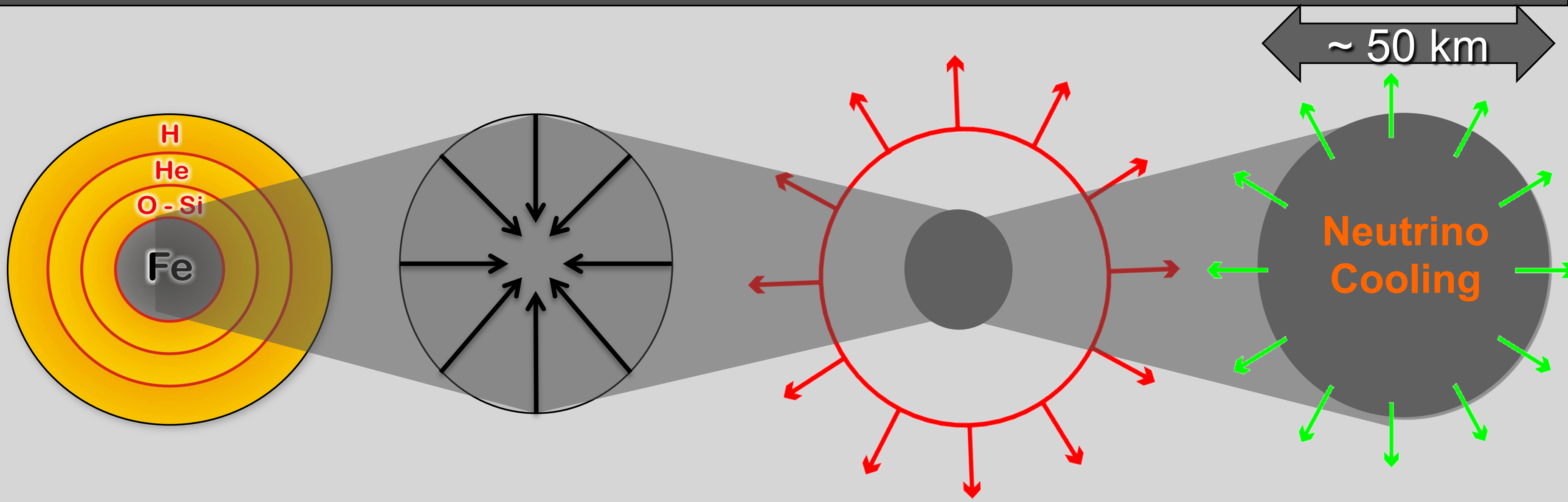


Supernova (SN) as Neutrino Source

SN Neutrino Oscillation: Initial Symmetries

Linear Stability Analysis: Symmetry Breaking

STELLAR COLLAPSE AND CORE-COLLAPSE SUPERNOVA



Onion structure at the end of stellar burning	Collapse (implosion) of the degenerate core	Bounce at nuclear density shock wave explosion	Neutrino Cooling Newborn Neutron Star

- ENERGY SCALES: ~ 10^{53} erg, 99% energy is emitted by Neutrinos, Energy 10 MeV
- TIME SCALE: The duration of the burst lasts ~10 s.
- DETECTION: Large volume detectors will see huge rate of MeV neutrinos in seconds.

NEUTRINO EMISSION PHASES

Neutronization burst ~ 50 ms

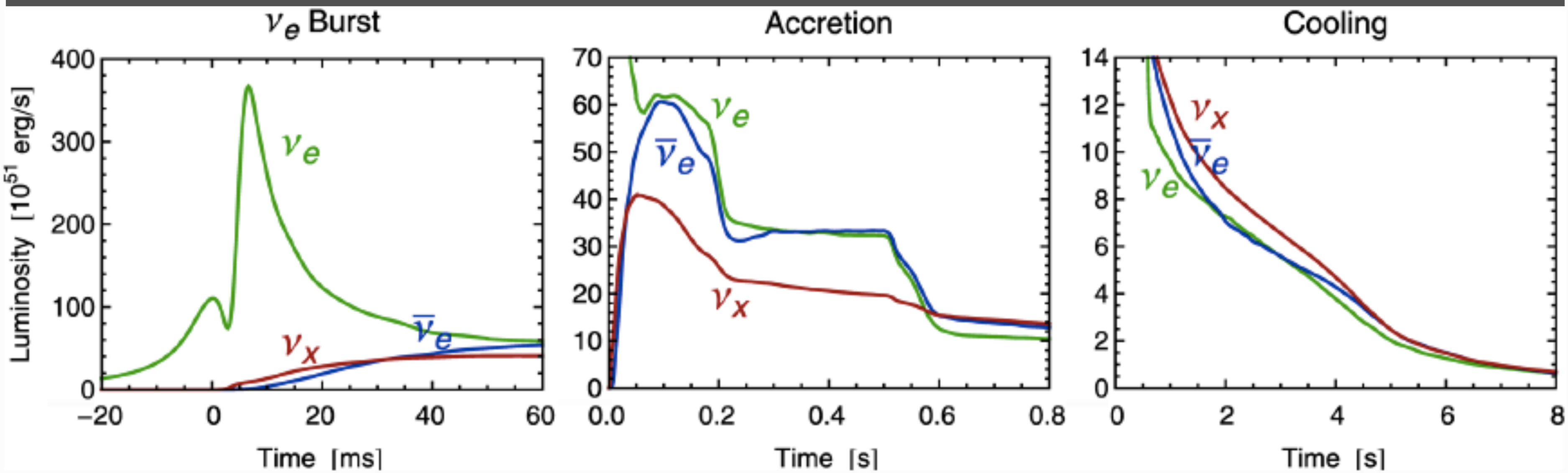
- Shock breakout
- De-leptonization of outer core layers

Accretion: ~ 0.5 s

- powered by infalling matter
- Stalled shock

Cooling~ 10 s

- Cooling by ν diffusion



- ν_e Burst and Accretion: Best phase to study oscillation.
- Cooling: Oscillation effects are negligible.
- Accretion: How to rejuvenate the stalled shock?

[Fischer et al. (Basel Simulations), A&A, 2010, 10. 8 M_{sun} progenitor mass]

STATUS OF SN EXPLOSION

- Neutrino-driven explosion

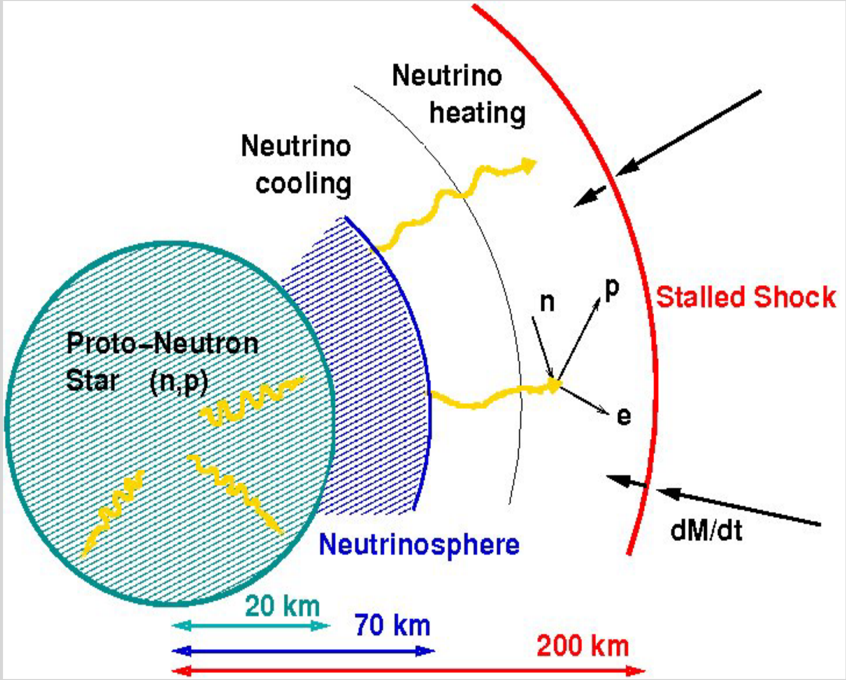
(Wilson mechanism)

- 1D (spherical sym) Numerical explosions successful for small-mass progenitors

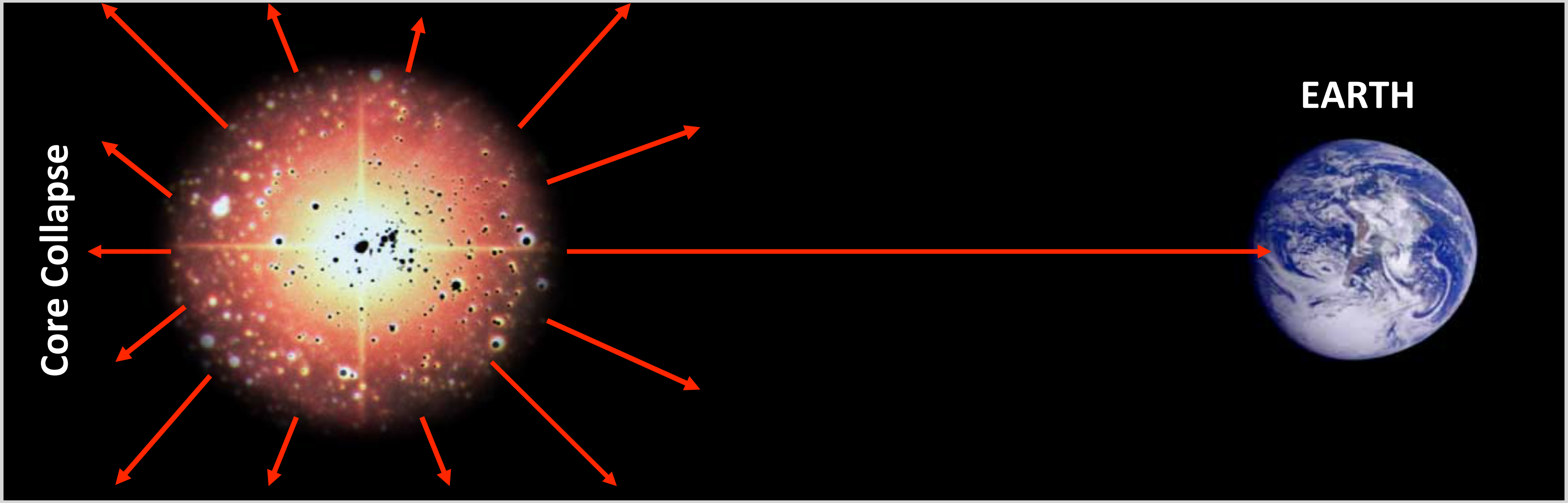
- 2D (axial sym) Numerical explosions okay for progenitors in wider mass range

- 3D simulations showing interesting features

Delayed Mechanism



TYPICAL PROBLEMS IN SUPERNOVA NEUTRINOS

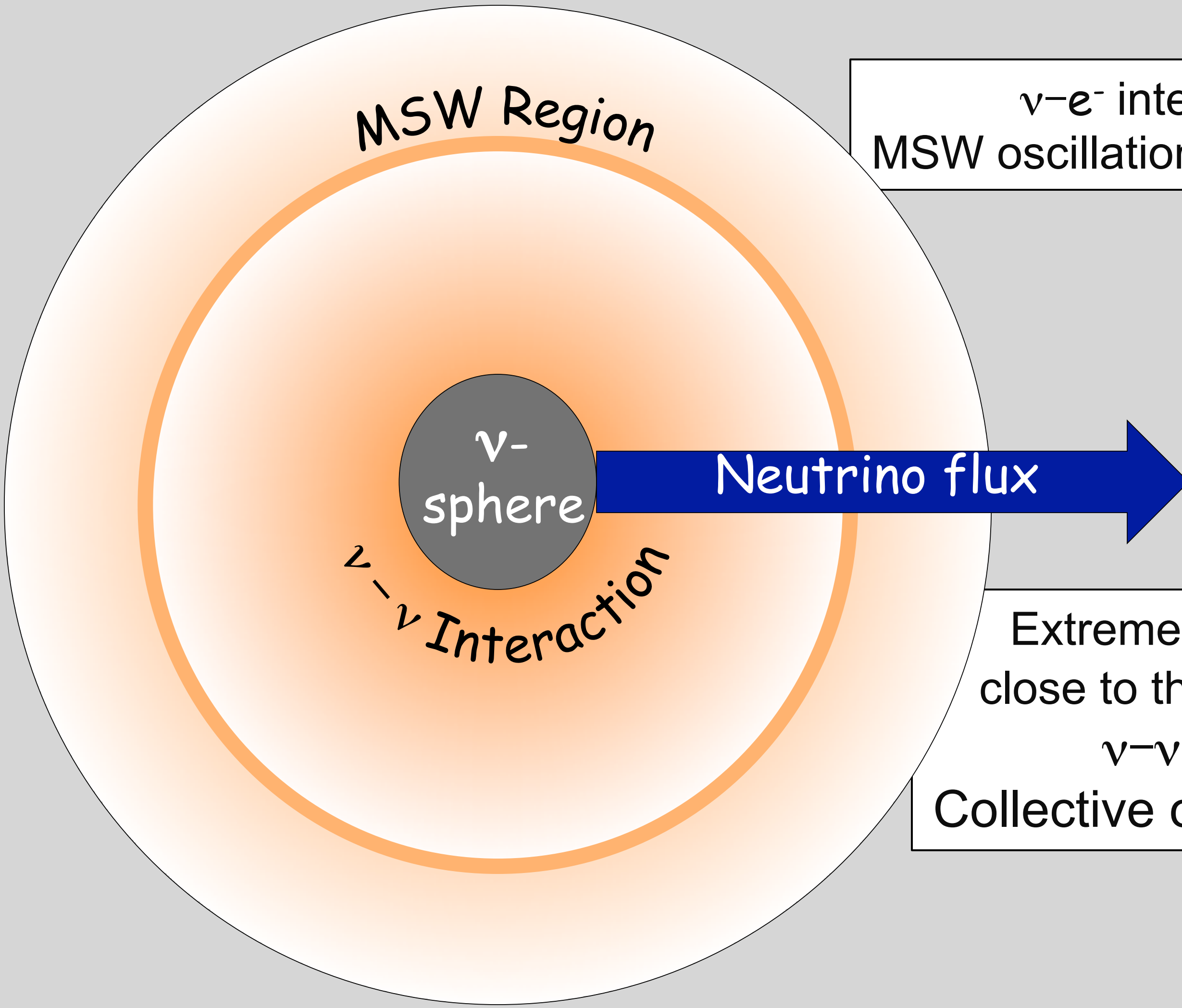


Supernova (SN) as Neutrino Source

SN Neutrino Oscillation: Initial Symmetries

Linear Stability Analysis: Symmetry Breaking

SN ν FLAVOR TRANSITIONS: COLLECTIVE OSCILLATION



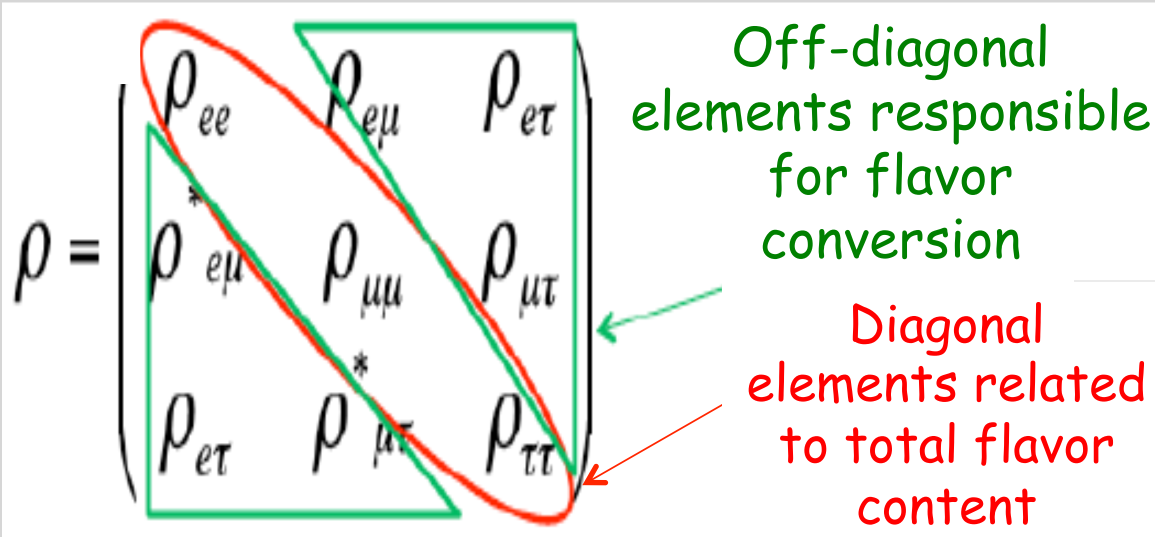
ν−e⁻ interaction
MSW oscillation ~ 10^4-10^5 km

Extreme neutrino density
close to the neutrino-sphere
ν−ν interaction
Collective oscillation~ 10^2 km

- Flavor Oscillation: In far separated regions, can be treated independently

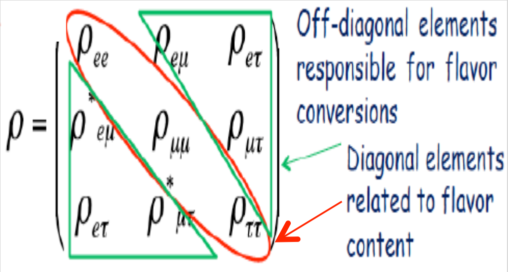
NEUTRINO TRANSPORT & FLAVOR OSCILLATIONS: 7D (1+3+3) PROBLEM

$$i(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{r}})\varrho = [\mathsf{H}, \varrho] \,, \quad \varrho = \varrho\,(t\,,\,\mathbf{r},\,\mathbf{p})$$



NEUTRINO TRANSPORT & FLAVOR OSCILLATIONS: 7D PROBLEM

$$i(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{r}})\varrho = [\mathsf{H}, \varrho], \quad \varrho = \varrho(t, \mathbf{r}, E, \mathbf{v}),$$



$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_{\mathrm{F}}\left[\mathsf{N}_\ell + \int d\Gamma' \, \frac{(\mathbf{v}-\mathbf{v}')^2}{2}\, \varrho_{t,\mathbf{r},E',\mathbf{v}'}\right]$$

Kinematical mass-mixing term	Dynamical MSW term (in matter)	Neutrino-neutrino interactions term (non-linear)
	r^{-3} dependence, at around 10^4 km	$r^{-2}\times r^{-2} \sim r^{-4}$ dependence, 10^2 km

- Flavor Evolution: Non-Linear, coupled system
- Coupling: Between neutrino-antineutrino, different energy & angular modes
- Numerical Solution: Intensive, even with assumptions on the system

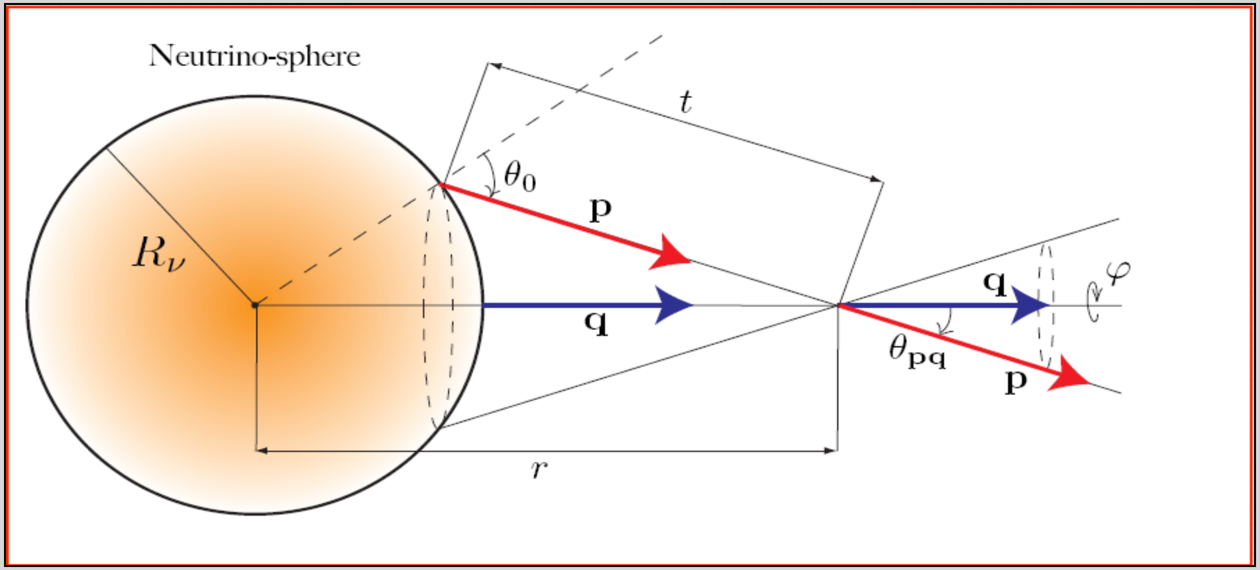
MULTI ANGLE PROBLEM (0+1+2):

Stationary, spherically symmetric, evolving with radius

$$v_r \partial_r \rho(r, E, \theta) = -i \left[H(r, E, \theta), \rho(r, E, \theta)\right]$$

'θ' Zenith angle of nu momentum $\vec{p}\,(E)$,
azimuthal symmetry in momentum: no ϕ

'v_r' Radial velocity depends on θ, leads to multi-angle matter effect



SINGLE ANGLE APPROXIMATION: (0+1+1) spherical symm in both space and velocity

Stationary, spherically symmetric, evolving with radius

$$\dot{\rho}(r, E) = -i\,[H(r, E), \rho(r, E)]$$

SINGLE ANGLE APPROXIMATION: (0+1+1)
spherical symm in both space and velocity

Stationary, spherically symmetric, evolving with radius

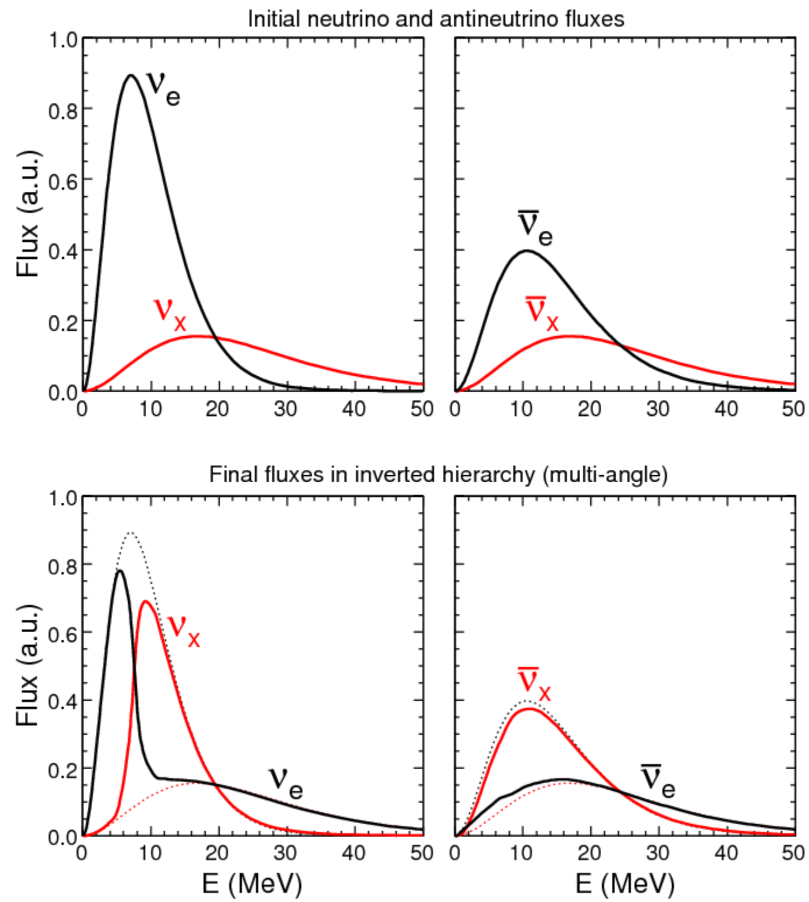
$$\dot{\rho}(r, E) = -i\,[H(r, E), \rho(r, E)]$$

Initial fluxes at neutrinosphere (r ~10 km)

Spectral Splits

IH

Fluxes at the end of collective effects (r ~200 km)



[Fogli, Lisi, Marrone, Mirizzi, arXiV: 0707.1998]

MULTI ANGLE PROBLEM (0+1+2): Matter Multi-angle Effect

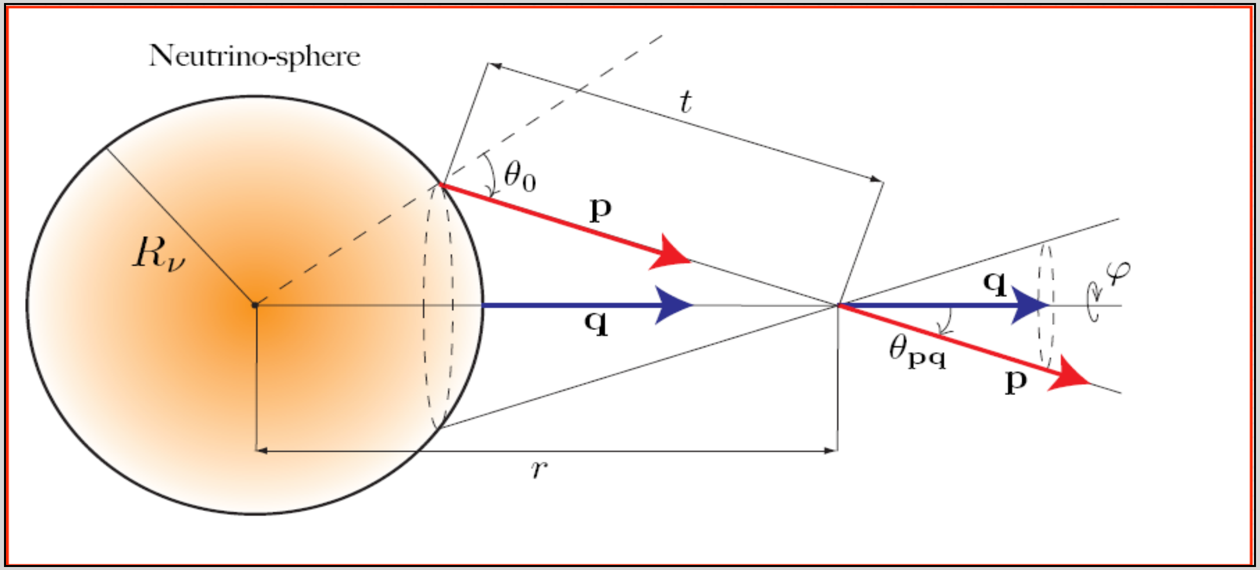
Stationary, spherically symmetric, evolving with radius

$$v_r \partial_r \rho(r, E, \theta) = -i\,[H(r, E, \theta), \rho(r, E, \theta)]$$

'θ' Zenith angle of nu momentum $\vec{p}$

'v_r' Radial velocity depends on θ, leads to multi-angle matter effect

Ignore matter: Matter induced resonance happens far away from collective, however…….



[Esteban-Pretel, Mirizzi, Pastor, Tomas, Raffelt, Serpico & Sigl, arxiv: 0807.0659]

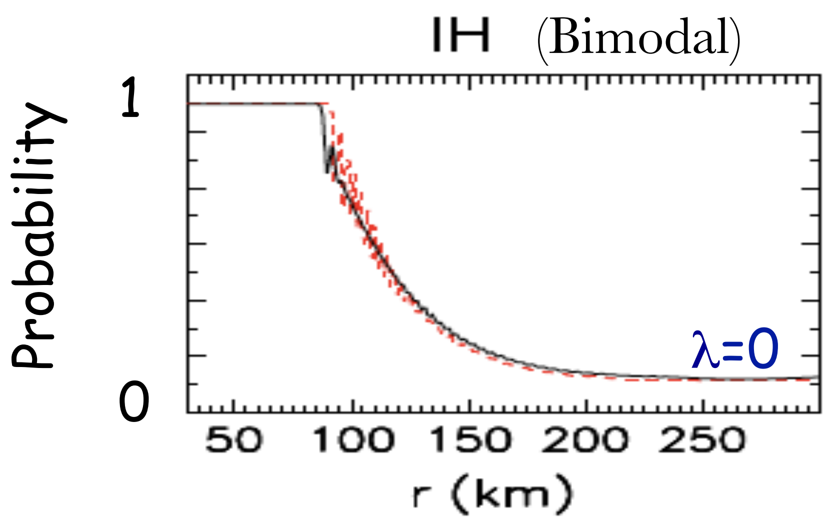
MATTER MULTI ANGLE SOLUTION (0+1+2):

Stationary, spherically symmetric, evolving with radius

$$v_r \partial_r \rho(r, E, \theta) = -i\, [H(r, E, \theta), \rho(r, E, \theta)]$$

'θ' Zenith angle of nu momentum $\vec{p}$

'v_r' Radial velocity depends on θ, leads to multi-angle matter effect



$\lambda \propto N_e$

S.C, Fischer, Mirizzi, Saviano &Tomas PRL, 2011; PRD, 2011

MATTER MULTI ANGLE SOLUTION (0+1+2):

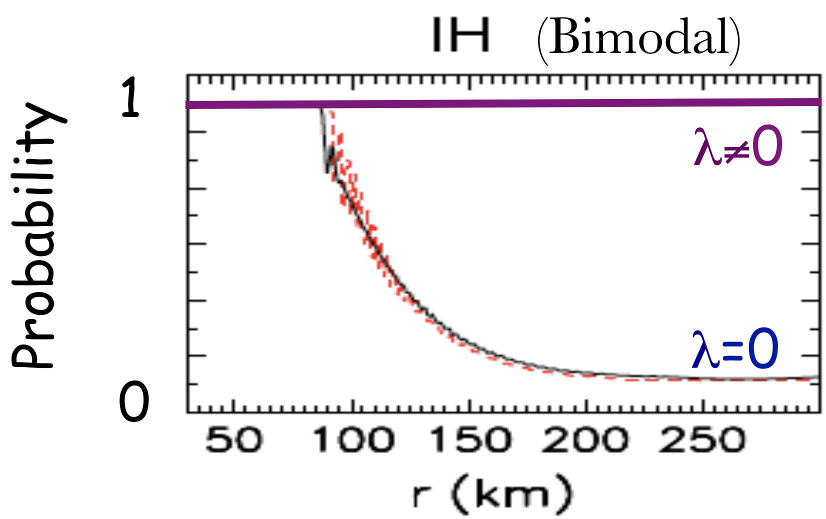
Stationary, spherically symmetric, evolving with radius

$$v_r \partial_r \rho(r,E,\theta) = -i\,[H(r,E,\theta), \rho(r,E,\theta)]$$

'θ' Zenith angle of nu momentum $\vec{p}$

'v_r' Radial velocity depends on θ, leads to multi-angle matter effect

Matter Multi-angle Effect



$\lambda \propto N_e$

Early accretion phase: No Collective Oscillations

S.C, Fischer, Mirizzi, Saviano &Tomas PRL, 2011; PRD, 2011

MATTER MULTI ANGLE SOLUTION (0+1+3):

Stationary, spherically symmetric, evolving with radius

$$v_r \partial_r \rho(r, E, \theta) = -i\,[H(r, E, \theta), \rho(r, E, \theta)]$$

'θ' Zenith angle of nu momentum $\vec{p}$

'v_r' Radial velocity depends on θ, leads to multi-angle matter effect

Axial symmetry in velocity / momentum distribution

What if this symmetry is broken? Multi Azimuthal Angle (MAA), φ

Flavor conversion in NH, MAA instability

Sarikas, Raffelt & Seixas PRL, 2013

MATTER MULTI ANGLE SOLUTION (0+1+3):

Stationary, spherically symmetric, evolving with radius

$$v_r \partial_r \rho(r,E,\theta) = -i\,[H(r,E,\theta), \rho(r,E,\theta)]$$

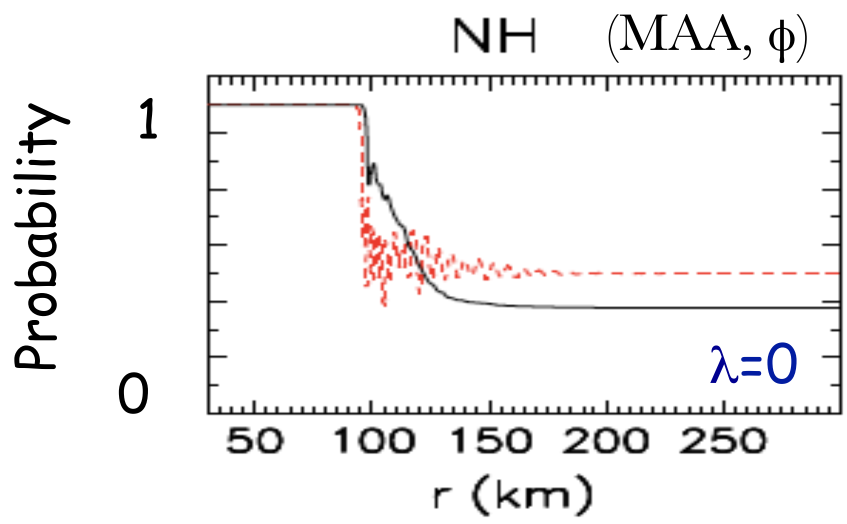
'θ' Zenith angle of nu momentum $\vec{p}$

'v_r' Radial velocity depends on θ, leads to multi-angle matter effect

Axial symmetry in velocity / momentum distribution

What if this symmetry is broken? Multi Azimuthal Angle, ϕ

MAA instability



S.C, Mirizzi, Saviano & Seixas PRD, 2014

Sarikas, Raffelt & Seixas PRL, 2013

MATTER MULTI ANGLE SOLUTION (0+1+3):

Stationary, spherically symmetric, evolving with radius

$$v_r \partial_r \rho(r, E, \theta) = -i\, [H(r, E, \theta), \rho(r, E, \theta)]$$

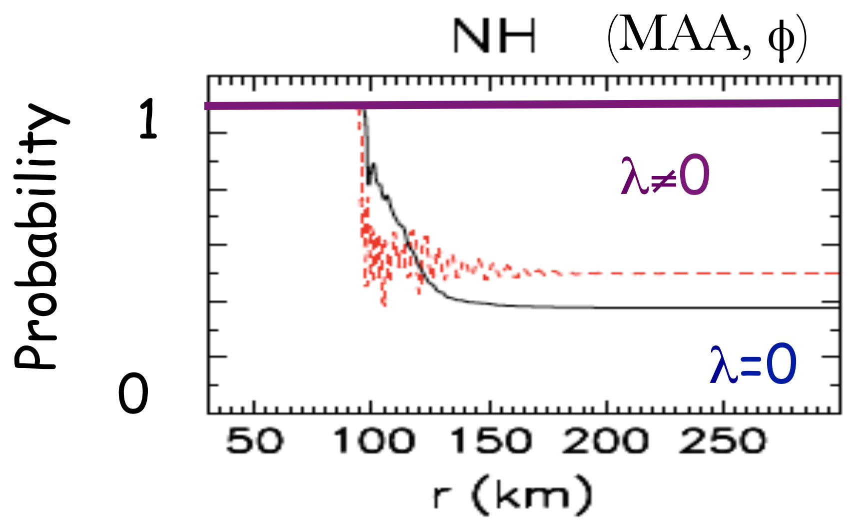
'θ' Zenith angle of nu momentum $\vec{p}$

'v_r' Radial velocity depends on θ, leads to multi-angle matter effect

Axial symmetry in velocity / momentum distribution

What if this symmetry is broken?
Multi Azimuthal Angle, ϕ

MAA instability



S.C, Mirizzi, Saviano & Seixas PRD, 2014

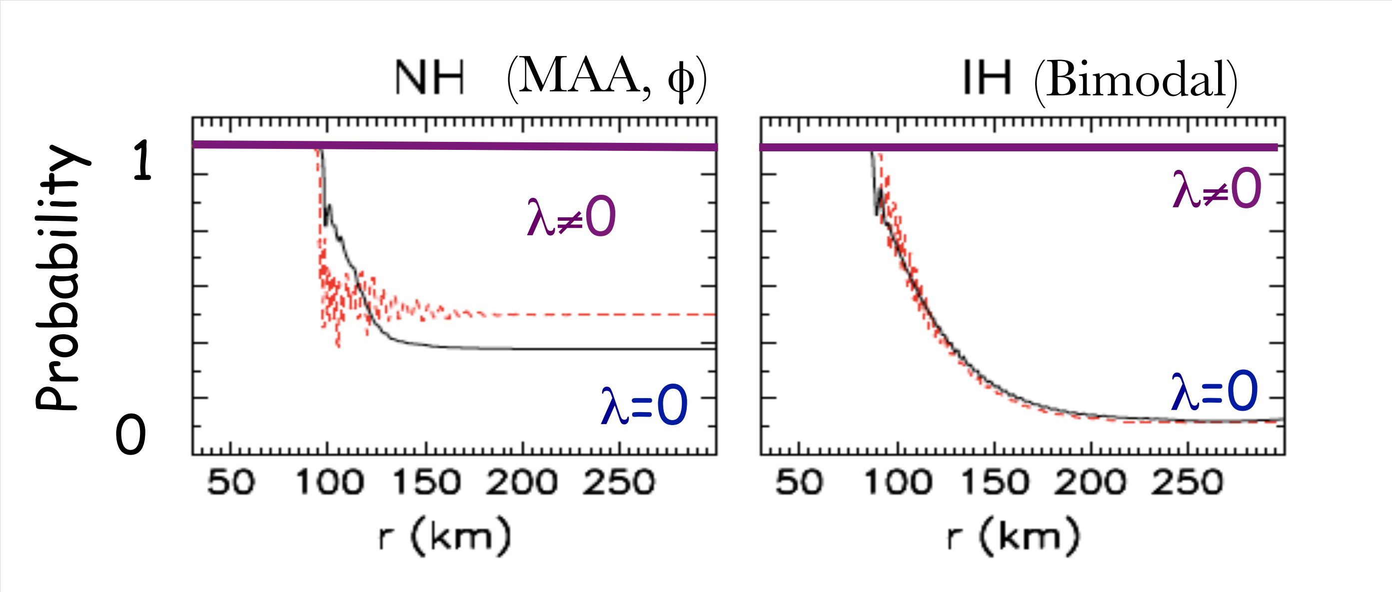
MATTER MULTI ANGLE SOLUTION (0+1+3):

Stationary, spherically symmetric, evolving with radius

$$v_r \partial_r \rho(r, E, \theta) = -i\,[H(r, E, \theta), \rho(r, E, \theta)]$$

'θ' Zenith angle of nu momentum $\vec{p}$

'v_r' Radial velocity depends on θ, leads to multi-angle matter effect



Early accretion phase: No Collective Oscillations

S.C, Mirizzi, Saviano & Seixas PRD, 2014

S.C, Fischer, Mirizzi, Saviano &Tomas PRL, 2011; PRD, 2011

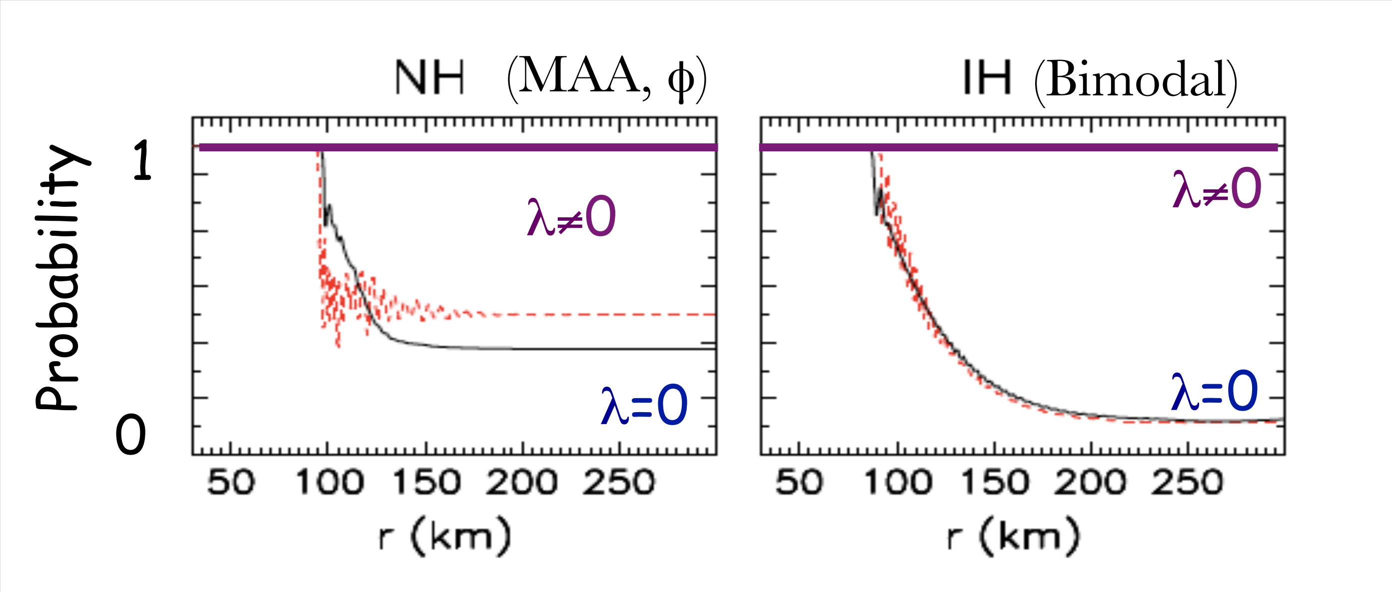
MATTER MULTI ANGLE SOLUTION (0+1+3):

Stationary, spherically symmetric, evolving with radius

$$v_r \partial_r \rho(r, E, \theta) = -i\,[H(r, E, \theta), \rho(r, E, \theta)]$$

'θ' Zenith angle of nu momentum $\vec{p}$

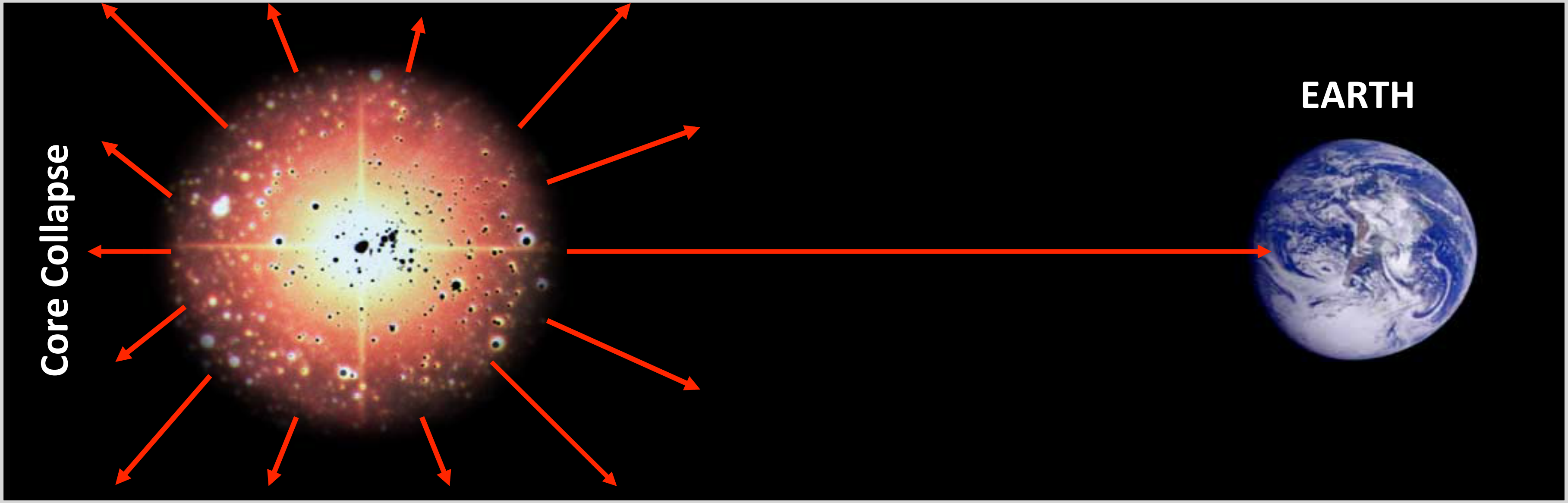
'v_r' Radial velocity depends on θ, leads to multi-angle matter effect



Early accretion phase: No Collective Oscillations

Ordinary differential equations with Maximal symmetries can Miss the dominant solutions

TYPICAL PROBLEMS IN SUPERNOVA NEUTRINOS



Supernova (SN) as Neutrino Source

SN Neutrino Oscillation: Initial Symmetries

Linear Stability Analysis: Symmetry Breaking

LINEARIZED STABILITY ANALYSIS

Neutrino transport and flavor oscillations with $\omega = \Delta m^2/2E$

$$v_r \partial_r \rho(r,E,\theta) = -i\,[H(r,E,\theta),\rho(r,E,\theta)]$$

$$\rho(r,\omega,u) = g_{\omega,u} \begin{pmatrix} s & S \\ S^* & -s \end{pmatrix}_{r,\omega,u} \leftarrow |S| \ll 1$$

Linearized equation of motion

$u = \sin^2(\theta)$

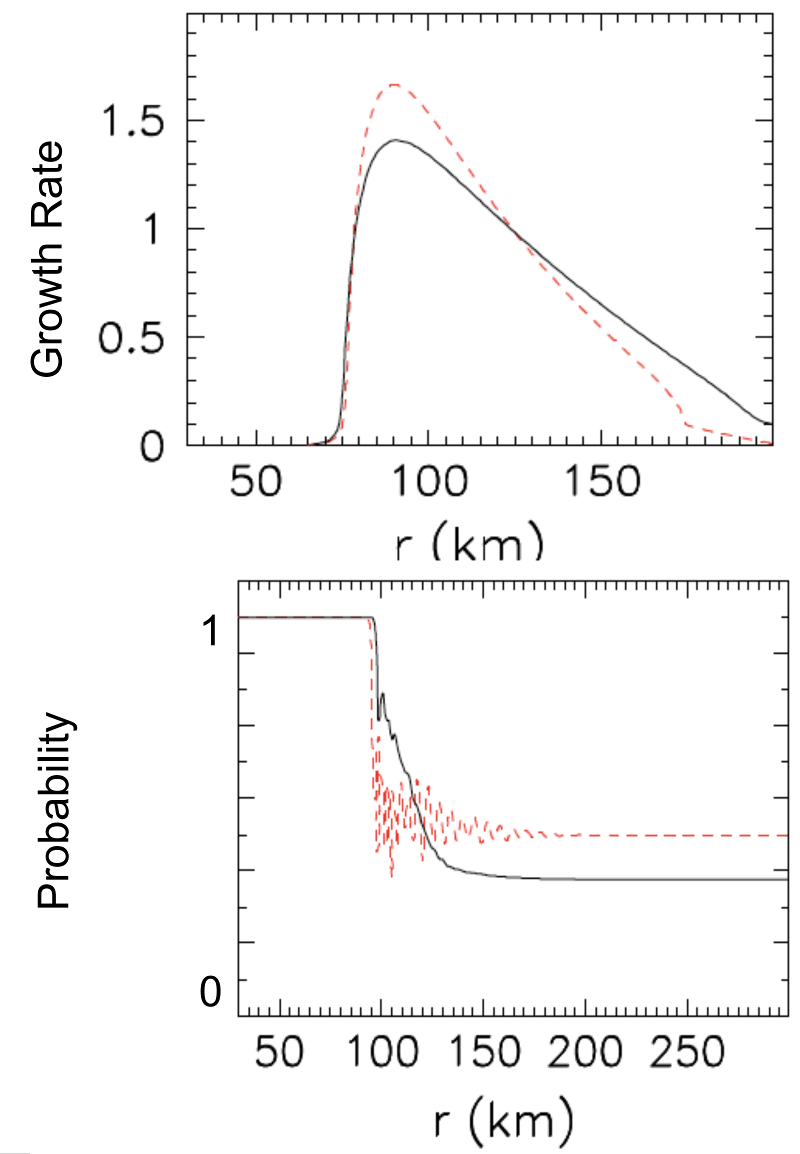
Banerjee, Dighe & Raffelt PRD, 2011

eigenmodes $S_{r,\omega,u} = Q_{\omega,u} \; e^{-i\Omega r}$

$$\left[\omega + u\left(\lambda + \int d\omega' du'\, g_{\omega',u'}\right) - \Omega\right] Q_{\omega,u} = \mu \int d\omega' du'\,(u+u') g_{\omega',u'} Q_{\omega',u'}$$

$\Omega = \gamma + \mathrm{i}\,\kappa$ solve for exponential growth rate κ

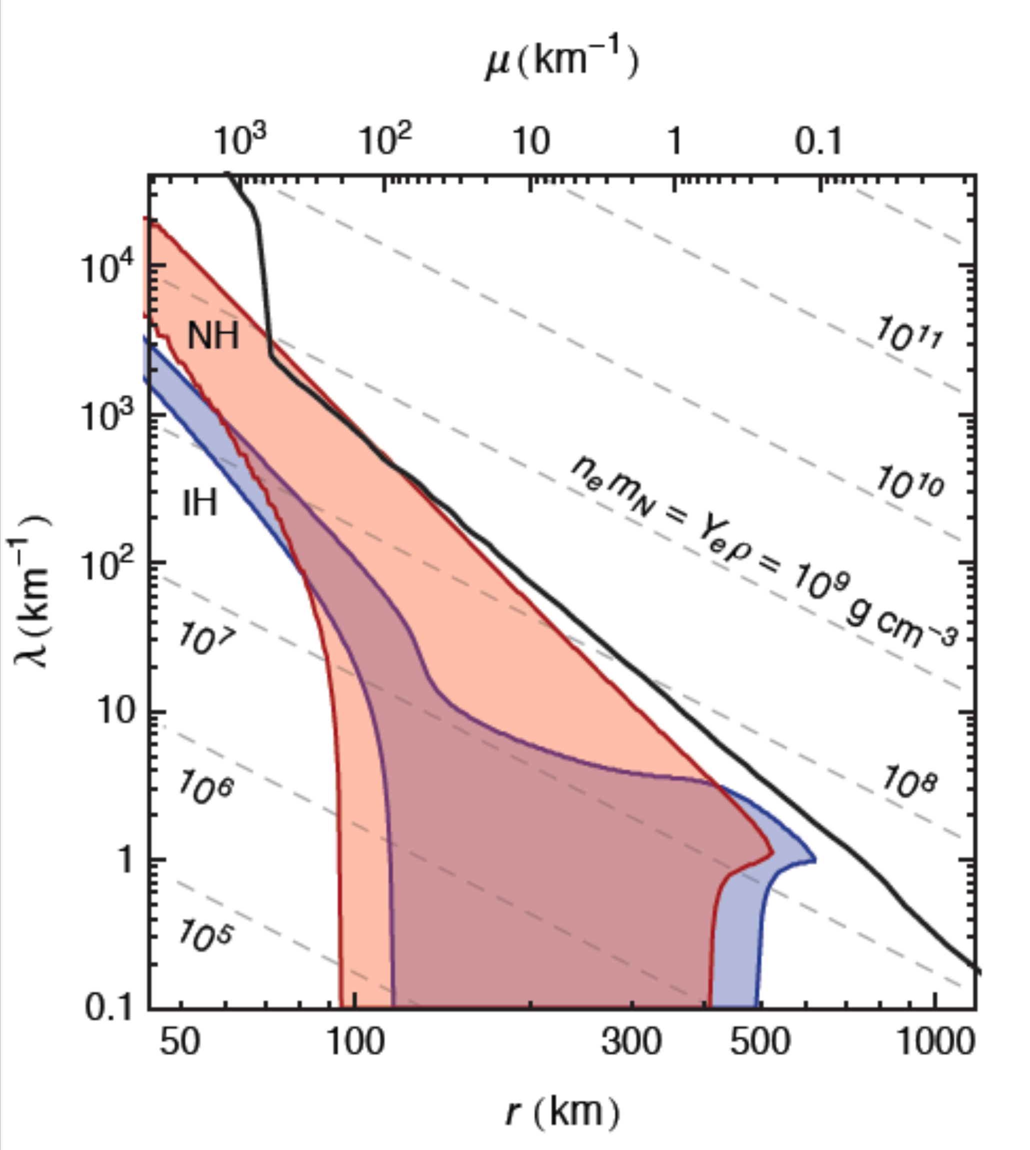
LINEARIZED STABILITY ANALYSIS (0+1+3)



Onset of the conversion:
Peak of the growth rate curve

S.C & Mirizzi, PRD, 2014

FOOT PRINT PLOT (0+1+3)



Contours of instability parameter 'k' in the (λ, μ) plane.
&
SN density profile at 280 ms

Axial-symmetry breaking (MAA) instability (normal ordering NH)

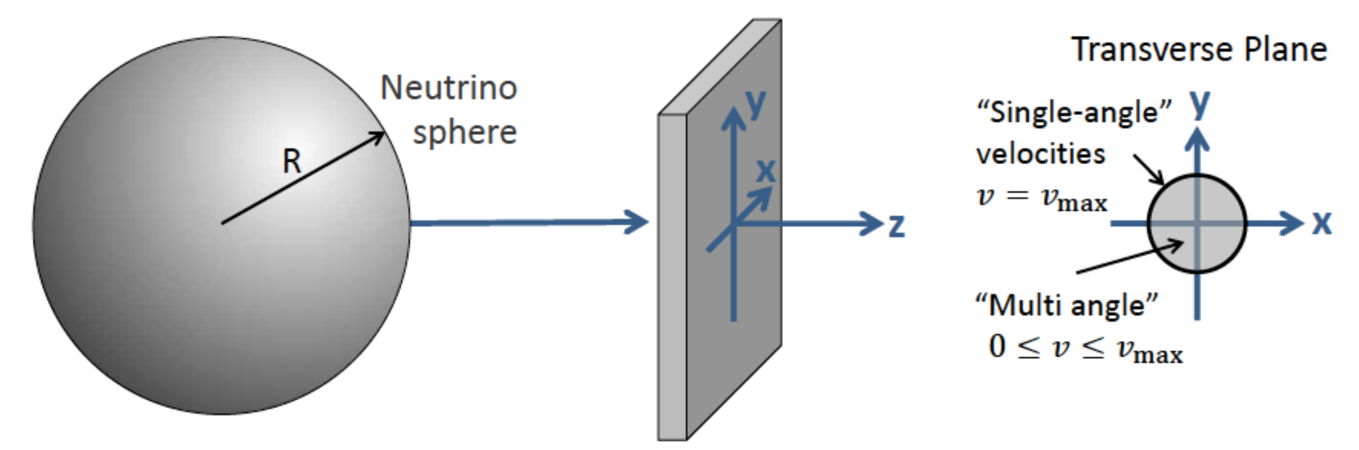
"bimodal" instability (inverted mass ordering IH)

Sarikas, Raffelt & Seixas PRL. 2013

SPATIAL SYMMETRY BREAKING (0+3+3)

Spatial symmetry breaking: Spatial Inhomogeneity

Colliding beam: stability analysis
Duan & Shalgar, PLB 2015
see also
Mirizzi, Mangano & Saviano, PRD 2015



Neutrino transport and flavor oscillations with $\omega = \Delta m^2/2E$

$$i(\partial_z + \mathbf{v}\cdot\nabla_{\mathbf{x}})\varrho(z,\mathbf{x},\omega,\mathbf{v}) = [\mathsf{H}(z,\mathbf{x},\omega,\mathbf{v}), \varrho(z,\mathbf{x},\omega,\mathbf{v})]$$

S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

SPATIAL SYMMETRY BREAKING(0+3+3)

$$\rho(z, x, \omega, v) = g(\omega, \vec{v}) \begin{pmatrix} s & S \\ S^* & -s \end{pmatrix}_{(z, x, \omega, v)} \leftarrow |S| \ll 1$$

Linearized equation of motion

$$i(\partial_z + \vec{v} \cdot \vec{\nabla}_x) S_{z,\mathbf{x},\omega,\mathbf{v}} = \left[\omega + \frac{\lambda + \epsilon\mu}{2} v^2\right] S_{z,\mathbf{x},\omega,\mathbf{v}} - \mu \int d\Gamma' \, g_{\omega',\vec{v}'} \frac{(\vec{v} - \vec{v}')^2}{2} S_{z,\mathbf{x},\omega',\mathbf{v}'}$$

Spatial Fourier transform $\vec{v} \cdot \vec{\nabla}_x \to i\vec{k} \cdot \vec{v}$

eigenmodes $S_{z,\mathbf{k},\omega,\mathbf{v}} = Q_{\Omega,\mathbf{k},\omega,\mathbf{v}} e^{-i\Omega z}$

$$\left[\frac{\lambda + \epsilon\mu}{2} v^2 + \vec{k} \cdot \vec{v} + \omega - \Omega\right] Q_{\Omega,\vec{k},\omega,\vec{v}} = \mu \int d\Gamma' \, g_{\omega',\vec{v}'} \frac{(\vec{v} - \vec{v}')^2}{2} Q_{\Omega,\vec{k},\omega',\vec{v}'}$$

LINEARIZED STABILITY ANALYSIS: INHOMOGENEITY IN TRANSVERSE PLANE

$$\left[\frac{\lambda+\epsilon\mu}{2}v^2+\vec{k}\cdot\vec{v}+\omega-\Omega\right]Q_{\Omega,\vec{k},\omega,\vec{v}}=\mu\int d\Gamma'\,g_{\omega',\vec{v}'}\frac{(\vec{v}-\vec{v}')^2}{2}Q_{\Omega,\vec{k},\omega',\vec{v}'}$$

nu-nu interaction energy

$$\sqrt{2}G_F n_\nu R^2/r^2$$

Matter effect

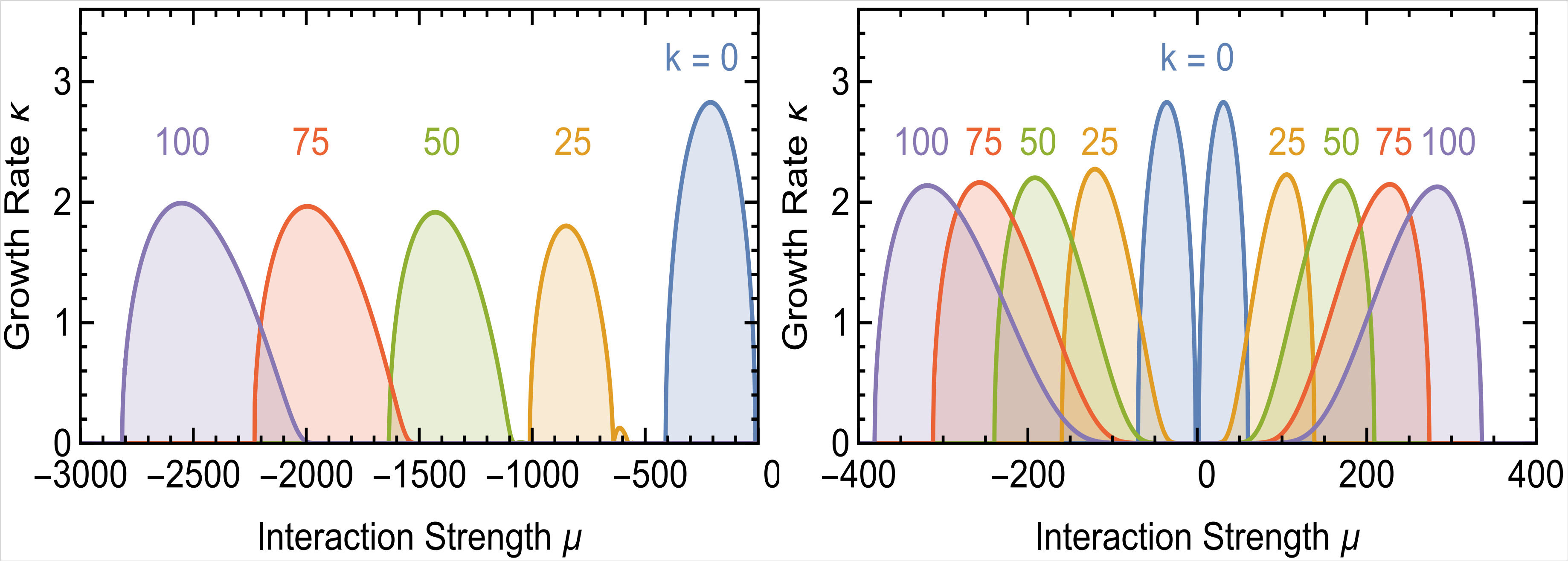
$$\sqrt{2}G_F n_e R^2/r^2$$

μ & λ defines the parameter space
Relative sign of μ, λ and ω defines the mass ordering

S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

SPATIAL SYMMETRY BREAKING

$$\left[\frac{\lambda+\epsilon\mu}{2}v^2+\vec{k}\cdot\vec{v}+\omega-\Omega\right]Q_{\Omega,\vec{k},\omega,\vec{v}}=\mu\int d\Gamma'\,g_{\omega',\vec{v}'}\frac{(\vec{v}-\vec{v}')^2}{2}Q_{\Omega,\vec{k},\omega',\vec{v}'}$$



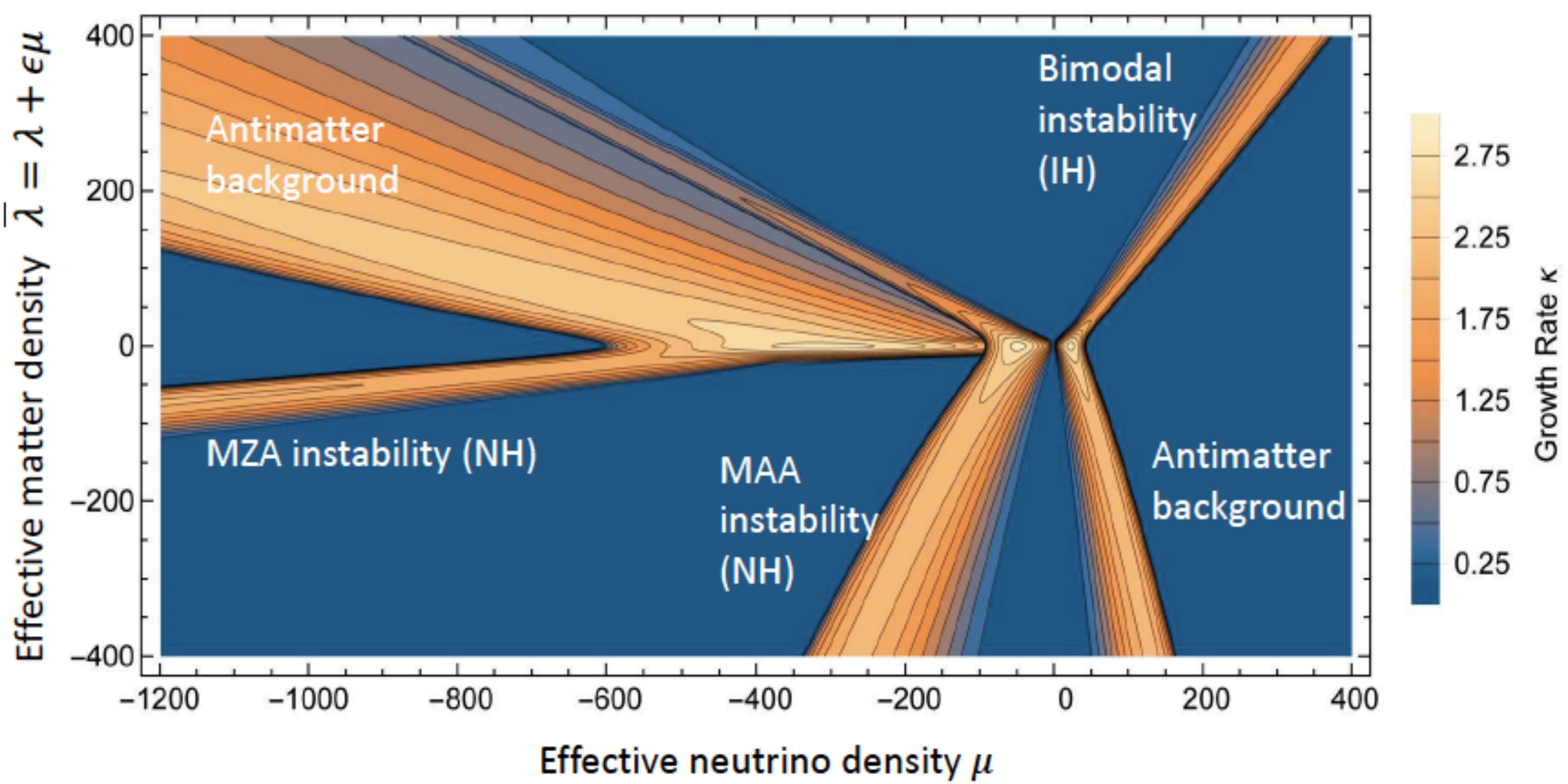
MZA

MAA (μ<0), Bimodal (μ>0)

S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

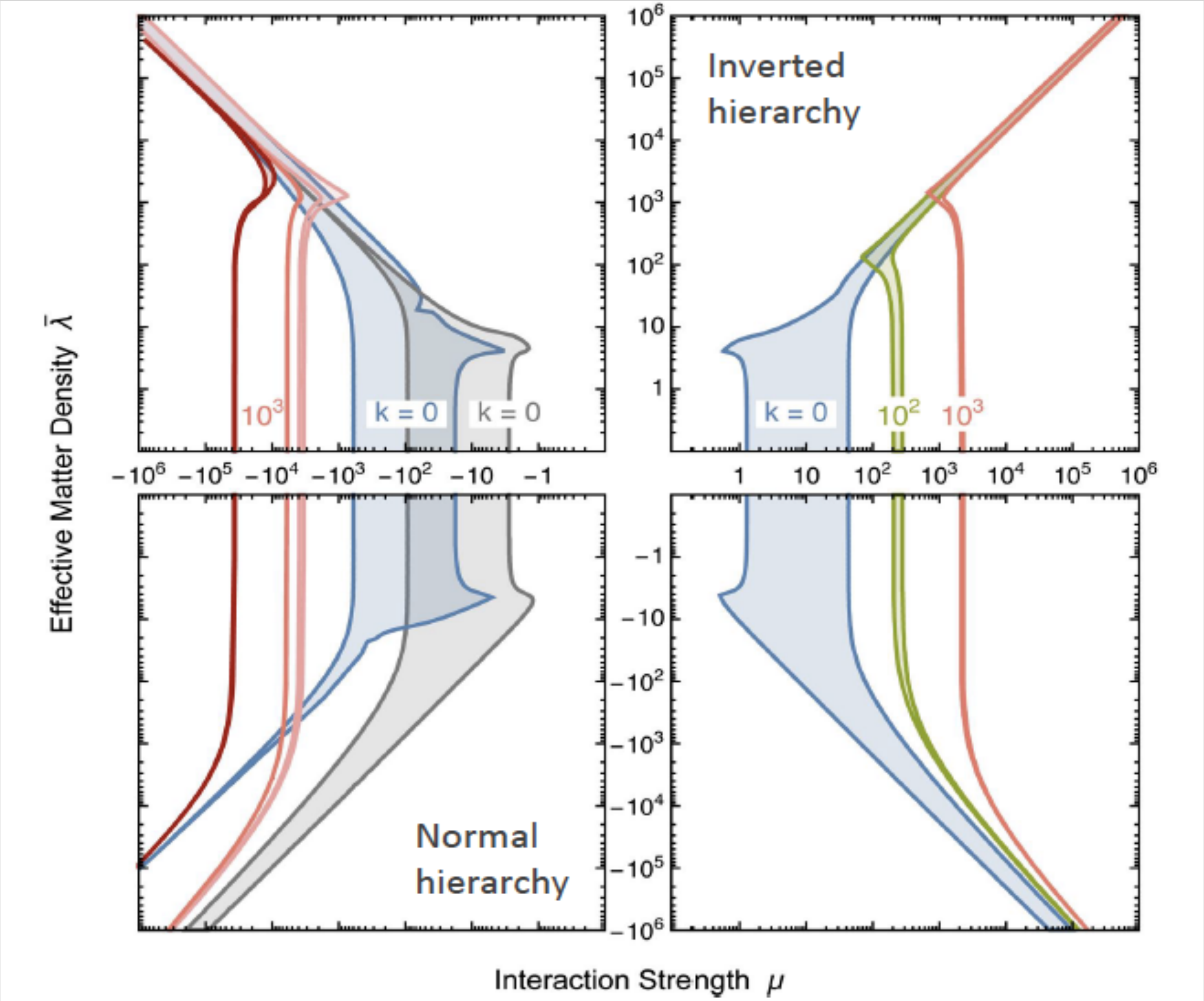
SPATIAL SYMMETRY BREAKING: BUTTERFLY DIAGRAM

Growth rate in 2D parameter space of effective matter and neutrino density
("Butterfly diagram")



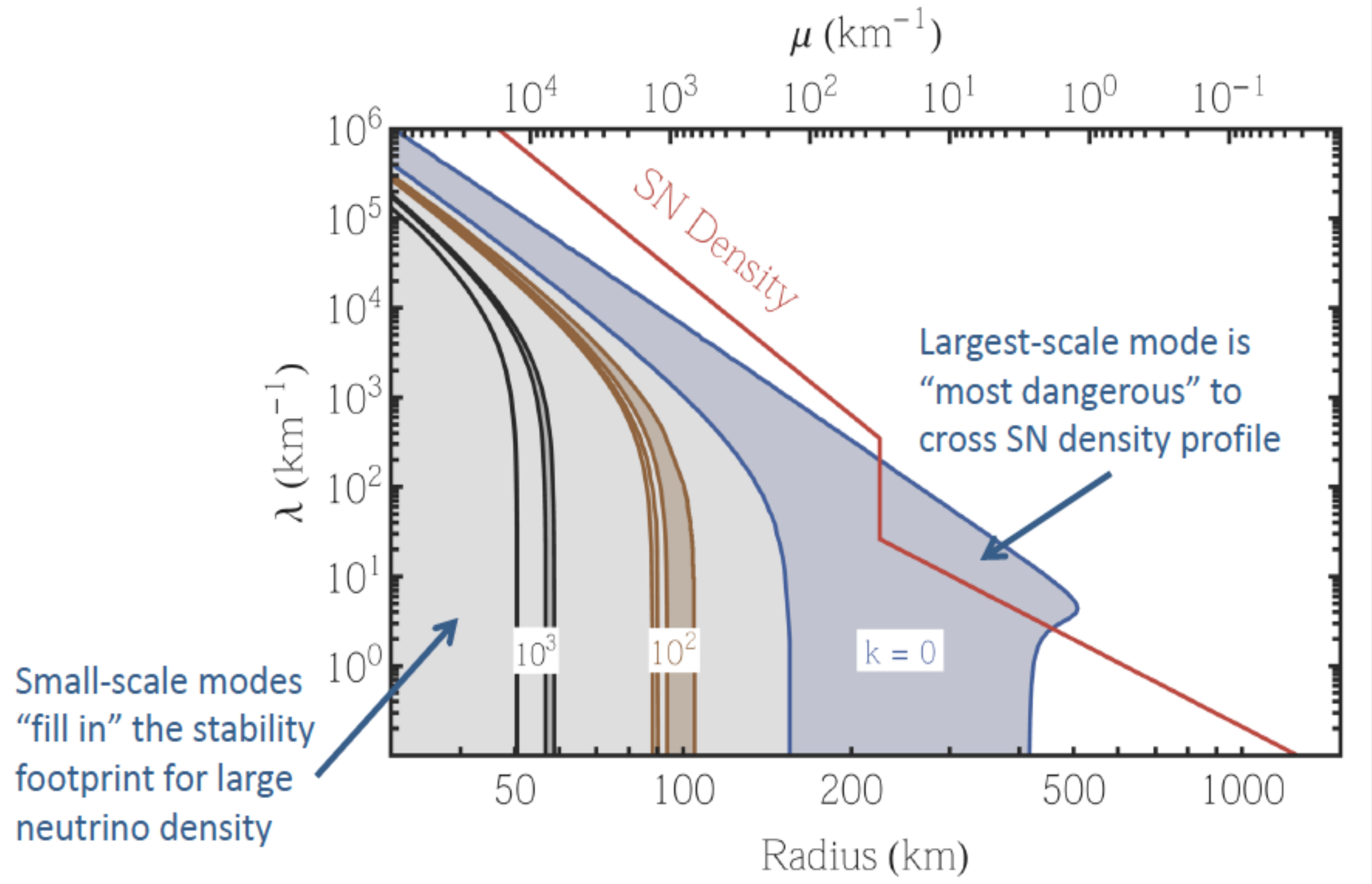
S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

LINEARIZED STABILITY ANALYSIS: INHOMOGENEITY IN TRANSVERSE PLANE



S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

LINEARIZED STABILITY ANALYSIS: INHOMOGENEITY IN TRANSVERSE PLANE (MAA, NO)



S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

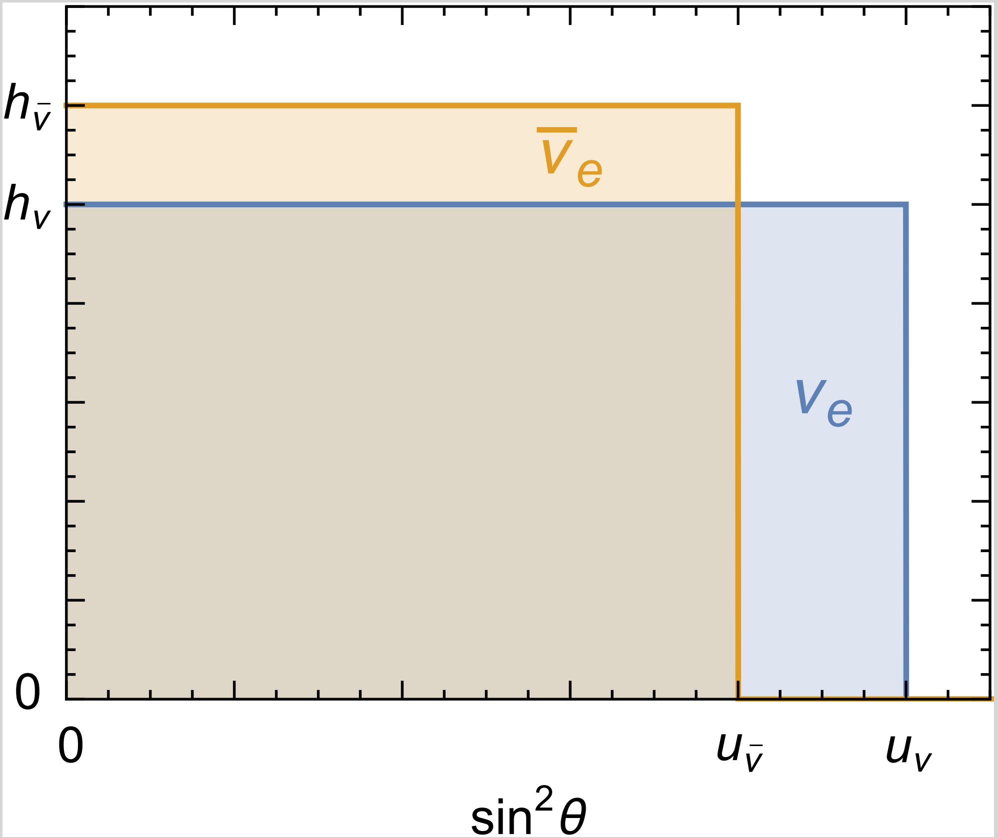
FAST INSTABILITY: ORDER μ GROWTH

- Unstable modes grow with rates of order μ instead of ω ($\mu \ll \omega$)
- This requires different angle distribution for different flavors.
- Thus the difference spectrum $g_{\omega,v}$ is flavor dependent

R. F. Sawyer, PRD 2005, PRL 2016

$$\left[\frac{\lambda + \epsilon\mu}{2} v^2 + \vec{k}\cdot\vec{v} + \omega - \Omega\right] Q_{\Omega,\vec{k},\omega,\vec{v}} = \mu \int d\Gamma' \, g_{\omega',\vec{v}'} \frac{(\vec{v} - \vec{v}')^2}{2} Q_{\Omega,\vec{k},\omega',\vec{v}'}$$

FAST INSTABILITY



$$h_{\nu_e}(u) = \int_0^\infty d\omega\, g(\omega, u)$$

$$h(u) = \frac{1 \pm a}{1 \pm b} \times \begin{cases} 1 & \text{for } 0 \le u \le 1 \pm b\,, \\ 0 & \text{otherwise}\,, \end{cases}$$

Uniform but different distribution
For neutrinos and antineutrinos,
width parameter (b)

$$-1 < b < +1$$

The distribution also connected
to the lepton asymmetry of the system,
asymmetry parameter (a)

$$-1 < a < +1$$

SN: *a > 0, b > 0*

S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

FAST INSTABILITY (0+1+3)

$$\left[\frac{\lambda+\epsilon\mu}{2}v^2+\vec{k}\cdot\vec{v}+\omega-\Omega\right]Q_{\Omega,\vec{k},\omega,\vec{v}}=\mu\int d\Gamma'\, g_{\omega',\vec{v}'}\frac{(\vec{v}-\vec{v}')^2}{2}Q_{\Omega,\vec{k},\omega',\vec{v}'}$$

- $k = 0,\ \omega = 0$
- Calculate growth rate (Imaginary part of Ω) in units of μ
- For both axially symmetric and broken cases

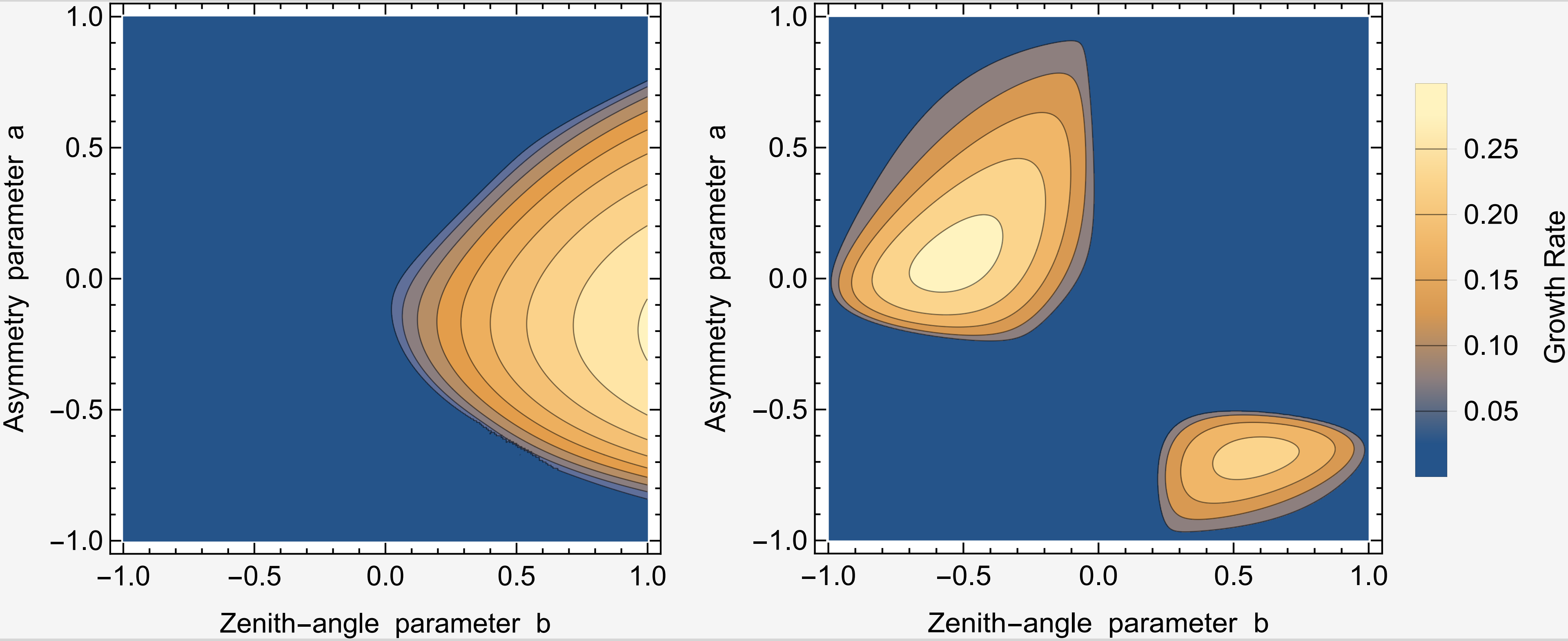
S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

FAST INSTABILITY (0+1+3)

SN: *a > 0, b > 0*

Axially symmetric solution **(0+1+2)**

Axial symmetry broken **(0+1+3)**



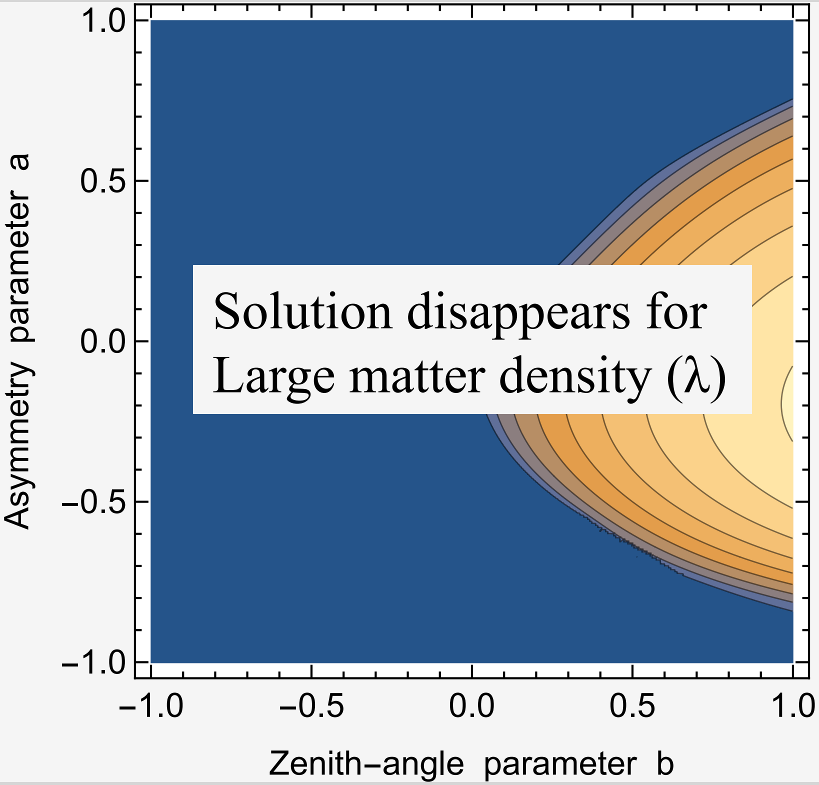
S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

FAST INSTABILITY (0+1+3)

SN: ***a > 0, b > 0***

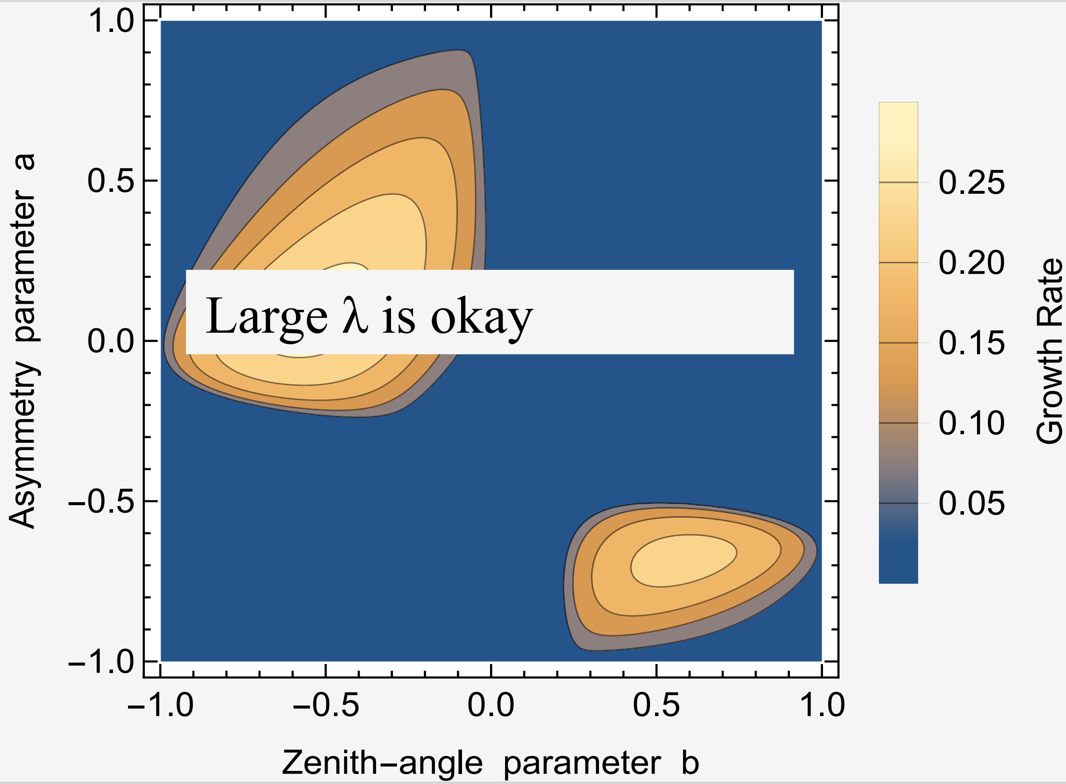
Axially symmetric solution

(0+1+2)



Axial symmetry broken

(0+1+3)



S.C, Hansen, Izaguirre & Raffelt, JCAP 2016

BREAKING OF STATIONARITY (1+3+3)

$$i(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{r}})\varrho = [\mathsf{H}, \varrho], \quad \varrho = \varrho(t, \mathbf{r}, E, \mathbf{v}),$$

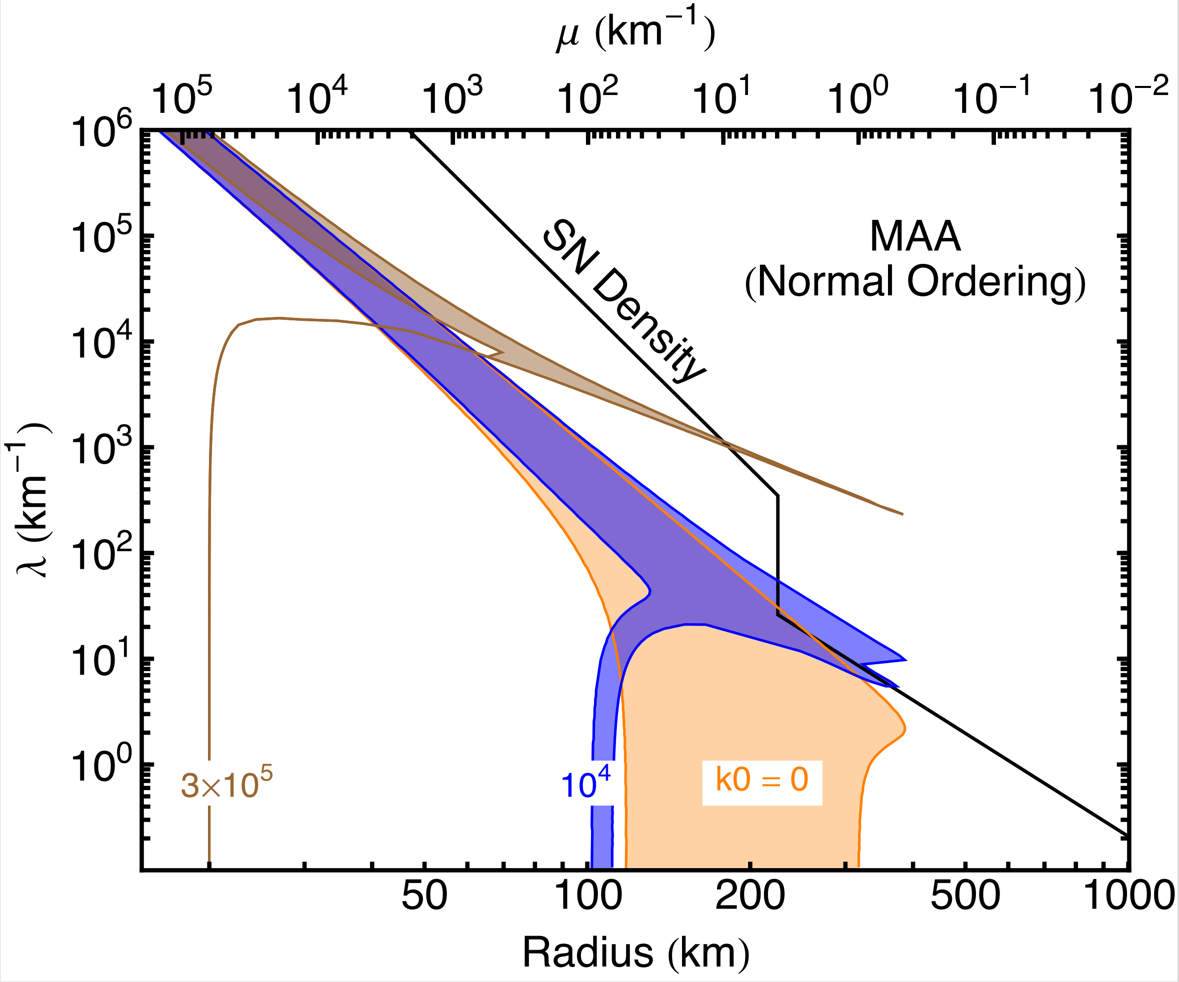
Abbar & Duan, PLB 2015
Dasgupta & Mirizzi, PRD 2015

$$\left(\bar{\lambda}_r \mathbf{v}^2 + k_0 \frac{R^2}{2r^2}\mathbf{v}^2 + \mathbf{k}\cdot\mathbf{v} + \omega - \Omega_r\right) Q_{\Omega,k_0,k,\omega,v} = \mu_r \int_{-\infty}^{+\infty} d\omega' \int d\mathbf{v}' \, (\mathbf{v}-\mathbf{v}')^2 \, Q_{\Omega,k_0,k,\omega',v'}$$

- k_0 from Fourier transform to the time part, i.e, frequency
- k_0 can be both +ve and -ve, thus can nullify matter effect

BREAKING OF STATIONARITY (1+1+3)

- For simplicity assume spatial homogeneity, $k = 0$, $K_0 \neq 0$



Cascading between different temporal modes would change the picture

However, that depends on the Duration of instability

Cappozi, Dasgupta & Mirizzi arXiv:1603.03288

S.C, Hansen, Izaguirre & Raffelt, PLB 2016

FUTURE OUTLOOK

SNe provide extreme conditions for neutrino oscillations, compareble only to,

- Early Universe
- Merging Compact objects

Scpecially neutrino evolution in stellar collapse is,

- Space-time dependent phenomenon (not stationary or homogeneous)
- Solutions do not respect initial symmetries (instabilities in all scales)

Thank you!

Extra Slides

PENDULUM IN FLAVOR SPACE

[*Hannestad, Raffelt, Sigl, Wong, astro-ph/0608695, Duan, Carlson, Fuller, Qian, astro-ph/0703776*]

Neutrino mass hierarchy (and θ_{13}) set initial condition and fate

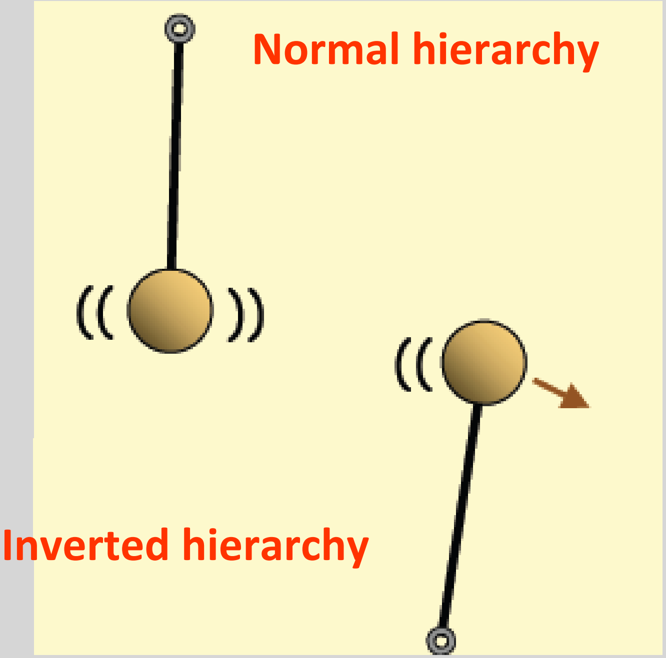
With only initial ν_e and $\bar{\nu}_e$:

- **Normal hierarchy**

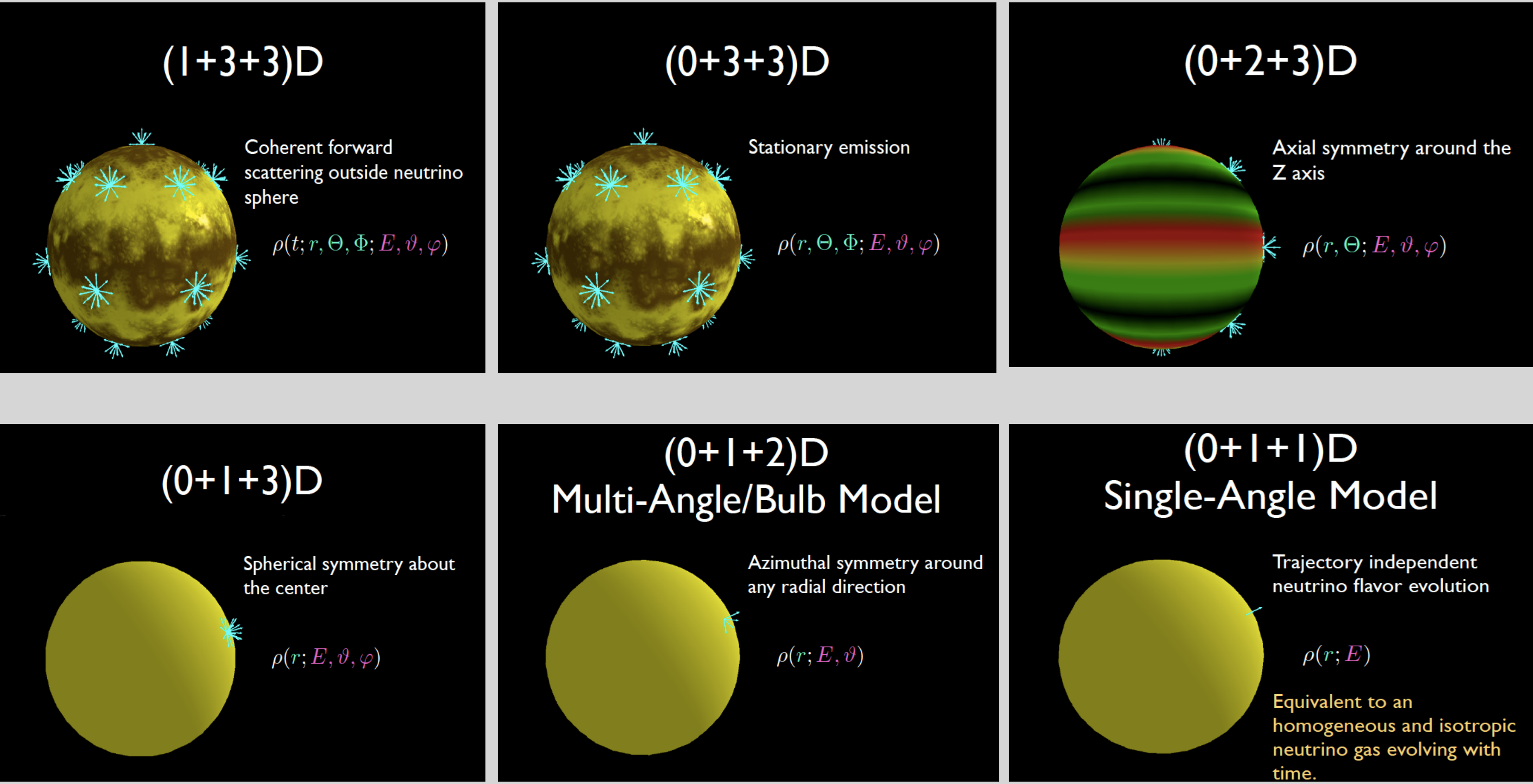
  Pendulum starts in ~ downard (stable) positions and stays nearby. No significant flavor change.

- **Inverted hierarchy**

  Pendulum starts in ~ upward (unstable) positions and eventually falls down. Significant flavor changes.



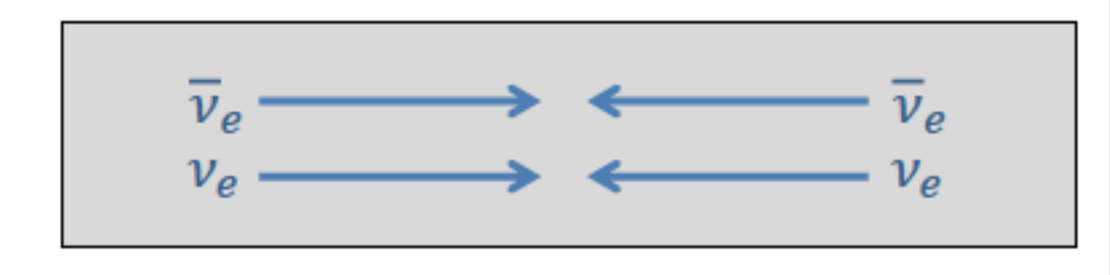
θ_{13} sets initial misalignment with vertical. Specific value not much relevant.

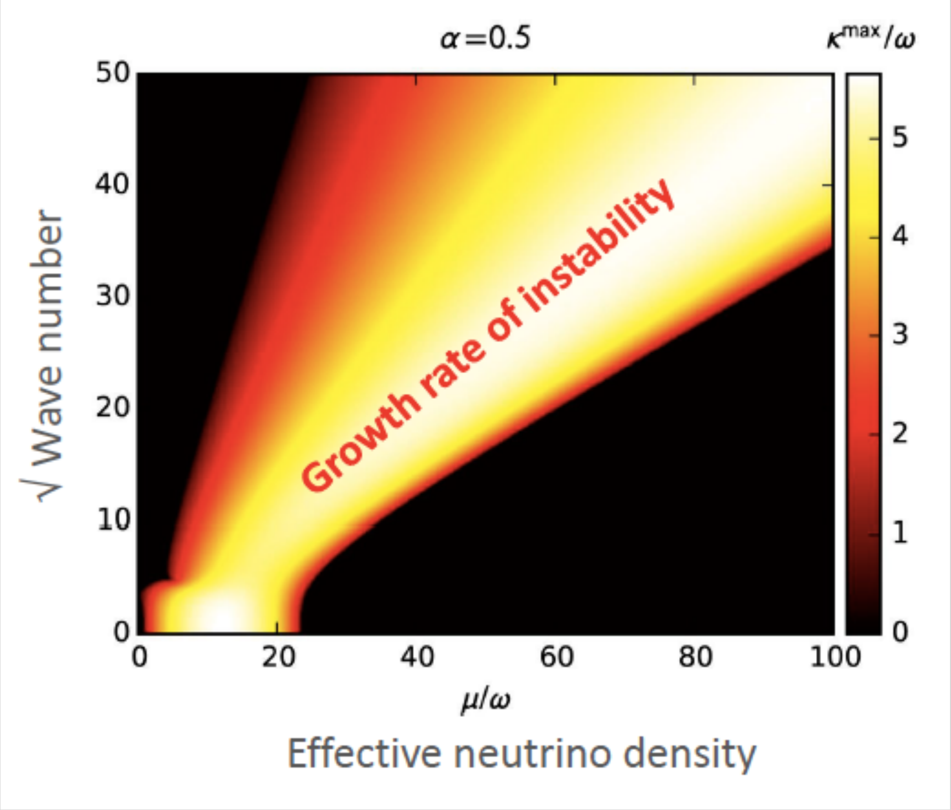


slide from H. Duan

Duan & Shalgar, PLB 2015
Mirizzi, Mangano & Saviano, PRD 2015

SPATIAL SYMMETRY BREAKING





Colliding beam: stability analysis

Duan & Shalgar, PLB 2015

see also

Mirizzi, Mangano & Saviano, PRD 2015